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HENCKS'S FIELD BOOK FOR ENGINEERS

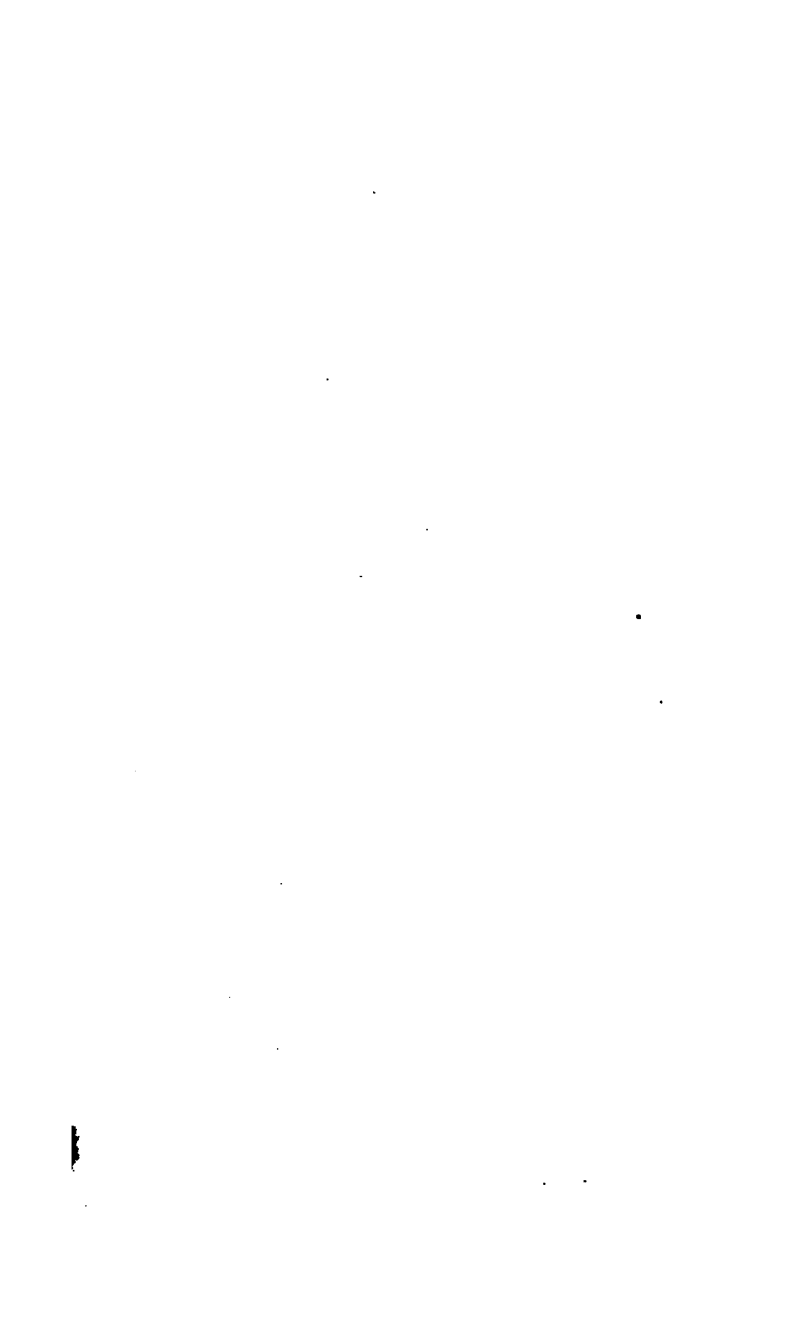
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Even then they apply only to ground which is level, though often used in a manner erroneous in § 128. When the cross-section is level, the place of the tables is supplied by § 119, and when several sections are calculated is usually the case, and the work is arranged in regular form, as in § 120, the calculation is at least as short as by the most extended tables. The method in excavation on curves (§ 129) is not yet been introduced elsewhere.

Of this kind, brevity is an essential feature. The "Problem" and "Solution" has, therefore, been as presenting most concisely the thing to be done, and the manner of doing it. Every solution, however, is accompanied with a demonstration, which is deemed an essential feature. These demonstrations, with a few notable exceptions, principally in Chapter II., presuppose knowledge of nothing beyond Algebra, Geometry, and Trigonometry. The result is in general expressed in a concise formula, and not in words. Those familiar with these symbols need not be told how much more easily and quickly apprehended a process becomes when it is expressed. Those not familiar with these symbols lose no time in acquiring the ready use of a direct and expressive. It may be remarked that no part of the author's design to furnish a collection of "rules," professing to require only an ability for their successful application. Rules can seldom be applied without a thorough understanding of the principles on which they rest, and such an understanding, in the present case, implies a knowledge of algebraic

As the rules presented will, it is hoped, prove rel



F I E L D - B O O K

FOR

RAILROAD ENGINEERS.

ble. Those specially prepared for this work have been computed with great care. The values have in some cases been carried out farther than ordinary practice requires, in order that interpolated values may be obtained from them more accurately. For the greater part of the material composing the Table of Magnetic Variation the author is indebted to Professor Bache, whose distinguished ability in conducting the operations of the Coast Survey is equalled only by his desire to diffuse its results. The remaining tables have been carefully examined by comparing them with others of approved reputation for accuracy. Many errors have in this way been detected in some of the tables of corresponding extent in general use, particularly in the Table of Squares, Cubes, &c., and the Tables of Logarithmic and Natural Sines, Cosines, &c. The number of tables might have been greatly increased, but for an unwillingness to insert any thing not falling strictly within the plan of the work or not resting on sufficient authority.

J. B. H.

Boston, February, 1854.

FIELD-BOOK

FOR

RAILROAD ENGINEERS.

CONTAINING

FORMULÆ

FOR LAYING OUT CURVES, DETERMINING FROG ANGLES, LEVELLING,
CALCULATING EARTH-WORK, ETC., ETC.,

TOGETHER WITH

TABLES

OF RADII, ORDINATES, DEFLECTIONS, LONG CHORDS, MAGNETIC VARIA-
TION, LOGARITHMS, LOGARITHMIC AND NATURAL SINES,
TANGENTS, ETC., ETC.

BY

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CIVIL ENGINEER.

STANFORD LIBRARY

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1874.

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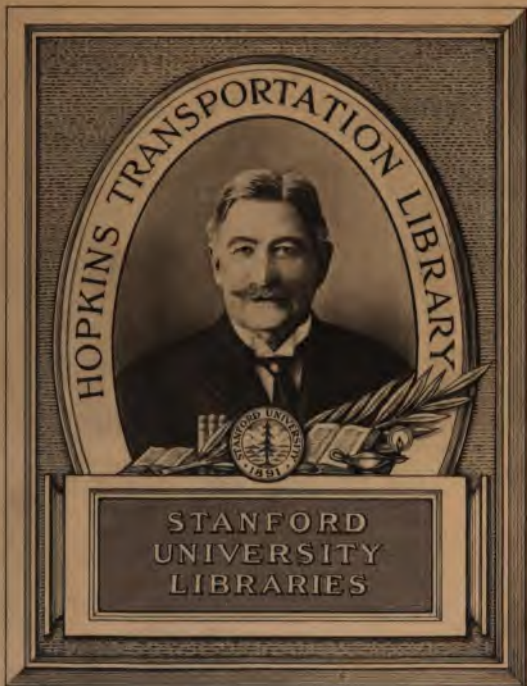
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GIFT OF

Dr. Charles David Marx

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EXPLANATION OF SIGNS.

THE sign $+$ indicates that the quantities between which it is placed are to be *added* together.

The sign $-$ indicates that the quantity before which it is placed is to be *subtracted*.

The sign $\times$ indicates that the quantities between which it is placed are to be *multiplied* together.

The sign $\div$ or $:$ indicates that the first of two quantities between which it is placed is to be *divided* by the second.

The sign $=$ indicates that the quantities between which it is placed are *equal*.

The sign $\propto$ indicates that the *difference* of the two quantities between which it is placed is to be taken.

The sign $\therefore$ stands for the word "hence" or "therefore."

The *ratio* of one quantity to another may be regarded as the quotient of the first *divided* by the second. Hence, the ratio of a to b is expressed by $a : b$, and the ratio of c to d by $c : d$. A *proportion* expresses the *equality* of two ratios. Hence, a proportion is represented by placing the sign $=$ between two ratios; as, $a : b = c : d$.

In the text and in the tables the foot has been taken as the unit of measure when no other unit is specified.

Fig. 1.

ing the straight lines GA and BH . The length of such a curve is measured by chords, each 100 feet long.* Thus, if the chords AD , DE , EF , and FB are each 100 feet in length, the whole curve is said to be 400 feet long. The straight lines GA and BH are always tangent to the curve at its extremities, which are called *tangent points*. If GA and BH are produced, until they meet in C , AC and BC are called the *tangents* of the curve. If AC is produced a little beyond C to K , the angle KCB , formed by one tangent with the other produced, is called the *angle of intersection*, and shows the *change of direction* in passing from one tangent to the other.

The following propositions relating to the circle are derived from Geometry.

I. A tangent to a circle is perpendicular to the radius drawn through the tangent point. Thus, AC is perpendicular to AO , and BC to BO .

II. Two tangents drawn to a circle from any point are equal, and if a chord be drawn between the two tangent points, the angles between this chord and the tangents are equal. Thus $AC = BC$, and the angle $BAC = ABC$.

III. An acute angle between a tangent and a chord is equal to half the central angle subtended by the same chord. Thus, $CAB = \frac{1}{2} AOB$.

IV. An acute angle subtended by a chord, and having its vertex in the circumference of a circle, is equal to half the central angle subtended by the same chord. Thus, $DAE = \frac{1}{2} DOE$.

V. Equal chords subtend equal angles at the centre of a circle, and also at the circumference, if the angles are inscribed in similar segments. Thus, $AOD = DOE$, and $DAE = EAF$.

VI. The angle of intersection of two tangents is equal to the central angle subtended by the chord which unites the tangent points. Thus, $KCB = AOB$.

3. In order to unite two straight lines, as GA and BH , by a curve, the angle of intersection is measured, and then a radius for the curve may be assumed, and the tangents calculated, or the tangents may be assumed of a certain length, and the radius calculated.

* Some engineers prefer a chain 50 feet in length, and measure the length of a curve by chords of 50 instead of 100 feet. The chord of 100 feet has been adopted throughout this article; but the formulæ deduced may be very readily modified to chords of any length. See also § 13.

Solution. In the right triangle AOC we have (Tab. X. 6)
 $\frac{AO}{AC} = \cot. AOC$, or $\frac{R}{T} = \cot. \frac{1}{2}I$;

$$\therefore R = T \cot. \frac{1}{2}I.$$

Example. Given $I = 31^\circ 16'$ and $T = 950$, to find R . Here

$T = 950$	2.977724
$\frac{1}{2}I = 15^\circ 38'$	cot. 0.553102
$R = 3394.89$	3.530826

6. The *degree* of a curve is determined by the angle subtended at its centre by a chord of 100 feet. Thus, if $AOD = 6^\circ$ (fig. 1), $ADEFB$ is a 6° curve.

7. The *deflection angle* of a curve is the acute angle formed at any point between a tangent and a chord of 100 feet. The deflection angle is, therefore (§ 2, III.), half the degree of the curve. Thus, CAD or CBF is the deflection angle of the curve $ADEFB$, and is half AOD or half FOB .

A. Method by Deflection Angles.

8. The usual method of laying out a curve on the ground is by means of deflection angles.

9. **Problem.** Given the radius $AO = R$ (fig. 1), to find the deflection angle $CBF = D$.

Solution. Draw OL perpendicular to BF . Then the angle $BO L = \frac{1}{2}BOF = D$, and $BL = \frac{1}{2}BF = 50$. But in the right triangle $OB L$ we have (Tab. X. 1) $\sin. BO L = \frac{BL}{BO}$;

$$\sin. D = \frac{50}{R}.$$

Example. Given $R = 5729.65$, to find D . Here

50	1.698970
$R = 5729.65$	3.758128
$D = 30'$	sin. 7.940842

Hence a curve of this radius is a 1° curve, and its deflection angle is $30'$.

10. **Problem.** Given the deflection angle $CBF = D$ (fig. 1), to find the radius $AO = R$.

Solution. By the preceding section we have $\sin. D = \frac{50}{R}$, whence $R \sin. D = 50$;

$$\therefore R = \frac{50}{\sin. D}.$$

By this formula the radii in Table I. are calculated.

Example. Given $D = 1^\circ$, to find R . Here

50	1.698970
$D = 1^\circ$	sin. 8 241855
$R = 2864.93$	3.457115

11. Problem. Given the angle of intersection $KCB = I$ (fig. 1), and the tangent $AC = T$, to find the deflection angle $CAD = D$.

Solution. From § 9 we have $\sin. D = \frac{50}{R}$, and from § 5, $R = T \cot. \frac{1}{2} I$. Substituting this value of R in the first equation, we get $\sin. D = \frac{50}{T \cot. \frac{1}{2} I}$;

$$\therefore \sin. D = \frac{50 \tan. \frac{1}{2} I}{T}.$$

Example. Given $I = 21^\circ$ and $T = 424.8$, to find D . Here

50	1.698970
$\frac{1}{2} I = 10^\circ 30'$	tan. 9.267967
	0.966937
$T = 424.8$	2.628185
$D = 1^\circ 15'$	sin. 8.338752

12. Problem. Given the angle of intersection $KCB = I$ (fig. 1) and the deflection angle $CAD = D$, to find the tangent $AC = T$.

Solution. From the preceding section we have $\sin. D = \frac{50 \tan. \frac{1}{2} I}{T}$.

Hence, $T \sin. D = 50 \tan. \frac{1}{2} I$;

$$\therefore T = \frac{50 \tan. \frac{1}{2} I}{\sin. D}.$$

Example. Given $I = 28^\circ$ and $D = 1^\circ$, to find T . Here

$$T = \frac{50 \tan. 14^\circ}{\sin. 1^\circ} = 714.31.$$

13. **Problem.** Given the angle of intersection $KCB = I$ (fig. 1), and the deflection angle $CAD = D$, to find the length of the curve.

Solution. By § 2 the length of a curve is measured by chords of 100 feet applied around the curve. Now the first chord AD makes with the tangent AC an angle $CAD = D$, and each succeeding chord DE , EF , &c. subtends at A an additional angle DAE , EAF , &c. each equal to D ; since each of these angles (§ 2, IV.) is half of a central angle subtended by a chord of 100 feet. The angle $CAB = \frac{1}{2} AOB = \frac{1}{2} I$ is, therefore, made up of as many times D , as there are chords around the curve. Then if n represents the number of chords, we have $nD = \frac{1}{2} I$;



$$\therefore n = \frac{\frac{1}{2} I}{D}.$$

If D is not contained an even number of times in $\frac{1}{2} I$, the quotient above will still give the length of the curve. Thus, in fig. 2, suppose D is contained $4\frac{3}{8}$ times in $\frac{1}{2} I$. This shows that there will be four whole chords and $\frac{3}{8}$ of a chord around the curve from A to B . The angle GAB , the fraction of D , is called a *sub-deflection angle*, and GB , the fraction of a chord, is called a *sub-chord*.*

The length of the curve thus found is not the actual length of the arc, but the length required in locating a curve. If the actual length of the arc is required, it may be found by means of Table VI.

Example. Given $I = 16^{\circ} 52'$ and $D = 1^{\circ} 20'$, to find the length of the curve. Here $n = \frac{\frac{1}{2} I}{D} = \frac{8^{\circ} 26'}{1^{\circ} 20'} = \frac{506'}{80'} = 6.325$, that is, the curve is 632.5 feet long.

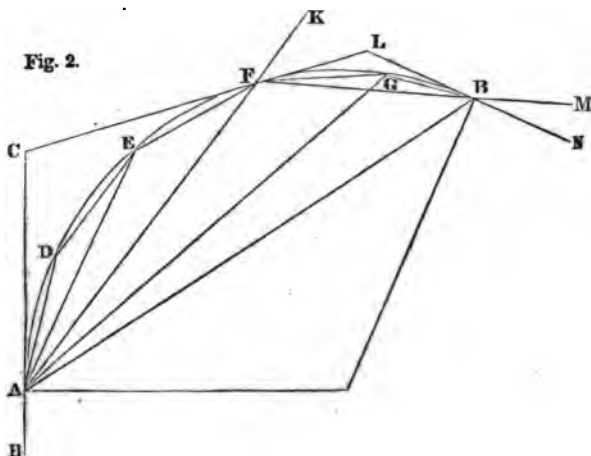
To find the arc itself in this example, we take from Table VI. the length of an arc of $16^{\circ} 52'$, since the central angle of the whole curve is equal to I (§ 2, VI), and multiply this length by the radius of the curve.

Arc 10°	= .1745329
" 6°	= .1047198
" $50'$	= .0145444
" $2'$	= .0005818
" $16^{\circ} 52'$	= .2943789

* This method of finding the length of a sub-chord is not mathematically accurate; for, by geometry, angles inscribed in a circle are proportional to the arcs on which they stand; whereas this method supposes them to be proportional to the sines of these arcs. In railroad curves, the error arising from this supposition is small to be regarded.

The radius of the curve is found from Table I. to be 2148.79, and this multiplied by .2943789 gives 632.558 feet for the length of the arc.

14. Problem. *Given the deflection angle D , to lay out a curve from a given tangent point.*



Solution. Let A (fig. 2) be the given tangent point in the tangent HC . Set the instrument at A , and lay off the given deflection angle D from AC . This will give the direction AD , and 100 feet being measured from A in this direction, the point D will be determined. Lay off in succession the additional angles DAE , EAF , &c., each equal to D , and make DE , EF , &c. each 100 feet, and the points E , F , &c. will be determined. The points D , E , F , &c., thus determined, are points on the required curve (§ 7, and § 2, III., IV.), and are called *stations*.

If there is a sub-chord at the end, as GB , the sub-deflection angle GAB must be the same part of D that GB is of a whole chord (§ 13).

15. It is often impossible to lay out the whole of a curve, without removing the instrument from its first position, either on account of the great length of the curve, or because some obstruction to the sight may be met with. In this case, after determining as many stations as possible, and removing the instrument to the last of these stations, we are able to find the tangent to the curve at this station; for

A line drawn through B and M will be the tangent required; for it has been proved (§ 18) that the angle CBM is in this case equal to $\frac{1}{2} BOC$, and BM is consequently (§ 2, III.) a tangent at B .

If B is at the end of the curve, the tangent at B may be found without first laying off HC . Thus, if a chain equal to the chord is extended to H on AB produced, the point H marked, and the chain then swung round, keeping the end at B fixed, until $HM = \frac{1}{2}d$, BM will be the direction of the required tangent.*

22. Problem. Given the chord deflection d , to lay out a curve from a given tangent point.

Solution. Let A (fig. 3) be the given tangent point, and suppose d has been calculated for a chord of 100 feet. Stretch a chain of 100 feet from A to G on the tangent EA produced, and mark the point G . Swing the chain round towards AB , keeping the end at A fixed, until BG is equal to the tangent deflection $\frac{1}{2}d$, and B will be the first station on the curve. Stretch the chain from B to H on AB produced, and having marked this point, swing the chain round, until HC is equal to the chord deflection d . C is the second station on the curve. Continue to lay off the chord deflection from the preceding chord produced, until the curve is finished.

Should a sub-chord DF occur at the end of the curve, find the tangent DL at D (§ 21), lay off from it the proper tangent deflection LF for the given sub-chord, making DF of the given length, and F will be a point on the curve. The proper tangent deflection for the sub-chord may be found thus. Represent the sub-chord by c' , and the corresponding chord deflection by d' , and we have (§ 18) $\frac{1}{2}d' = \frac{c'^2}{2R}$; but since $\frac{1}{2}d = \frac{c^2}{2R}$, we have $\frac{1}{2}d' : \frac{1}{2}d = c'^2 : c^2$. Therefore $\frac{1}{2}d' = \frac{1}{2}d \left(\frac{c'}{c}\right)^2$.

Example. Given the intersection angle I between two tangents equal to $16^\circ 30'$, and $R = 1250$, to find T , d , and the length of the curve in stations. Here

$$(\S 4) \quad T = R \tan. \frac{1}{2} I = 1250 \tan. 8^\circ 15' = 181.24;$$

$$(\S 18) \quad d = \frac{c^2}{R} = \frac{100^2}{1250} = 8,$$

* The distance BM is not exactly equal to the chord, but the error arising from making it equal is too small to be regarded in any curves but those of very small radius. If necessary, the true length of BM may be calculated; for $BM = HM$.

$$(\S 9) \sin. D = \frac{50}{R} = \frac{50}{1250} = .04 = \text{nat. sin. } 2^\circ 17\frac{1}{2}';$$

$$(\S 13) n = \frac{\frac{1}{2} I}{D} = \frac{8^\circ 15'}{2^\circ 17\frac{1}{2}'} = \frac{495'}{137.5'} = 3.60.$$

These results show, that the tangent point *A* (fig. 3) on the first tangent is 131.24 feet from the point of intersection, — that the tangent deflection $GB = \frac{1}{2} d = 4$ feet, — that the chord deflection *HC* or *KD* = 8 feet, — and that the curve is 360 feet long. The three whole stations *B*, *C*, and *D* having been found, and the tangent *DL* drawn, the tangent deflection for the sub-chord of 60 feet will be, as shown above, $\frac{1}{2} d' = 4 \left(\frac{60}{100} \right)^2 = 4 \times .6^2 = 4 \times .36 = 1.44$. *LF* = 1.44 feet being

laid off from *DL*, the point *F* will, if the work is correct, fall upon the second tangent point. A tangent at *F* may be found (§ 21) by producing *DF* to *P*, making *FP* = *DF* = 60 feet, and laying off *PN* = 1.44 feet. *FN* will be the direction of the required tangent, which should, of course, coincide with the given tangent.

23. Curves may be laid out with accuracy by tangent and chord deflections, if an instrument is used in producing the lines. But if an instrument is not at hand, and accuracy is not important, the lines may be produced by the eye alone. The radius of a curve to unite two given straight lines may also be found without an instrument by § 73, or, having assumed a radius, the tangent points may be found by § 74.

C. Ordinates.

24. The preceding methods of laying out curves determine points 100 feet distant from each other. These points are usually sufficient for grading a road; but when the track is laid, it is desirable to have intermediate points on the curve accurately determined. For this purpose the chord of 100 feet is divided into a certain number of equal parts, and the perpendicular distances from the points of division to the curve are calculated. These distances are called *ordinates*. If the chord is divided into eight equal parts, we shall have points on the curve at every 12.5 feet, and this will be often enough, if the rails, which are seldom shorter than 15 feet, have been properly curved (§ 28).

25. Problem. Given the deflection angle *D* or the radius *R* of a curve, to find the ordinates for any chord.

Solution. I. To find the middle ordinate. Let *AEB* (fig. 4) be a portion of a curve, subtended by a chord *AB*, which may be divided

noted by c . Draw the middle ordinate ED , and denote it by m . Produce ED to the centre F , and join AF and AE . Then (Tab. X. 3)

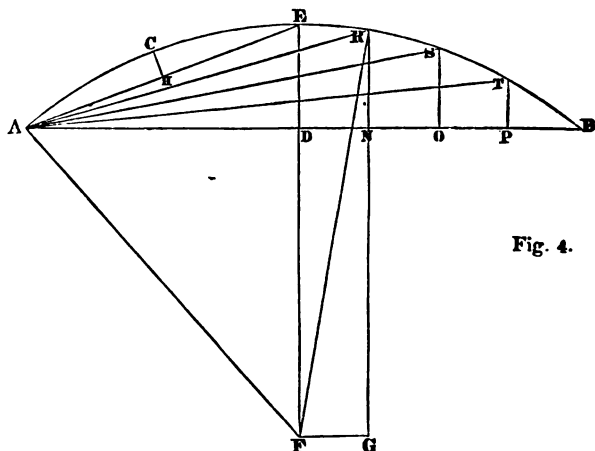


Fig. 4.

$\frac{ED}{AD} = \tan. EAD$, or $ED = AD \tan. EAD$. But, since the angle EAD is measured by half the arc BE , or by half the equal arc AE , we have $EAD = \frac{1}{2} AFE$. Therefore $ED = AD \tan. \frac{1}{2} AFE$, or

$$m = \frac{1}{2} c \tan. \frac{1}{2} AFE.$$

When $c = 100$, $AFE = D$ (§ 7), and $m = 50 \tan. \frac{1}{2} D$, whence m may be obtained from the table of natural tangents, by dividing $\tan. \frac{1}{2} D$ by 2, and removing the decimal point two places to the right.

The value of m may be obtained in another form thus. In the triangle ADF we have $DF = \sqrt{AF^2 - AD^2} = \sqrt{R^2 - \frac{1}{4}c^2}$. Then $m = EF - DF = R - DF$, or

$$m = R - \sqrt{R^2 - \frac{1}{4}c^2}.$$

II. To find any other ordinate, as RN , at a distance $DN = b$ from the centre of the chord. Produce RN until it meets the diameter parallel to AB in G , and join RF . Then $RG = \sqrt{RF^2 - FG^2} = \sqrt{R^2 - b^2}$, and $RN = RG - NG = RG - DF$. Substituting the value of RG and that of DF found above, we have

$$RN = \sqrt{R^2 - b^2} - \sqrt{R^2 - \frac{1}{4}c^2}.$$

By these formulæ the ordinates in Table I. are calculated.

The other ordinates may also be found from the middle ordinate by the following shorter, but not strictly exact method. It is founded on the supposition, that, if the half-chord BD be divided into any number of equal parts, the ordinates at these points will divide the arc EB into the same number of equal parts, and upon the further supposition, that the tangents of small angles are proportional to the angles themselves. These suppositions give rise to no material error in finding the ordinates of railroad curves for chords not exceeding 100 feet. Making, for example, four divisions of the chord on each side of the centre, and joining AR , AS , and AT , we have the angle $RAN = \frac{3}{4} EAD$, since RB is considered equal to $\frac{3}{4} EB$. But $EAD = \frac{1}{2} AFE$. Therefore, $RAN = \frac{3}{4} AFE$. In the same way we should find $SAO = \frac{1}{4} AFE$, and $TAP = \frac{1}{2} AFE$. We have then for the ordinates, $RN = AN \tan. RAN = \frac{3}{4} c \tan. \frac{3}{4} AFE$, $SO = AO \tan. SAO = \frac{1}{4} c \tan. \frac{1}{4} AFE$, and $TP = AP \tan. TAP = \frac{1}{2} c \tan. \frac{1}{2} AFE$. But, by the second supposition, $\tan. \frac{3}{4} AFE = \frac{3}{4} \tan. \frac{1}{2} AFE$, $\tan. \frac{1}{4} AFE = \frac{1}{2} \tan. \frac{1}{2} AFE$, and $\tan. \frac{1}{2} AFE = \frac{1}{4} \tan. \frac{1}{2} AFE$. Substituting these values, and recollecting that $\frac{1}{2} c \tan. \frac{1}{2} AFE = m$, we have

$$\left\{ \begin{array}{l} RN = \frac{15}{16} \times \frac{1}{2} c \tan. \frac{1}{2} AFE = \frac{15}{16} m, \\ SO = \frac{8}{4} \times \frac{1}{2} c \tan. \frac{1}{2} AFE = \frac{8}{4} m, \\ TP = \frac{7}{16} \times \frac{1}{2} c \tan. \frac{1}{2} AFE = \frac{7}{16} m. \end{array} \right.$$

In general, if the number of divisions of the chord on each side of the centre is represented by n , we should find for the respective ordinates, beginning nearest the centre, $\frac{(n+1)(n-1)m}{n^2}$, $\frac{(n+2)(n-2)m}{n^2}$, $\frac{(n+3)(n-3)m}{n^2}$, &c.

Example. Find the ordinates of an 8° curve to a chord of 100 feet. Here $m = 50 \tan. 2^\circ = 1.746$, $RN = \frac{15}{16} m = 1.637$, $SO = \frac{8}{4} m = 1.310$, and $TP = \frac{7}{16} m = 0.764$.

26. An approximate value of m also may be obtained from the formula $m = R - \sqrt{R^2 - \frac{1}{4}c^2}$. This is done by adding to the quantity under the radical the very small fraction $\frac{c^4}{64 R^2}$, making it a perfect

square, the root of which will be $R - \frac{c^2}{8R}$. We have, then, $m = R - \left(R - \frac{c^2}{8R}\right)$;

$$\therefore m = \frac{c^2}{8R}.$$

27. From this value of m we see that the middle ordinates of any two chords in the same curve are to each other nearly as the squares of the chords. If, then, AE (fig. 4) be considered equal to $\frac{1}{2} AB$, its middle ordinate $CH = \frac{1}{4} ED$. Intermediate points on a curve may, therefore, be very readily obtained, and generally with sufficient accuracy, in the following manner. Stretch a cord from A to B , and by means of the middle ordinate determine the point E . Then stretch the cord from A to E , and lay off the middle ordinate $CH = \frac{1}{4} ED$, thus determining the point C , and so continue to lay off from the successive half-chords one fourth the preceding ordinate, until a sufficient number of points is obtained.

D. Curving Rails.

28. The rails of a curve are usually curved before they are laid. To do this properly, it is necessary to know the middle ordinate of the curve for a chord of the length of a rail.

29. **Problem.** Given the radius or deflection angle of a curve, to find the middle ordinate for curving a rail of given length.

Solution. Denote the length of the rail by l , and we have (§ 25) the exact formula $m = R - \sqrt{R^2 - \frac{1}{4} l^2}$, and (§ 26) the approximate formula

$$m = \frac{\frac{1}{4} l^2}{2R}.$$

This formula is always near enough for chords of the length of a rail. If we substitute for R its value (§ 10) $R = \frac{50}{\sin. D}$, we have,

$$m = \frac{1}{4} l^2 \times \frac{\sin. D}{100}.$$

Example. In a 1° curve find the ordinate for a rail of 18 feet in length. Here R is found by Table I. to be 5729.65, and therefore,

by the first formula, $m = \frac{9^2}{11459.3} = .00707$. By the second formula, $m = .81 \sin. 30' = .00707$. The exact formula would give the same result even to the fifth decimal.

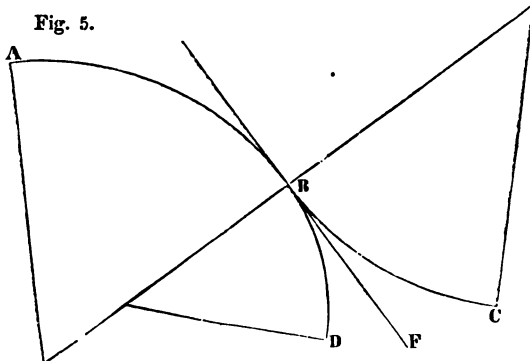
By keeping in mind, that the ordinate for a rail of 18 feet in a 1° curve is .007, the corresponding ordinate in a curve of any other degree may be found with sufficient accuracy, by multiplying this decimal by the number expressing the degree of the curve. Thus, for a curve of $5^\circ 36'$ or 5.6° , the ordinate would be $.007 \times 5.6 = .039$ ft. = .468 in.

For a rail of 20 feet we have $\frac{1}{4} l^2 = 100$, and, consequently, $m = \sin. D$. This gives for a 1° curve, $m = .0087$. The corresponding ordinate in a curve of any other degree may be found with sufficient accuracy, by multiplying this decimal by the number expressing the degree of the curve.

By the above formula for m , the ordinates for curving rails in Table I. are calculated.

ARTICLE II.—REVERSED AND COMPOUND CURVES.

30. Two curves often succeed each other having a common tangent at the point of junction. If the curves lie on *opposite* sides of the common tangent, they form a *reversed* curve, and their radii may be the same or different. If they lie on the *same* side of the common tangent,



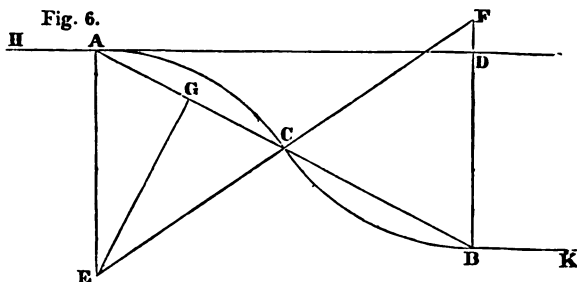
they have different radii, and form a compound curve. Thus ABC (fig. 5) is a reversed curve, and ABD a compound curve.

31. Problem. To lay out a reversed or a compound curve, when the radii or deflection angles and the tangent points are known.

Solution. Lay out the first portion of the curve from A to B (fig. 5), by one of the usual methods. Find BF , the tangent to AB , at the point B (§ 16 or § 21). Then BF will be the tangent also of the second portion BC of a reversed, or BD of a compound curve, and from this tangent either of these portions may be laid off in the usual manner.

A. Reversed Curves.

32 Theorem. The reversing point of a reversed curve between parallel tangents is in the line joining the tangent points.



Demonstration. Let ACB (fig. 6) be a reversed curve, uniting the parallel tangents HA and BK , having its radii equal or unequal, and reversing at C . If now the chords AC and CB are drawn, we have to prove that these chords are in the same straight line. The radii EC and CF , being perpendicular to the common tangent at C (§ 2, I.), are in the same straight line, and the radii AE and BF , being perpendicular to the parallel tangents HA and BK , are parallel. Therefore, the angle $AEC = CFB$, and, consequently, ECA , the half supplement of AEC , is equal to FCB , the half supplement of CFB ; but these angles cannot be equal, unless AC and CB are in the same straight line.

33. Problem. Given the perpendicular distance between two parallel tangents $BD = b$ (fig. 6), and the distance between the two tangent points $AB = a$, to determine the reversing point C and the common radius $EC = CF = R$ of a reversed curve uniting the tangents HA and BK .

Solution. Let ACB be the required curve. Since the radii are

equal, and the angle $AEC = BFC$, the triangles AEC and BFC are equal, and $AC = CB = \frac{1}{2}a$. The reversing point C is, therefore, the middle point of AB .

To find R , draw EG perpendicular to AC . Then the right triangles AEG and BAD are similar, since (§ 2, III.) the angle $BAD = \frac{1}{2}AEC = AEG$. Therefore $AE : AG = AB : BD$, or $R : \frac{1}{4}a = a : b$;

$$\therefore R = \frac{a^2}{4b}.$$

Corollary. If R and b are given, to find a , the equation $R = \frac{a^2}{4b}$ gives $a^2 = 4Rb$;

$$\therefore a = 2\sqrt{Rb}.$$

Examples. Given $b = 12$, and $a = 200$, to determine R . Here $R = \frac{200^2}{4 \times 12} = \frac{10000}{12} = 833\frac{1}{3}$.

Given $R = 675$, and $b = 12$, to find a . Here $a = 2\sqrt{675 \times 12} = 2\sqrt{8100} = 2 \times 90 = 180$.

34. Problem. Given the perpendicular distance between two parallel tangents $BD = b$ (fig. 7), the distance between the two tangent points $AB = a$, and the first radius $EC = R$ of a reversed curve uniting the tangents HA and BK , to find the chords $AC = a'$ and $CB = a''$, and the second radius $CF = R'$.

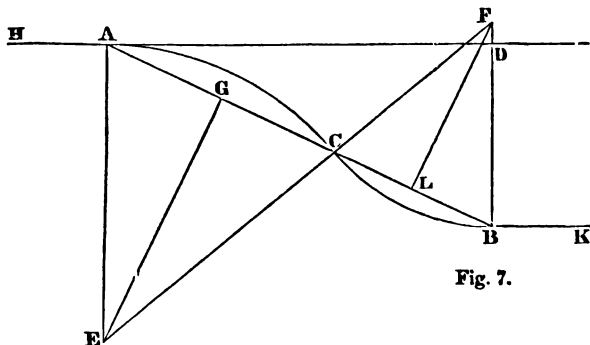
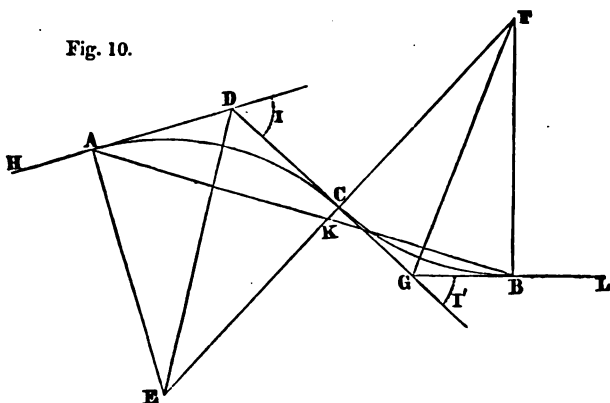


Fig. 7.

Solution. Draw the perpendiculars EG and FL . Then the right triangles ABD and EAG are similar, since the angle $BAD =$

39. Problem. Given the length of the common tangent $DG = a$, and the angles of intersection I and P (fig. 10), to determine the common radius $CE = CF = R$ of a reversed curve to unite the tangents HA and BL .

Fig. 10.



Solution. By § 4 we have $DC = R \tan. \frac{1}{2} I$, and $CG = R \tan. \frac{1}{2} P$; whence $R (\tan. \frac{1}{2} I + \tan. \frac{1}{2} P) = DC + CG = a$, or

$$R = \frac{a}{\tan. \frac{1}{2} I + \tan. \frac{1}{2} P}.$$

This formula may be adapted to calculation by logarithms; for we have (Tab. X. 35) $\tan. \frac{1}{2} I + \tan. \frac{1}{2} P = \frac{\sin. \frac{1}{2} (I + P)}{\cos. \frac{1}{2} I \cos. \frac{1}{2} P}$. Substituting this value, we get

$$R = \frac{a \cos. \frac{1}{2} I \cos. \frac{1}{2} P}{\sin. \frac{1}{2} (I + P)}$$

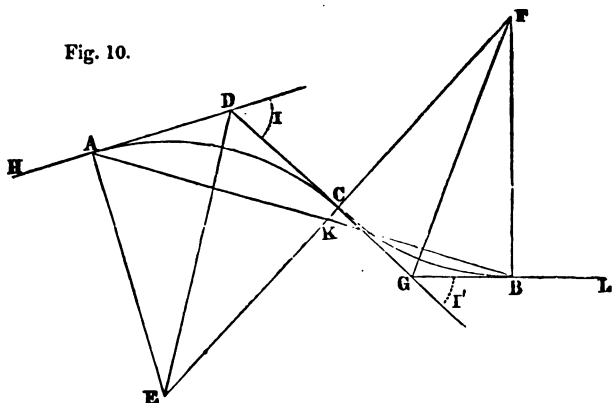
The tangent points A and B are obtained by measuring from D a distance $AD = R \tan. \frac{1}{2} I$, and from G a distance $BG = R \tan. \frac{1}{2} P$.

Example. Given $a = 600$, $I = 12^\circ$, and $P = 8^\circ$, to find R . Here

$a = 600$	2.778151
$\frac{1}{2} I = 6^\circ$	cos. 9.997614
$\frac{1}{2} P = 4^\circ$	cos. 9.998941
	2.774706
$\frac{1}{2} (I + P) = 10^\circ$	sin. 9.239670
$R = 3427.96$	3.535036

40. Problem. Given the line $AB = a$ (fig. 10), which joins the fixed tangent points A and B , the angle $DAB = A$, and the angle $ABG = B$, to find the common radius $EC = CF = R$ of a reversed curve to unite the tangents HA and BL .

Fig. 10.



Solution. Find first the auxiliary angle $AKE = BKF$, which may be denoted by K . For this purpose the triangle AEK gives $AE : EK = \sin. K : \sin. EAK$. Therefore $EK \sin. K = AE \sin. EAK = R \cos. A$, since $EAK = 90^\circ - A$. In like manner, the triangle BFK gives $FK \sin. K = BF \sin. FBK = R \cos. B$. Adding these equations, we have $(EK + FK) \sin. K = R (\cos. A + \cos. B)$, or, since $EK + FK = 2R$, $2R \sin. K = R (\cos. A + \cos. B)$. Therefore, $\sin. K = \frac{1}{2} (\cos. A + \cos. B)$. For calculation by logarithms, this becomes (Tab. X. 28)

$$\sin. K = \cos. \frac{1}{2} (A + B) \cos. \frac{1}{2} (A - B).$$

Having found K , we have the angle $AEK = E = 180^\circ - K - EAK = 180^\circ - K - (90^\circ - A) = 90^\circ + A - K$, and the angle $BFK = F = 180^\circ - K - FBK = 180^\circ - K - (90^\circ - B) = 90^\circ + B - K$. Moreover, the triangle AEK gives $AE : AK = \sin. K : \sin. E$, or $R \sin. E = AK \sin. K$, and the triangle BFK gives $BF : BK = \sin. K : \sin. F$, or $R \sin. F = BK \sin. K$. Adding these equations, we have $R (\sin. E + \sin. F) = (AK + BK) \sin. K = \sin. K$. Substituting for $\sin. E + \sin. F$ its value $2 \sin. \frac{1}{2} (E + F)$

$\cos. \frac{1}{2} (E - F)$ (Tab. X. 26), we have $2 R \sin. \frac{1}{2} (E + F) \cos. \frac{1}{2} (E - F) = a \sin. K$. Therefore $R = \frac{\frac{1}{2} a \sin. K}{\sin. \frac{1}{2} (E + F) \cos. \frac{1}{2} (E - F)}$. Finally, substituting for E its value $90^\circ + A - K$, and for F its value $90^\circ + B - K$, we get $\frac{1}{2} (E + F) = 90^\circ - [K - \frac{1}{2} (A + B)]$, and $\frac{1}{2} (E - F) = \frac{1}{2} (A - B)$; whence

$$R = \frac{\frac{1}{2} a \sin. K}{\cos. [K - \frac{1}{2} (A + B)] \cos. \frac{1}{2} (A - B)}$$

Example. Given $a = 1500$, $A = 18^\circ$, and $B = 6^\circ$, to find R . Here

$$\frac{1}{2} (A + B) = 12^\circ \quad \cos. 9.990404$$

$$\frac{1}{2} (A - B) = 6^\circ \quad \cos. 9.997614$$

$$K = 76^\circ 36' 10'' \quad \sin. 9.988018$$

$$\frac{1}{2} a = 750 \quad \quad \quad 2.875061$$

$$\hline 2.863079$$

$$K - \frac{1}{2} (A + B) = 64^\circ 36' 10'' \quad \cos. 9.632347$$

$$\frac{1}{2} (A - B) = 6^\circ \quad \cos. 9.997614$$

$$\hline 9.629961$$

$$R = 1710.48 \quad \quad \quad 3.233118$$

B. Compound Curves.

41. Theorem. *If one branch of a compound curve be produced, until the tangent at its extremity is parallel to the tangent at the extremity of the second branch, the common tangent point of the two arcs is in the straight line produced, which passes through the tangent points of these parallel tangents.*

Demonstration. Let ACB (fig. 11) be a compound curve, uniting the tangents HA and BK . The radii CE and CF , being perpendicular to the common tangent at C (§ 2, I.), are in the same straight line. Continue the curve AC to D , where its tangent OD becomes parallel to BK , and consequently the radius DE parallel to BF . Then if the chords CD and CB be drawn, we have the angle $CED = CFB$; whence ECD , the half-supplement of CEB , is equal to FCB , the half-supplement of CFB . But ECD cannot be equal to FCB , unless CD coincides with CB . Therefore the line BD produced passes through the common tangent point C .

42. Problem. *To find a limit in one direction of each radius of a compound curve.*

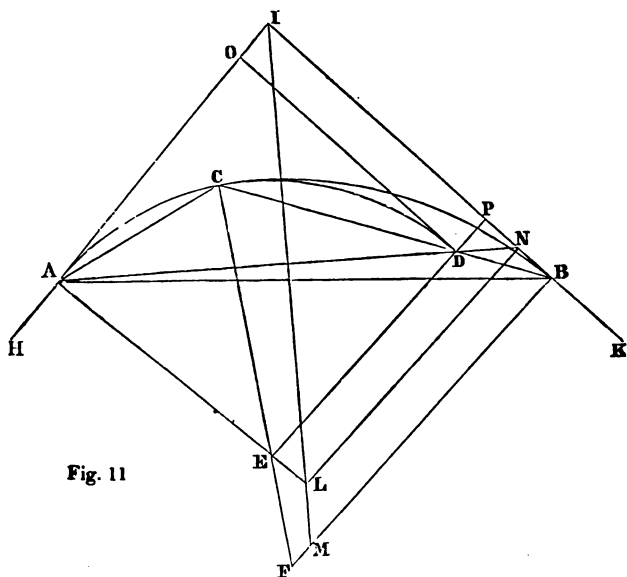


Fig. 11

Solution. Let AI and BI (fig. 11) be the tangents of the curve. Through the intersection point I , draw IM bisecting the angle AIB . Draw AL and BM perpendicular respectively to AI and BI , meeting IM in L and M . Then the radius of the branch commencing on the shorter tangent AI must be *less* than AL , and the radius of the branch commencing on the longer tangent BI must be *greater* than BM . For suppose the shorter radius to be made equal to AL , and make $IN = AI$, and join LN . Then the equal triangles AIL and NIL give $AL = LN$; so that the curve, if continued, will pass through N , where its tangent will coincide with IN . Then (§ 41) the common tangent point would be the intersection of the straight line through B and N with the first curve; but in this case there can be no intersection, and therefore no common tangent point. Suppose next, that this radius is *greater* than AL , and continue the curve, until its tangent becomes parallel to BI . In this case the extremity of the

curve will fall outside the tangent BI in the line AN produced, and a straight line through B and this extremity will again fail to intersect the curve already drawn. As no common tangent point can be found when this radius is taken equal to AL or greater than AL , no compound curve is possible. This radius must, therefore, be *less* than AL . In a similar manner it might be shown, that the radius of the other branch of the curve must be *greater* than BM . If we suppose the tangents AI and BI and the intersection angle I to be known, we have (§ 5) $AL = AI \cot. \frac{1}{2} I$, and $BM = BI \cot. \frac{1}{2} I$. These values are, therefore, the limits of the radii in one direction.

43. If nothing were given but the position of the tangents and the tangent points, it is evident that an indefinite number of different compound curves might connect the tangent points; for the shorter radius might be taken of any length less than the limit found above, and a corresponding value for the greater could be found. Some other condition must, therefore, be introduced, as is done in the following problems.

44. Problem. Given the line $AB = a$ (fig. 11), which joins the fixed tangent points A and B , the angle $B A I = A$, the angle $A B I = B$, and the first radius $AE = R$, to find the second radius $BF = R'$ of a compound curve to unite the tangents HA and BK .

Solution. Suppose the first curve to be run with the given radius from A to D , where its tangent DO becomes parallel to BI , and the angle $IAD = \frac{1}{2}(A + B)$. Then (§ 41) the common tangent point C is in the line BD produced, and the chord $CB = CD + BD$. Now in the triangle ABD we have $AB = a$, $AD = 2R \sin. \frac{1}{2}(A + B)$ (§ 69), and the included angle $DAB = IAB - IAD = A - \frac{1}{2}(A + B) = \frac{1}{2}(A - B)$. Find in this triangle (Tab. X. 14 and 12) the angle ABD and the side BD . Find also the angle $CBI = B - ABD$.

Then (§ 69) the chord $CB = 2R' \sin. CBI$, and the chord $CD = 2R \sin. CDO = 2R \sin. CBI$. Substituting these values of CB and CD in the equation found above, $CB = CD + BD$, we have $2R' \sin. CBI = 2R \sin. CBI + BD$;

$$\therefore R' = R + \frac{BD}{2 \sin. CBI}$$

When the angle B is greater than A , that is, when the greater radius is given, the solution is the same, except that the angle $DAB =$

$\frac{1}{2}(B - A)$, and CBI is found by subtracting the supplement of ABD from B . We shall also find $CB = CD - BD$, and consequently $R' = R - \frac{BD}{2 \sin. CBI}$.

If more convenient, the point D may be determined in the field, by laying off the angle $IAD = \frac{1}{2}(A + B)$, and measuring the distance $AD = 2R \sin. \frac{1}{2}(A + B)$. BD and CBI may then be measured, instead of being calculated as above.

Example. Given $a = 950$, $A = 8^\circ$, $B = 7^\circ$, and $R = 3000$, to find R' . Here $AD = 2 \times 3000 \sin. \frac{1}{2}(8^\circ + 7^\circ) = 783.16$, and $DAB = \frac{1}{2}(8^\circ - 7^\circ) = 30'$. Then to find ABD we have

$$\begin{array}{rcl} AB - AD & = & 166.84 \quad 2.222300 \\ \frac{1}{2}(ADB + ABD) & = & 89^\circ 45' \quad \tan. 2.360180 \\ & & 4.582480 \\ AB + AD & = & 1733.16 \quad 3.238839 \\ \frac{1}{2}(ADB - ABD) & = & 87^\circ 24' 17'' \quad \tan. 1.343641 \\ \therefore ABD & = & 2^\circ 20' 43'' \end{array}$$

Next, to find BD ,

$$\begin{array}{rcl} AD & = & 783.16 \quad 2.893849 \\ DAB & = & 30' \quad \sin. 7.940842 \\ & & 0.834691 \\ ABD & = & 2^\circ 20' 43'' \quad \sin. 8.611948 \\ BD & = & 167.01 \quad 2.222743 \\ B - ABD = CBI & = & 4^\circ 39' 17'' \quad \sin. 8.909292 \\ 2(R' - R) & = & 2058.03 \quad 3.313451 \\ \therefore R' - R & = & 1029.01 \\ \therefore R' & = & 3000 + 1029.01 = 4029.01 \end{array}$$

To find the central angle of each branch, we have $CFB = 2CBI = 9^\circ 18' 34''$, which is the central angle of the second branch; and $AEC = AED - CED = A + B - 2CBI = 5^\circ 41' 26''$, which is the central angle of the first branch.

45. Problem. Given (fig. 11) the tangents $AI = T$, $BI = T'$, the angle of intersection $= I$, and the first radius $AE = R$, to find the second radius $BF = R'$.

Solution. Suppose the first curve to be run with the given radius

D draw *DP* parallel to *AI*, and we have $IP = DO = AO = R \tan. \frac{1}{2} I$ (§ 4). Then in the triangle *DPB* we have $DP = IO = AI - AO = T - R \tan. \frac{1}{2} I$, $BP = BI - IP = T' - R \tan. \frac{1}{2} I$, and the included angle $DPB = AIB = 180^\circ - I$. Find in this triangle the angle *CBI*, and the side *BD*. The remainder of the solution is the same as in § 44. The determination of the point *D* in the field is also the same, the angle *IAD* being here $= \frac{1}{2} I$. When *B* is greater than *A*, that is, when the greater radius is given, the solution is the same, except that $DP = R \tan. \frac{1}{2} I - T$, and $BP = R \tan. \frac{1}{2} I - T'$.

Example. Given $T = 447.32$, $T' = 510.84$, $I = 15^\circ$, and $R = 3000$, to find R' . Here $R \tan. \frac{1}{2} I = 3000 \tan. 7\frac{1}{2}^\circ = 394.96$, $DP = 447.32 - 394.96 = 52.36$, $BP = 510.84 - 394.96 = 115.88$, and $DPB = 180^\circ - 15^\circ = 165^\circ$. Then (Tab. X. 14 and 12)

$BP - DP = 63.52$	1.602910
$\frac{1}{2} (BDP + PBD) = 7^\circ 30'$	tan. 9.119429
	0.922339
$BP + DP = 168.24$	2.225929
$\frac{1}{2} (BDP - PBD) = 2^\circ 50' 44''$	tan. 8.696410
$\therefore PBD = CBI = 4^\circ 39' 16''$	

Next, to find *BD*,

$DP = 52.36$	1.719000
$DPB = 15^\circ$	sin. 9.412996
	1.131996
$PBD = 4^\circ 39' 16''$	sin. 8.909266
$BD = 167.005$	2.222730

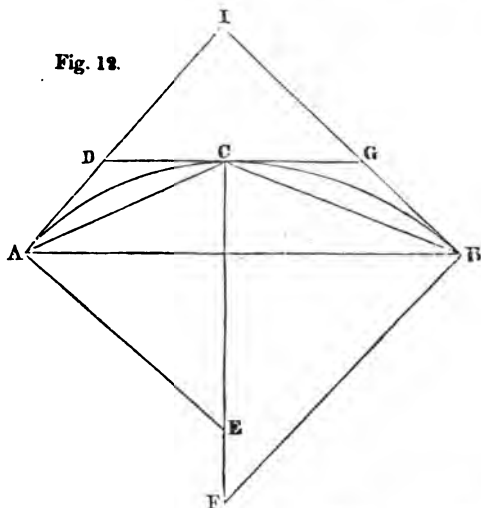
The tangents in this example were calculated from the example in § 44. The values of *CBI* and *BD* here found differ slightly from those obtained before. In general, the triangle *DBP* is of better form for accurate calculation than the triangle *ADB*.

46. If no circumstance determines either of the radii, the condition may be introduced, that the common tangent shall be parallel to the line joining the tangent points.

Problem. Given the line $AB = a$ (fig. 12), which unites the fixed tangent points *A* and *B*, the angle $IAB = A$, and the angle $ABI = B$, to find the radii $AE = R$ and $BF = R'$ of a compound curve, having the common tangent *DG* parallel to *AB*.

ution. Let AC and BC be the two branches of the required $\angle$, and draw the chords AC and BC . These chords bisect the

Fig. 12.



s A and B ; for the angle $DA C = \frac{1}{2} ID G = \frac{1}{2} IAB$, and the $GB C = \frac{1}{2} DGI = \frac{1}{2} ABI$. Then in the triangle ACB we $AC : AB = \sin. ABC : \sin. ACB$. But $ACB = 180^\circ - B + CBA = 180^\circ - \frac{1}{2}(A + B)$, and as the sine of the supplement of an angle is the same as the sine of the angle itself, $1CB = \sin. \frac{1}{2}(A + B)$. Therefore $AC : a = \sin. \frac{1}{2} B : \sin. \frac{1}{2}(A + B)$, or $AC = \frac{a \sin. \frac{1}{2} B}{\sin. \frac{1}{2}(A + B)}$. In a similar manner we should $BC = \frac{a \sin. \frac{1}{2} A}{\sin. \frac{1}{2}(A + B)}$. Now we have (§ 68) $R = \frac{\frac{1}{2} AC}{\sin. \frac{1}{2} A}$, and $\frac{\frac{1}{2} BC}{\sin. \frac{1}{2} B}$, or, substituting the values of AC and BC just found.

$$R = \frac{\frac{1}{2} a \sin. \frac{1}{2} B}{\sin. \frac{1}{2} A \sin. \frac{1}{2} (A + B)}; R' = \frac{\frac{1}{2} a \sin. \frac{1}{2} A}{\sin. \frac{1}{2} B \sin. \frac{1}{2} (A + B)}.$$

Ex. 1. Given $a = 950$, $A = 8^\circ$, and $B = 7^\circ$, to find R and R .

$\frac{1}{2} a = 475$		2.676394
$\frac{1}{2} B = 3^{\circ} 30'$	sin.	8.785675
		1.462369
$\frac{1}{2} A = 4^{\circ}$	sin.	8.843585
$\frac{1}{2} (A + B) = 7^{\circ} 30'$	sin.	9.115698
		7.959283
$R = 3184.83$		3.503086

Transposing these same logarithms according to the formula for R' we have

$\frac{1}{2} a = 475$		2.676694
$\frac{1}{2} A = 4^{\circ}$	sin.	8.843585
		1.520279
$\frac{1}{2} B = 3^{\circ} 30'$	sin.	8.785675
$\frac{1}{2} (A + B) = 7^{\circ} 30'$	sin.	9.115698
		7.901373
$R' = 4158.21$		3.618906

47. Problem. Given the line $AB = a$ (fig. 12), which unites the fixed tangent points A and B , and the tangents $AI = T$ and $BI = T'$, to find the tangents $AD = x$ and $BG = y$ of the two branches of a compound curve, having its common tangent DG parallel to AB .

Solution. Since $DC = AD = x$, and $CG = BG = y$, we have $DG = x + y$. Then the similar triangles IDG and IAB give $ID : IA = DG : AB$, or $T - x : T = x + y : a$. Therefore $aT - ax = Tx + Ty$ (1). Also $a : AI = BG : BI$, or $x : T = y : T'$. Therefore $Ty = T'x$ (2). Substituting in (1) the value of Ty in (2), we have $aT - ax = Tx + T'x$, or $ax + Tx + T'x = aT$;

$$\therefore x = \frac{aT}{a + T + T'};$$

and, since from (2), $y = \frac{T'x}{T}$,

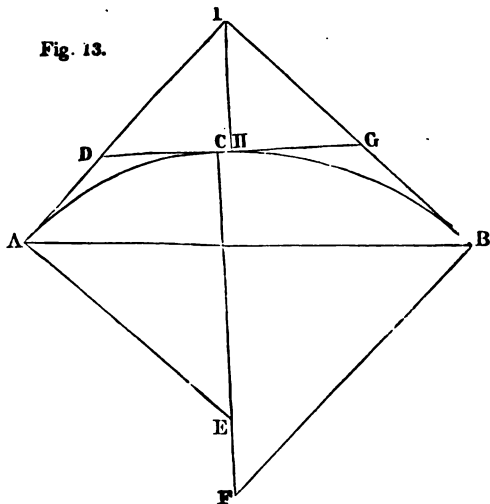
$$y = \frac{aT'}{a + T + T'}.$$

The intersection points D and G and the common tangent point C are now easily obtained on the ground, and the radii may be found by the usual methods. Or, if the angles $IAB = A$ and $ABI = B$

have been measured or calculated, we have (§ 5) $R = x \cot. \frac{1}{2} A$, and $R' = y \cot. \frac{1}{2} B$. Substituting the values of x and y found above, we have $R = \frac{a T \cot. \frac{1}{2} A}{a + T + T'}$, and $R' = \frac{a T' \cot. \frac{1}{2} B}{a + T + T'}$.

Example. Given $a = 500$, $T = 250$, and $T' = 290$, to find x and y . Here $a + T + T' = 500 + 250 + 290 = 1040$; whence $x = 500 \times 250 \div 1040 = 120.19$, and $y = 500 \times 290 \div 1040 = 139.42$.

48. Problem. Given the tangents $AI = T$, $BI = T'$, and the angle of intersection I , to unite the tangent points A and B (fig. 13) by a compound curve, on condition that the two branches shall have their angles of intersection IDG and IGD equal.



Solutum. Since $IDG = IGD = \frac{1}{2} I$, we have $ID = IG$. Represent the line $ID = IG$ by x . Then if the perpendicular IH be let

* The radii of an oval of given length and breadth, or of a three-centre arch of given span and rise, may also be found from these formulæ. In these cases $A + B = 90^\circ$, and the values of R and R' may be reduced to $R = \frac{a T}{a + T' - T}$ and $R' = \frac{a T'}{a + T - T'}$. These values admit of an easy construction, or they may be readily calculated.

fall from I , we have (Tab. X. 11) $DH = ID \cos. IDG = x \cos. \frac{1}{2} I$, and $DG = 2x \cos. \frac{1}{2} I$. But $DG = DC + CG = AD + BG = T - x + T' - x = T + T' - 2x$. Therefore $2x \cos. \frac{1}{2} I = T + T' - 2x$, or $2x + 2x \cos. \frac{1}{2} I = T + T'$; whence $x = \frac{\frac{1}{2}(T+T')}{1 + \cos. \frac{1}{2} I}$, or (Tab. X. 25)

$$x = \frac{\frac{1}{2}(T+T')}{\cos.^2 \frac{1}{2} I}.$$

The tangents $AD = T - x$ and $BG = T' - x$ are now readily found. With these and the known angles of intersection, the radii or deflection angles may be found (§ 5 or § 11). This method answers very well, when the given tangents are nearly equal; but in general the preceding method is preferable.

Example. Given $T = 480$, $T' = 500$, and $I = 18^\circ$, to find x . Here

$$\begin{array}{rcl} \frac{1}{2}(T+T') & = & 245 \qquad \qquad \qquad 2.389166 \\ \frac{1}{2}I = 4^\circ 30' & 2 \cos. & 9.997318 \\ \hline x & = & 246.52 \qquad \qquad \qquad 2.391848 \end{array}$$

Then $AD = 480 - 246.52 = 233.48$, and $BG = 500 - 246.52 = 253.48$. The angle of intersection for both branches of the curve being 9° , we find the radii $AE = 233.48 \cot. 4^\circ 30' = 2^\circ 56.65$, and $BF = 253.48 \cot. 4^\circ 30' = 3220.77$.

ARTICLE III.—TURNOUTS AND CROSSINGS.

49. THE usual mode of turning off from a main track is by switching a pair of rails in the main track, and putting in a turnout curve tangent to the switched rails, with a frog placed where the outer rail of the turnout crosses the rail of the main track. AB (fig. 14) represents one of the rails of the main track switched, BF represents the outer rail of the turnout curve, tangent to AB , and F shows the position of the frog. The switch angle, denoted by S , is the angle DAB , formed by the switched rail AB with AD , its former position in the main track. The frog angle, denoted by F' , is the angle GFM made by the crossing rails, the direction of the turnout rail at F being the tangent FM at that point. In the problems of this article the gauge of the track DC , denoted by g , and the distance DB , denoted by d , are supposed to be known. The switch angle S is also supposed to be known, since its sine (Tab. X. 1) is equal to d divided by the length

of the switched rail. If, for example, the rail is 18 feet in length and $d = .42$, we have $S = 1^\circ 20'$.

A. Turnout from Straight Lines.

50. Problem. Given the radius R of the centre line of a turnout (fig. 14), to find the frog angle $GFM = F$ and the chord BF .

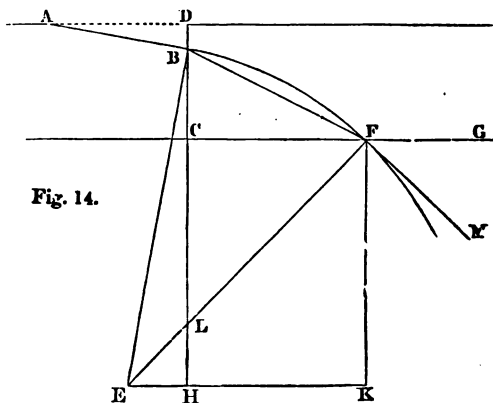


Fig. 14.

Solution. Through the centre E draw EK parallel to the main track. Draw BH and FK perpendicular to EK , and join EF . Then, since EF is perpendicular to FM and FK is perpendicular to FG , the angle $EFK = GFM = F$; and since EB and BH are respectively perpendicular to AB and AD , the angle $EBH = DAB = S$. Now the triangle EFK gives (Tab. X. 2) $\cos. EFK = \frac{FK}{EF}$. But EF , the radius of the outer rail, is equal to $R + \frac{1}{2}g$, and $FK = CH = BH - BC = BE \cos. EBH - BC = (R + \frac{1}{2}g) \cos. S - (g - d)$. Substituting these values, we have $\cos. EFK = \frac{(R + \frac{1}{2}g) \cos. S - (g - d)}{R + \frac{1}{2}g}$, or

$$\cos. F = \cos. S - \frac{g - d}{R + \frac{1}{2}g}.$$

From this formula F may be found by the table of natural cosines. To adapt it to calculation by logarithms, we may consider $g - d$ to be equal to $(g - d) \cos. S$, which will lead to no material error since

$g - d$ is very small, and $\cos. S$ almost equal to unity. The value of $\cos. F$ then becomes

$$\cos. F = \frac{(R - \frac{1}{2}g + d) \cos. S}{R + \frac{1}{2}g}.$$

To find BF , the right triangle BCF gives (Tab. X. 9) $BF = \frac{BC}{\sin. BFC}$. But $BC = g - d$ and the angle $BFC = BFE - CFE = (90^\circ - \frac{1}{2}BEF) - (90^\circ - F) = F - \frac{1}{2}BEF$. But $BEF = BLF - EBL = F - S$. Therefore $BFC = F - \frac{1}{2}(F - S) = \frac{1}{2}(F + S)$. Substituting these values in the formula for BF , we have

$$BF = \frac{g - d}{\sin. \frac{1}{2}(F + S)}.$$

By the above formulæ the columns headed F and BF in Table V. are calculated.

Example. Given $g = 4.7$, $d = .42$, $S = 1^\circ 20'$, and $R = 500$, to find F and BF . Here $\text{nat. cos. } S = .999729$, $g - d = 4.28$, $R + \frac{1}{2}g = 502.35$, and $4.28 \div 502.35 = .008520$. Therefore $\text{nat. cos. } F = .999729 - .008520 = .991209$, which gives $F = 7^\circ 36' 10''$. Next, to find BF ,

$g - d = 4.28$	0.631444
$\frac{1}{2}(F + S) = 4^\circ 28' 5''$	sin. 8.891555
$BF = 54.94$	1.739889

51. Problem. Given the frog angle $GF M = F$ (fig. 14), to find the radius R of the centre line of a turnout, and the chord BF .

Solution. From the preceding solution we have $\cos. F = \frac{R + \frac{1}{2}g \cos. S - (g - d)}{R + \frac{1}{2}g}$. Therefore $(R + \frac{1}{2}g) \cos. F = (R + \frac{1}{2}g) \cos. S - (g - d)$, or

$$R + \frac{1}{2}g = \frac{g - d}{\cos. S - \cos. F}.$$

For calculation by logarithms this becomes (Tab. X. 29)

$$R + \frac{1}{2}g = \frac{\frac{1}{2}(g - d)}{\sin. \frac{1}{2}(F + S) \sin. \frac{1}{2}(F - S)}.$$

Having thus found $R + \frac{1}{2}g$, we find R by subtracting $\frac{1}{2}g$. BF is found, as in the preceding problem, by the formula

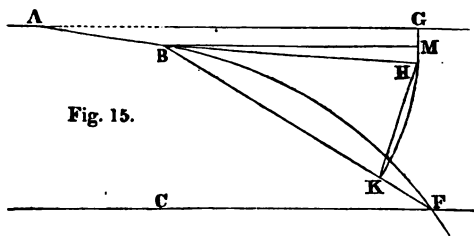
$$BF = \frac{g - d}{\sin. \frac{1}{2}(F + S)}.$$

Example. Given $g = 4.7$, $d = .42$, $S = 1^\circ 20'$, and $F = 7^\circ$, to find R . Here

$\frac{1}{2}(g - d) = 2.14$		0.330414
$\frac{1}{2}(F + S) = 4^\circ 10'$	sin. 8.861283	
$\frac{1}{2}(F - S) = 2^\circ 50'$	sin. 8.693998	
		7.555281
$R + \frac{1}{2}g = 595.85$		2.775133
$\therefore R = 593.5$		

52. Problem. To find mechanically the proper position of a given frog.

Solution. Denote the length of the switch rail by l , the length of the frog by f , and its width by w . From B as a centre with a radius $BH = 2l$, describe on the ground an arc $G H K$ (fig. 15), and from the inside of the rail at G measure $GH = 2d$, and from H measure HK such that $HK : BH = \frac{1}{2}w : f$, or $HK : 2l = \frac{1}{2}w : f$; that is, $HK = \frac{wl}{f}$. Then a straight line through B and the point K will strike the inside of the other rail at F , the place for the point of the



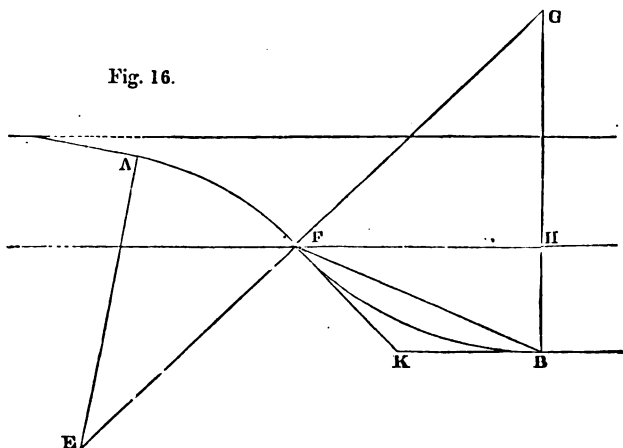
frog. For the angle HBK has been made equal to $\frac{1}{2}F$, and if BM be drawn parallel to the main track, the angle MBH is seen to be equal to $\frac{1}{2}S$. Therefore, $MBK = BFC = \frac{1}{2}(F + S)$, and this was shown (§ 50) to be the true value of BFC .

53. If the turnout is to reverse, and become parallel to the main track, the problems on reversed curves already given will in general be sufficient. Thus, if the tangent points of the required curve are fixed, the common radius may be found by § 40. If the tangent point at the switch is fixed, and the common radius given, the reversing *chord* and the other tangent point may be found by § 37, the change of direction of the two tangents being here equal to S . But when the

frog angle is given, or determined from a given first radius, and the point of the frog is taken as the reversing point, the radius of the second portion may be found by the following method.

54. Problem. *Given the frog angle F and the distance $HB = b$ (fig. 16) between the main track and a turnout, to find the radius R' of the second branch of the turnout, the reversing point being taken opposite F , the point of the frog.*

Fig. 16.



Solution. Let the arc FB be the inner rail of the second branch, $FG = R' - \frac{1}{2}g$ its radius, and B the tangent point where the turnout becomes parallel to the main track. Now since the tangent FK is one side of the frog produced, the angle $HFK = F$, and since the angle of intersection at K is also equal to F , $BFK = \frac{1}{2}F$ (§ 2, II.); whence $BFH = \frac{1}{2}F$. Then (§ 68) $FG = \frac{\frac{1}{2}BF}{\sin. \frac{1}{2}BFK}$, or $R' - \frac{1}{2}g = \frac{\frac{1}{2}BF}{\sin. \frac{1}{2}BF}$. But $BF = \frac{HB}{\sin. BFH}$ (Tab. X. 9), or $\frac{1}{2}BF = \frac{\frac{1}{2}b}{\sin. \frac{1}{2}F}$. Substituting this value of $\frac{1}{2}BF$, we have

$$R' - \frac{1}{2}g = \frac{\frac{1}{2}b}{\sin. \frac{1}{2}F}$$

In measuring the distance $HB = b$, it is to be observed, that the widths of both rails must be included.

Example. Given $b = 62$ and $F' = 8^\circ$, to find R' . Here

$\frac{1}{2}b = 3.1$	0.491362
$\frac{1}{2}F = 4^\circ$	sin. 8.843585
$\frac{1}{2}BF = 44.44$	1.647777
$\frac{1}{2}F = 4^\circ$	sin. 8.843585
$R' - \frac{1}{2}g = 637.08$	2.804192
$\therefore R' = 639.43$	

B. Crossings on Straight Lines.

55. When a turnout enters a parallel main track by a second switch it becomes a *crossing*. As the switch angle is the same on both tracks a crossing on a straight line is a reversed curve between parallel tangents. Let HD and NK (fig. 17) be the centre lines of two parallel tracks, and HA and BK the direction of the switched rails. If now the tangent points A and B are fixed, the distance $AB = a$ may be measured, and also the perpendicular distance $BP = b$ between the tangents HP and BK . Then the common radius of the crossing ACB may be found by § 33; or if the radius of one part of the crossing is fixed, the second radius may be found by § 34. But if both frog angles are given, we have the two radii or the common radius of a crossing given, and it will then be necessary to determine the distance AB between the two tangent points.

56. **Problem.** Given the perpendicular distance $GN = b$ (fig. 17) between the centre lines of two parallel tracks, and the radii $EC = R$ and $CF = R'$ of a crossing, to find the chords AC and BC .

Solution. Draw EG perpendicular to the main track, and AL , CM , and BL' parallel to it. Denote the angle AEC by E . Then, since the angle $AEL = AHG = S$, we have $CEL = E + S$, and in the right triangle CEM (Tab. X. 2), $CE \cos. CEM = R \cos. (E + S) = EM = EL - LM$. But $EL = AE \cos. AEL = R \cos. S$, and $LM : L'M = AC : BC$. Now $AC : BC = EC : CF = R : R'$. Therefore, $LM : L'M = R : R'$, or $LM : LM + L'M = R : R + R'$; that is, $LM : b - 2d = R : R + R'$, whence $LM = \frac{R(b - 2d)}{R + R'}$. Substituting these values of EL and LM in the equation for $R \cos. (E + S)$, we have $R \cos. (E + S) = R \cos. S - \frac{R(b - 2d)}{R + R'}$,

$$\therefore \cos. (E + S) = \cos. S - \frac{b - 2d}{R + R'}$$

Having thus found $E + S$, we have the angle E and also its equal CFB . Then (§ 69)

$$\text{Fig. 10.} \quad AC = 2R \sin. \frac{1}{2} E; \quad BC = 2R' \sin. \frac{1}{2} E.$$

We have also $AB = AC + BC$, since AC and BC are in the same straight line (§ 32), or $AB = 2(R + R') \sin. \frac{1}{2} E$.

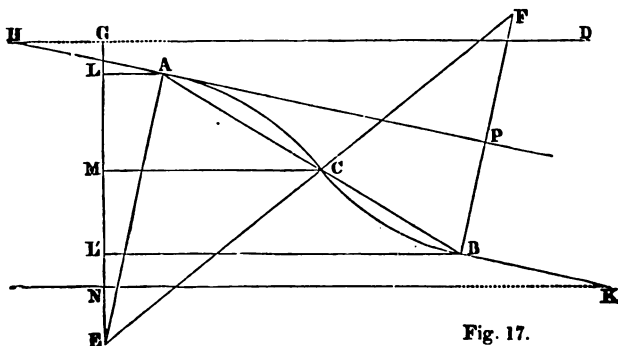


Fig. 17.

When the two radii are equal, the same formulæ apply by making $R' = R$. In this case, we have

$$\cos. (E + S) = \cos. S - \frac{b - 2d}{2R};$$

 $A C = B C = 2 R \sin. \frac{1}{2} E.$

Example. Given $d = .42$, $g = 4.7$, $S = 1^{\circ} 20'$, $b = 11$, and the angles of the two frogs each 7° , to find $AC = BC = \frac{1}{2} AB$. The common radius R , corresponding to $F = 7^{\circ}$, is found (§ 51) to be 593.5. Then $2R = 1187$, $b - 2d = 10.16$, and $10.16 \div 1187 = .00856$. Therefore, $\text{nat. cos.}(E + S) = .99973 - .00856 = .99117$; whence $E + S = 7^{\circ} 37' 15''$. Subtracting S , we have $E = 6^{\circ} 17' 15''$.
Next

$$\begin{array}{rcl} 2R = 1187 & & 3.074451 \\ \frac{1}{2}E = 3^{\circ}8'37\frac{1}{2}'' & \sin. & 8.739106 \\ \hline AC = 65.1 & & 1.813557 \end{array}$$

C. Turnout from Curves.

57. Problem. Given the radius R of the centre line of the main track and the frog angle F , to determine the position of the frog by means of the chord BF (figs. 18 and 19), and to find the radius R' of the centre line of the turnout.

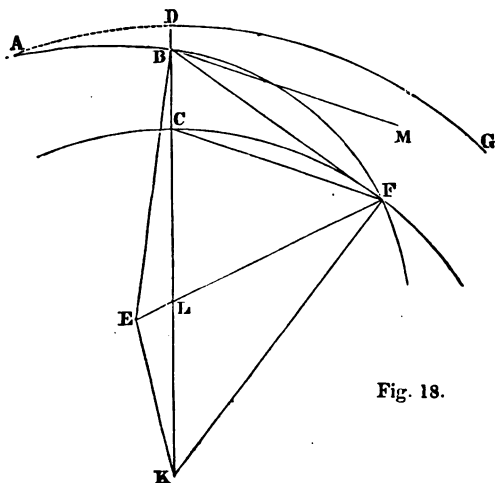


Fig. 18.

Solution. I. When the turnout is from the inside of the curve (fig. 18). Let AG and CF be the rails of the main track, AB the switch rail, and the arc BF the outer rail of the turnout, crossing the inside rail of the main track at F . Then, since the angle $E\hat{F}K$ has its sides perpendicular to the tangents of the two curves at F , it is equal to the acute angle made by the crossing rails, that is, $E\hat{F}K = F$. Also $E\hat{B}L = S$. The first step is to find the angle $B\hat{K}F$ denoted by K . To find this angle, we have in the triangle $B\hat{F}K$ (Tab. X. 14), $BK + KF : BK - KF = \tan. \frac{1}{2} (B\hat{F}K + F\hat{B}K) : \tan. \frac{1}{2} (B\hat{F}K - F\hat{B}K)$. But $BK = R + \frac{1}{2}g - d$, and $KF = R - \frac{1}{2}g$. Therefore, $BK + KF = 2R - d$, and $BK - KF = g - d$. Moreover, $B\hat{F}K = B\hat{F}E + E\hat{F}K = B\hat{F}E + F$, and $F\hat{B}K = E\hat{B}F - E\hat{B}K = B\hat{F}E - S$. Therefore, $B\hat{F}K - F\hat{B}K = F + S$. Lastly, $B\hat{F}K + F\hat{B}K = 180^\circ - K$. Substituting these values in the preceding proportion, we have $2R - d : g - d = \tan. (90^\circ - \frac{1}{2}K) : \tan. \frac{1}{2}(F + S)$.

or $\tan. (90^\circ - \frac{1}{2} K) = \frac{(2R - d) \tan. \frac{1}{2} (F + S)}{g - d}$. But $\tan. (90^\circ - \frac{1}{2} K) = \cot. \frac{1}{2} K = \frac{1}{\tan. \frac{1}{2} K}$;

$$\therefore \tan. \frac{1}{2} K = \frac{g - d}{(2R - d) \tan. \frac{1}{2} (F + S)}.$$

Next, to find the chord BF , we have, in the triangle BFC (Tab. X. 12), $BF = \frac{BC \sin. BCF}{\sin. BFC}$. But $BC = g - d$, and $BCF = 180^\circ - FCK = 180^\circ - (90^\circ - \frac{1}{2} K) = 90^\circ + \frac{1}{2} K$, or $\sin. BCF = \cos. \frac{1}{2} K$. Moreover, $BFC = \frac{1}{2} (F + S)$; for $BFK = KFC + BFC$, and $FBK = KCF - BFC = KFC - BFC$. Therefore, $BFK - FBK = 2BFC$. But, as shown above, $BFK - FBK = F + S$. Therefore, $2BFC = F + S$, or $BFC = \frac{1}{2} (F + S)$. Substituting these values in the expression for BF , we have

$$BF = \frac{(g - d) \cos. \frac{1}{2} K}{\sin. \frac{1}{2} (F + S)}.$$

Lastly, to find R' , we have (§ 68) $R' + \frac{1}{2} g = EF = \frac{\frac{1}{2} BF}{\sin. \frac{1}{2} BEF}$. But $BEF = BLF - EBL$, and $BLF = LFK + LKF = F + K$. Therefore, $BEF = F + K - S$, and

$$R' + \frac{1}{2} g = \frac{\frac{1}{2} BF}{\sin. \frac{1}{2} (F + K - S)}.$$

II. When the turnout is from the *outside* of the curve, the preceding solution requires a few modifications. In the present case, the angle $EFK' = F$ (fig. 19) and $EBL = S$. To find K , we have in the triangle BFK , $KF + BK : KF - BK = \tan. \frac{1}{2} (FBK + BFK) : \tan. \frac{1}{2} (FBK - BFK)$. But $KF = R + \frac{1}{2} g$, and $BK = R - \frac{1}{2} g + d$. Therefore, $KF + BK = 2R + d$, and $KF - BK = g - d$. Moreover, $FBK = 180^\circ - FBL = 180^\circ - (EBF - EBL) = 180^\circ - (EBF - S)$, and $BFK = 180^\circ - BFK' = 180^\circ - (BFE + EFK') = 180^\circ - (EBF + F)$. Therefore, $FBK - BFK = F + S$. Lastly, $FBK + BFK = 180^\circ - K$. Substituting these values in the preceding proportion, we have $2R + d : g - d = \tan. (90^\circ - \frac{1}{2} K) : \tan. \frac{1}{2} (F + S)$, or $\tan. (90^\circ - \frac{1}{2} K) = \frac{(2R + d) \tan. \frac{1}{2} (F + S)}{g - d}$. But $\tan. (90^\circ - \frac{1}{2} K) = \cot. \frac{1}{2} K = \frac{1}{\tan. \frac{1}{2} K}$;

$$\therefore \tan. \frac{1}{2} K = \frac{g - d}{(2R + d) \tan. \frac{1}{2} (F + S)}.$$

But $BEF = BLF - EBL$, and $BLF = LFK - LKF = F - K$. Therefore, $BEF = F - K - S$, and

$$R' + \frac{1}{2}g = \frac{\frac{1}{2}BF}{\sin. \frac{1}{2}(F - K - S)}.$$

Example. Given $g = 4.7$, $d = .42$, $S = 1^\circ 20'$, $R = 4583.75$, and $F = 7^\circ$, to find the chord BF and the radius R' of a turnout from the outside of the curve. Here

$g - d = 4.28$		0.631444	0.631444
$2R + d = 9167.92$	3.962271		
$\frac{1}{2}(F + S) = 4^\circ 10'$	tan. 8.862433		sin. 8.861283
		<u>2.824704</u>	<u>1.770161</u>
$\frac{1}{2}K = 22' 1.8''$	tan. 7.806740	cos. 9.999991	
$BF = 58.905$			<u>1.770152</u>
2		0.301030	
$\frac{1}{2}(F - K - S) = 2^\circ 27' 58.2''$	sin. 8.633766		
			<u>8.934796</u>
$R' + \frac{1}{2}g = 684.47$			<u>2.835356</u>
$\therefore R' = 682.12$			

58. Problem. To find mechanically the proper position of a given frog.

Solution. The method here is similar to that already given, when the turnout is from a straight line (§ 52). Draw BM (figs. 18 and 19) parallel to FC , and we have $FBM = BFC = \frac{1}{2}(F + S)$, as just shown (§ 57). This angle is to be laid off from BM ; but as F is the point to be found, the chord FC can be only estimated at first, and BM taken parallel to it, from which the angle $\frac{1}{2}(F + S)$ may be laid off by the method of § 52. In this case, however, the first measure on the arc is d , and not $2d$; since we have here to start from BM , and not from the rail. Having thus determined the point F approximately, BM may be laid off more accurately, and F found anew.

59. When frogs are cast to be kept on hand, it is desirable to have them of such a pattern that they will fall at the beginning or end of a certain rail; that is, the chord BF is known, and the angle F is required.

Problem. Given the position of a frog by means of the chord BF (figs. 14, 18, and 19), to determine the frog angle F .

Solution. The formula $BF = \frac{s-d}{\sin. \frac{1}{2}(F+S)}$, which is exact on straight lines (§ 50), and near enough on ordinary curves (§ 57, note), gives

$$\sin. \frac{1}{2}(F+S) = \frac{s-d}{BF}.$$

By this formula $\frac{1}{2}(F+S)$ may be found, and consequently F .

60. Problem. Given the radius R of the centre line of the main track, and the radius R' of the centre line of a turnout, to find the frog angle F , and the chord BF (figs. 18 and 19).

Solution. I. When the turnout is from the inside of the curve (fig. 18). In the triangle BEK find the angle BEK and the side EK . For this purpose we have $BE = R' + \frac{1}{2}g$, $BK = R + \frac{1}{2}g - d$, and the included angle $EBK = S$. Then in the triangle EFK we have EK , as just found, $EF = R' + \frac{1}{2}g$, and $FK = R - \frac{1}{2}g$. The frog angle $EFK = F$ may, therefore, be found by formula 15, Tab. X., which gives

$$\tan. \frac{1}{2}F = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

where s is the half sum of the three sides, a the side EK , and b and c the remaining sides.

Find also in the triangle EFK the angle FEK , and we have the angle $BEF = BEK - FEK$. Then in the triangle BEF we have (§ 69)

$$BF = 2(R' + \frac{1}{2}g) \sin. \frac{1}{2}BEF.*$$

II. When the turnout is from the outside of the curve (fig. 19). In the triangle BEK find the angle BEK and the side EK . For this purpose we have $BE = R' + \frac{1}{2}g$, $BK = R - \frac{1}{2}g + d$, and the included angle $EBK = 180^\circ - S$. Then in the triangle EFK we have EK , as just found, $EF = R' + \frac{1}{2}g$, and $FK = R + \frac{1}{2}g$. The angle EFK may, therefore, be found by formula 15, Tab. X., which gives $\tan. \frac{1}{2}EFK = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$. But the angle $EFK' = F$

* The value of BF may be more easily found by the approximate formula $BF = \frac{s-d}{\sin. \frac{1}{2}(F+S)}$, and generally with sufficient accuracy. See note to § 57. This remark applies also to BF in the second part of this solution.

$= 180^\circ - EFK$. Therefore $\frac{1}{2}F = 90^\circ - \frac{1}{2}EFK$, and $\cot. \frac{1}{2}F = \tan. \frac{1}{2}EFK$;

$$\therefore \cot. \frac{1}{2}F = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

where s is the half sum of the three sides, a the side EK , and b and c the remaining sides.

Find also in the triangle EFK the angle FEK , and we have the angle $BEF = FEK - BEK$. Then in the triangle BEF we have (§ 69)

$$BF = 2(R' + \frac{1}{2}g) \sin. \frac{1}{2}BEF.$$

Example. Given $g = 4.7$, $d = .42$, $S = 1^\circ 20'$, $R = 4583.75$, and $R' = 682.12$, to find F and the chord BF of a turnout from the outside of the curve. Here in the triangle BEK (fig. 19) we have $BE = R' + \frac{1}{2}g = 684.47$, $BK = R - \frac{1}{2}g + d = 4581.82$, and the angles $BEK + BKE = S = 1^\circ 20'$. Then

$$\begin{array}{rcl} BK - BE & = & 3897.35 \qquad 3.590769 \\ \frac{1}{2}(BEK + BKE) & = & 40' \qquad \tan. 8.065806 \\ & & \underline{1.656575} \\ BK + BE & = & 5266.29 \qquad 3.721505 \\ \frac{1}{2}(BEK - BKE)^* & = & 29.6029' \qquad \tan. 7.935070 \\ \therefore BEK & = & 1^\circ 9.6029' \end{array}$$

EK is now found by the formula $EK = \frac{BK \sin. EBK}{\sin. BEK}$, or $\log. EK = \log. 4581.82 + \log. \sin. 178^\circ 40' - \log. \sin. 1^\circ 9.6029' = 3.721491$, whence $EK = 5266.12$.

Then to find F , we have, in the triangle EFK , $s = \frac{1}{2}(5266.12 + 684.47 + 4586.10) = 5268.34$, $s - a = 2.22$, $s - b = 4583.87$, and $s - c = 682.24$.

$$\begin{array}{rcl} s - b & = & 4583.87 \qquad 3.661233 \\ s - c & = & 682.24 \qquad 2.833937 \\ & & \underline{6.495170} \\ s & = & 5268.34 \qquad 3.721674 \\ s - a & = & 2.22 \qquad \underline{0.346353} \\ & & 4.068027 \\ & & 2)2.427143 \\ \frac{1}{2}F & = & 3^\circ 36' \qquad \cot. 1.213571 \\ \therefore F & = & 7^\circ \end{array}$$

* This angle and the sine of $1^\circ 9.6029'$ below, are found by the method given in connection with Table XIII. If the ordinary interpolations had been used, we should have found $F = 7^\circ 7'$, whereas it should be 7° the answer is

To find FEK , we have s as before, but as a is here the side FK opposite the angle sought, we have $s - a = 682.24$, $s - b = 4583.87$, and $s - c = 2.22$. Then by means of the logarithms just used, we find $\frac{1}{2}FEK = 3^\circ 2' 45''$. Subtracting $\frac{1}{2}BEK = 34' 48''$, we have $\frac{1}{2}BEF = 2^\circ 27' 57''$. Lastly, $BF' = 1368.94 \sin. 2^\circ 27' 57'' = 58.897$.

The formula $BF' = \frac{s-d}{\sin. \frac{1}{2}(F+S)}$ (§ 57, note) would give $BF = 58.906$, and this value is even nearer the truth than that just found, owing, however, to no error in the formulæ, but to inaccuracies incident to the calculation.

61. If the turnout is to reverse, in order to join a track parallel to the main track, as ACB (fig. 20), it will be necessary to determine the reversing points C and B . These points will be determined, if we find the angles AEC and BFC , and the chords AC and CB .

62. **Problem.** Given the radius $DK = R$ (fig 20) of the centre line of the main track, the common radius $EC = CF = R'$ of the centre line of a turnout, and the distance $BG = b$ between the centre lines of the parallel tracks, to find the central angles AEC and BFC and the chords AC and BC .

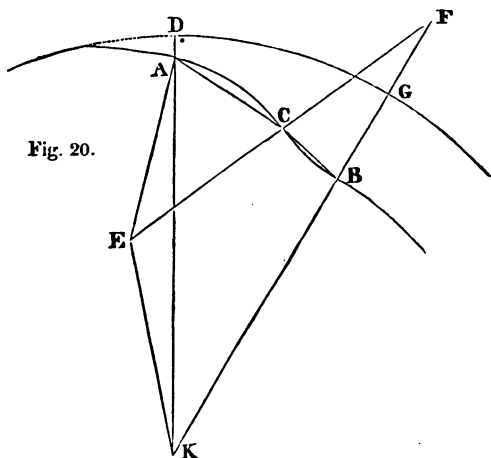


Fig. 20.

Solution. In the triangle AEK find the angle AEK and the side

E K For this purpose we have $AE = R'$, $AK = R - d$, and the included angle $EAK = S$. Or, if the frog angle has been previously calculated by § 60, the values of AEK and EK are already known.*

Find in the triangle EFK the angles EFK and FEK . For this purpose we have EK , as just found, $EF = 2R'$, and $FK = R + R' - b$. Then $AEC = AEK - FEK$, and $BFC = FEK$. Lastly, (§ 69)

$$AC = 2R \sin \frac{1}{2} AEC; \quad CB = 2R' \sin \frac{1}{2} BFC.$$

This solution, with a few obvious modifications, will apply, when the turnout is from the outside of a curve.

D. Crossings on Curves.

63. When a turnout enters a parallel main track by a second switch, it becomes a *crossing*. Then if the tangent points A and B (fig. 21) are fixed, the distance AB must be measured, and also the angles which AB makes with the tangents at A and B . The common radius of the crossing may then be found by § 40; or if one radius of the crossing is given, the other may be found by § 38. But if one tangent point A is fixed, and the common radius of the crossing is given, it will be necessary to determine the reversing point C and the tangent point B . These points will be determined, if we find the angles AEC and BFC , and the chords AC and CB .

64. Problem. Given the radius $DK = R$ (fig. 21) of the centre line of the main track, the common radius $EC = CF = R'$ of the centre line of a crossing, and the distance $DG = b$ between the centre lines of the parallel tracks, to find the central angles AEC and BFC and the chords AC and CB .

Solution. In the triangle AEK find the angle AEK and the side EK . For this purpose we have $AE = R'$, $AK = R - d$, and the included angle $EAK = S$.

Find in the triangle BFK the angle BFK and the side FK . For this purpose we have $BF = R'$, $BK = R - b + d$, and the included angle $FBK = 180^\circ - S$.

Find in the triangle EFK the angles FEK and EFK . For this

* The triangle AEK does not correspond precisely with BEK in § 60, A being on the centre line and B on the outer rail; but the difference is too slight to affect the calculations.

purpose we have $E K$ and $F K$ as just found, and $E F = 2 R'$. Then $A E C = A E K - F E K$, and $B F C = E F K - B F K$. Lastly (§ 69,)

$$\boxed{\text{P}} \quad A C = 2 R' \sin. \frac{1}{2} A E C; \quad C B = 2 R' \sin. \frac{1}{2} B F C.$$

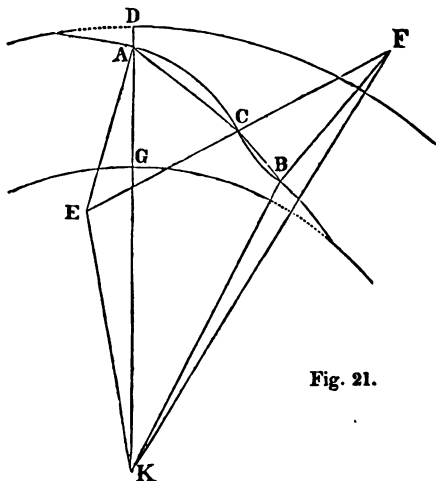


Fig. 21.

ARTICLE IV.—MISCELLANEOUS PROBLEMS.

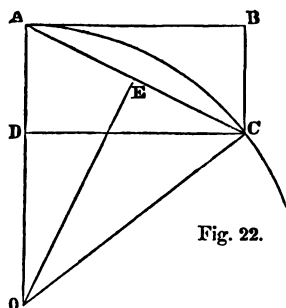
65. Problem. Given $A B = a$ (fig. 22) and the perpendicular $B C = b$, to find the radius of a curve that shall pass through C and the tangent point A .

Solution. Let O be the centre of the curve, and draw the radii $A O$ and $C O$ and the line $C D$ parallel to $A B$. Then in the right triangle $C O D$ we have $O C^2 = C D^2 + O D^2$. But $O C = R$, $C D = a$, and $O D = A O - A D = R - b$. Therefore, $R^2 = a^2 + (R - b)^2 = a^2 + R^2 - 2 R b + b^2$, or $2 R b = a^2 + b^2$;

$$\boxed{\text{P}} \quad \therefore R = \frac{a^2}{2b} + \frac{1}{2}b.$$

Example. Given $a = 204$ and $b = 24$, to find R . Here $R = \frac{204^2}{2 \times 24} + \frac{24}{2} = 867 + 12 = 879$.

66. Corollary 1. If R and b are given to find $AB = a$, that is, to determine the tangent point from which a curve of given radius



must start to pass through a given point, we have (§ 65) $2Rb = a^2 + b^2$, or $a^2 = 2Rb - b^2$;

$$\therefore a = \sqrt{b(2R - b)}.$$

Example. Given $b = 24$ and $R = 879$, to find a . Here $a = \sqrt{24(1758 - 24)} = \sqrt{41616} = 204$.

67. Corollary 2. If R and a are given, and b is required, we have (§ 65) $2Rb = a^2 + b^2$, or $b^2 - 2Rb = -a^2$. Solving this equation, we find for the value of b here required,

$$b = R - \sqrt{R^2 - a^2}.$$

68. Problem. Given the distance $AC = c$ (fig. 22) and the angle $BAC = A$, to find the radius R or deflection angle D of a curve, that shall pass through C and the tangent point A .

Solution. Draw OE perpendicular to AC . Then the angle $AOE = \frac{1}{2}AOC = BAC = A$ (§ 2, III.), and the right triangle AOE gives (Tab. X. 9) $AO = \frac{AE}{\sin. AOE}$;

$$\therefore R = \frac{\frac{1}{2}c}{\sin. A}.$$

To find D , we have (§ 9) $\sin. D = \frac{50}{R}$. Substituting for R its value just found, we have $\sin. D = 50 \div \frac{\frac{1}{2}c}{\sin. A}$;



$$\therefore \sin. D = \frac{100 \sin. A}{c}.$$

Example. Given $c = 285.4$ and $A = 5^\circ$, to find R and D . Here $R = \frac{142.7}{\sin. 5^\circ} = 1637.3$; and $\sin. D = \frac{100 \sin. 5^\circ}{285.4} = \frac{\sin. 5^\circ}{2.854} = \sin. 1^\circ 45'$ or $D = 1^\circ 45'$.

69. Problem. Given the radius R or the deflection angle D of a curve, and the angle $BAC = A$ (fig. 22), made by any chord with the tangent at A , to find the length of the chord $AC = c$.

Solution. If R is given, we have (§ 68) $R = \frac{\frac{1}{2}c}{\sin. A}$;



$$\therefore c = 2 R \sin. A.$$

If D is given, we have (§ 68) $\sin. D = \frac{100 \sin. A}{c}$;



$$c = \frac{100 \sin. A}{\sin. D}.$$

This formula is useful for finding the length of chords, when a curve is laid out by points two, three, or more stations apart. Thus, suppose that the curve AC is four stations long, and that we wish to find the length of the chord AC . In this case the angle $A = 4D$ and $c = \frac{100 \sin. 4D}{\sin. D}$. By this method Table II. is calculated.

Example. Given $R = 2455.7$ or $D = 1^\circ 10'$, and $A = 4^\circ 40'$, to find c . Here, by the first formula, $c = 4911.4 \sin. 4^\circ 40' = 399.59$. By the second formula, $c = \frac{100 \sin. 4^\circ 40'}{\sin. 1^\circ 10'} = 399.59$.

70. Problem. Given the angle of intersection $KCB = I$ (fig. 23), and the distance $CD = b$ from the intersection point to the curve in the direction of the centre, to find the tangent $AC = T$, and the radius $AO = R$.

Solution. In the triangle ADC we have $\sin. CAD : \sin. ADC = CD : AC$. But $CAD = \frac{1}{2} AOD = \frac{1}{2} I$ (§ 2, III. and VI.), and as the sine of an angle is the same as the sine of its supplement, $\sin. ADC = \sin. ADE = \cos. DAE = \cos. \frac{1}{2} I$. Moreover, $CD = b$ and $AC = T$. Substituting these values in the preceding proportion, we have $\sin. \frac{1}{2} I : \cos. \frac{1}{2} I = b : T$, or $T = \frac{b \cos. \frac{1}{2} I}{\sin. \frac{1}{2} I}$; whence (Tab. X. 33)

7

$$\sin D = \frac{100 \sin A}{c}.$$

Example. Given $r = 285.4$ and $A = 3^\circ$, to find R and D . Here $R = \frac{100}{\sin 3^\circ} = 1917.3$; and $\sin D = \frac{100 \sin 3^\circ}{285.4} = \frac{\sin 3^\circ}{2.854} = \sin 1^\circ 49'$ or $D = 1^\circ 49'$.

59. Problem. Given the radius R or the deflection angle D of a curve, and the angle $EAC = A$ (fig. 22), made by any chord with the tangent at A , to find the length of the chord $AC = c$.

Solution. If R is given, we have (§ 48) $R = \frac{\frac{1}{2}c}{\sin A}$;

8

$$\therefore c = 2R \sin A.$$

If D is given, we have (§ 68) $\sin D = \frac{100 \sin A}{c}$;

9

$$\therefore c = \frac{100 \sin A}{\sin D}.$$

This formula is useful for finding the length of chords, when a curve is laid out by points two, three, or more stations apart. Thus, suppose that the curve AC is four stations long, and that we wish to find the length of the chord AC . In this case the angle $A = 4D$ and $c = \frac{100 \sin 4D}{\sin D}$. By this method Table II. is calculated.

Example. Given $R = 2455.7$ or $D = 1^\circ 10'$, and $A = 4^\circ 40'$, to find c . Here, by the first formula, $c = 4911.4 \sin 4^\circ 40' = 399.59$.

By the second formula, $c = \frac{100 \sin 4^\circ 40'}{\sin 1^\circ 10'} = 399.59$.

70. Problem. Given the angle of intersection $KCB = I$ (fig. 23), and the distance $CD = b$ from the intersection point to the curve in the direction of the centre, to find the tangent $AC = T$, and the radius $AO = R$.

Solution. In the triangle ADC we have $\sin CAD : \sin ADC = CD : AC$. But $CAD = \frac{1}{2} AOD = \frac{1}{2} I$ (§ 2, III. and VI.), and as the sine of an angle is the same as the sine of its supplement, $\sin ADC = \sin ADE = \cos DAE = \cos \frac{1}{2} I$. Moreover, $CD = b$ and $AC = T$. Substituting these values in the preceding proportion, we have $\sin \frac{1}{2} I : \cos \frac{1}{2} I = b : T$, or $T = \frac{b \cos \frac{1}{2} I}{\sin \frac{1}{2} I}$; whence (Tab. X. 33)

Example. Given $I = 20^\circ 16'$, $a = 600$, and $b = 80$, to find x and R . Here $c = 600 \cos. 10^\circ 8' = 590.64$, $2c - b = 1101.28$, and $x = \sqrt{80 \times 1101.28} = 296.82$. Then $T = 600 + 296.82 = 896.82$, and $R = 896.82 \cot. 10^\circ 8' = 5017.82$.

73. Problem. (Given the tangent AC (fig. 25), and the chord AB , uniting the tangent points A and B , to find the radius $AO = R$.)

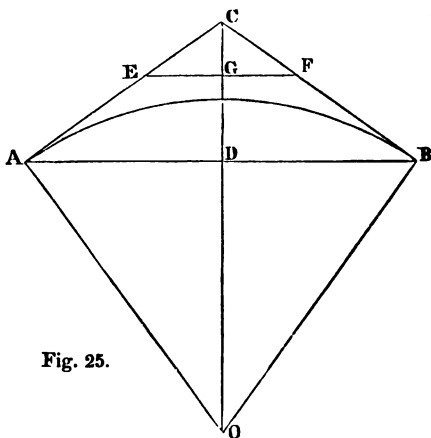


Fig. 25.

Solution. Measure or calculate the perpendicular CD . Then if CD be produced to the centre O , the right triangles ADC and CAO , having the angle at C common, are similar, and give $CD : AD = AC : AO$, or



$$R = \frac{AD \times AC}{CD}.$$

If it is inconvenient to measure the chord AB , a line EF , parallel to it, may be obtained by laying off from C equal distances CE and CF . Then measuring EG and GC , we have, from the similar triangles EGC and CAO , $CG : GE = AC : AO$, or $R = \frac{GE \times AC}{CG}$.

Example. Given $AC = 246$ and $AD = 240$, to find R . Here $CD = 54$, and $R = \frac{240 \times 246}{54} = 1093.33$.

74. Problem. Given the radius $AO = R$ (fig. 25), to find the tangent $AC = T$ of a curve to unite two straight lines given on the ground.

Solution. Lay off from the intersection C of the given straight lines any equal distances CE and CF . Draw the perpendicular CG to the middle of EF , and measure GE and CG . Then the right triangles EGC and CAO , having the angle at C common, are similar, and give $GE : CG = AO : AC$, or

$$T = \frac{CG \times AO}{GE}.$$

By this problem and the preceding one, the radius or tangent points of a curve may be found without an instrument for measuring angles.

Example. Given $R = 1093\frac{1}{2}$, $GE = 80$, and $CG = 18$, to find T .

Here $T = \frac{18 \times 1093\frac{1}{2}}{80} = 246$.

75. Problem. To find the angle of intersection I of two straight lines, when the point of intersection is inaccessible, and to determine the tangent points, when the length of the tangents is given.

Solution. I. To find the angle of intersection I . Let AC and CV (fig. 26) be the given lines. Sight from some point A on one line to a point B on the other, and measure the angles CAB and TBV . These angles make up the change of direction in passing from one tangent to the other. But the angle of intersection (§ 2) shows the change of direction between two tangents, and it must, therefore, be equal to the sum of CAB and TBV , that is,

$$I = CAB + TBV$$

But if obstacles of any kind render it necessary to pass from AC to BV by a broken line, as $ADEFB$, measure the angles CAD , NDE , PEF , RFB , and SBV , observing to note those angles as minus which are laid off contrary to the general direction of these angles. Thus the general direction of the angles in this case is to the right; but the angle PEF lies to the left of DE produced, and is therefore to be marked minus. The angles to be measured show the successive changes of direction in passing from one tangent to the other. Thus CAD shows the change of direction between the first tangent and AD , NDE shows the change between AD produced and DE , PEF the change between DE produced and EF , RFB the change between EF produced and FB , and, lastly, SBV the change between BF pro-

duced and the second tangent. But the angle of intersection (§ 2) shows the change of direction in passing from one tangent to another, and it must, therefore, be equal to the sum of the partial changes measured, that is,

$$I = CAD + NDE - PEF + RFB + SBV.$$

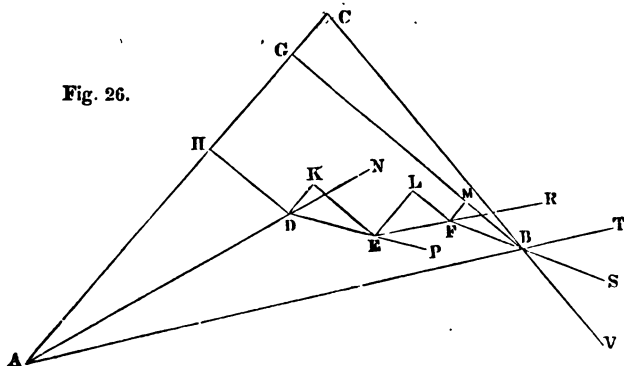


Fig. 26.

II. To determine the tangent points. This will be done if we find the distances AC and BC ; for then any other distances from C may be found. It is supposed that the distance AB , or the distances AD , DE , EF , and FB have been measured.

If one line AB connects A and B , find AC and BC in the triangle ABC . For this purpose we have one side AB and all the angles.

If a broken line $ADEFB$ connects A and B , let fall a perpendicular BG from B upon AC , produced if necessary, and find AG and BG by the usual method of working a traverse. Thus, if AC is taken as a meridian line, and DK , EL , and FM are drawn parallel to AC , and DH , EK , and FL are drawn parallel to BG , the difference of latitude AG is equal to the sum of the partial differences of latitude AH , DK , EL , and FM , and the departure BG is equal to the sum of the partial departures DH , EK , FL , and BM . To find these partial differences of latitude and departures, we have the distances AD , DE , EF , and FB , and the bearings may be obtained from the angles already measured. Thus the bearing of AD is CAD , the bearing of DE is $KDE = KDN + NDE = CAD + NDE$, the bearing of EF is $LEF = LEP - PEF = KDE - PEF$, and the

bearing of FB is $MFB = MFR + RFB = LEF + RFB$; that is, the bearing of each line is equal to the *algebraic sum* of the preceding bearing and its own change of direction. The differences of latitude and the departures may now be obtained from a traverse table, or more correctly by the formulæ:

Diff. of lat. = dist. $\times$ cos. of bearing; dep. = dist. $\times$ sin. of bearing

Thus, $AH = AD \cos. CAD$, and $DH = AD \sin. CAD$.

Having found AG and BG , we have, in the right triangle BGC , (Tab. X. 9) $GC = BG \cot. BCG$, and $BC = \frac{BG}{\sin. BCG}$. But $BCG = 180^\circ - I$. Therefore, $\cot. BCG = -\cot. I$, and $\sin. BCG = \sin. I$. Hence $GC = -BG \cot. I$, and $BC = \frac{BG}{\sin. I}$. Then, since $AC = AG + GC$, we have

$$\text{FIG. 1} \quad AC = AG - BG \cot. I; \quad BC = \frac{BG}{\sin. I}.$$

When I is between 90° and 180° , as in the figure, $\cot. I$ is negative, and $-BG \cot. I$ is, therefore, positive. When I is less than 90° , G will fall on the other side of I ; but the same formula for AC will still apply; for $\cot. I$ is now positive, and consequently, $-BG \cot. I$ is negative, as it should be, since, in this case, AC would equal AG minus GC .

Example. Given $AD = 1200$, $DE = 350$, $EF = 300$, $FB = 310$, $CAD = 20^\circ$, $NDE = 44^\circ$, $PEF = -25^\circ$, $RFB = 31^\circ$, and $SBV = 30^\circ$, to find the angle of intersection I , and the distances AC and BC .

Here $I = 20^\circ + 44^\circ - 25^\circ + 31^\circ + 30^\circ = 100^\circ$. To find AG and BG , the work may be arranged as in the following table:—

Angles to the Right.	Bearings.	Distances.	N.	E.
0				
20	N. 20° E.	1200	1127.63	410.42
44	64	350	153.43	314.58
-25	39	300	233.14	188.80
31	70	310	106.03	291.30
			1620.23	1205.10

The first column contains the observed angles. The second contains the bearings, which are found from the angles of the first column, i

the manner already explained. AC is considered as running north from A , and the bearings are, therefore, marked N. E. The other columns require no explanation. We find $AG = 1620.23$, and $BG = 1205.10$. Then $GC = -BG \cot. I = -1205.1 \times \cot. 100^\circ = 212.49$. This value is positive, because it is the product of two negative factors, $\cot. 100^\circ$ being the same as $-\cot. 80^\circ$, a negative quantity. Then $AC = AG + GC = 1620.23 + 212.49 = 1832.72$, and $BC = \frac{1205.1}{\sin. 100^\circ} = 1223.69$. Having thus found the distances of A and B from the point of intersection, we can easily fix the tangent points for tangents of any given length.

76. Problem. *To lay out a curve, when an obstruction of any kind prevents the use of the ordinary methods.*

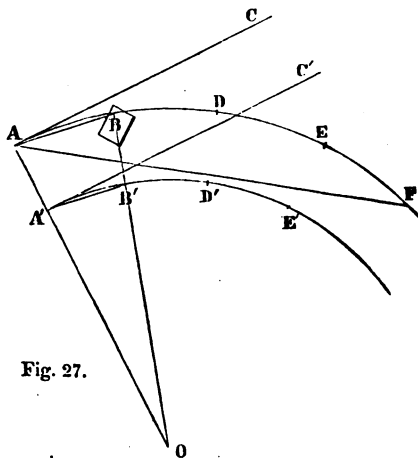


Fig. 27.

Solution. First Method. Suppose the instrument to be placed at A (fig. 27), and that a house, for instance, covers the station at B , and also obstructs the view from A to the stations at D and E . Lay off from A C , the tangent at A , such a multiple of the deflection angle D , as will be sufficient to make the sight clear the obstruction. In the figure it is supposed that $4D$ is the proper angle. The sight will then pass through F , the fourth station from A , and this station will be determined by measuring from A the length of the chord AF , found by

§ 69 or by Table II. From the station at F the stations at D and E may afterwards be fixed, by laying off the proper deflections from the tangent at F .

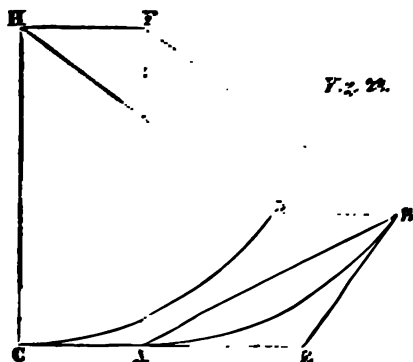
Second Method. This consists in running an auxiliary curve parallel to the true curve, either inside or outside of it. For this purpose lay off perpendicular to AC , the tangent at A , a line AA' of any convenient length, and from A' a line $A'C'$ parallel to AC . Then $A'C'$ is the tangent from which the auxiliary curve $A'E'$ is to be laid off. The stations on this curve are made to correspond to stations of 100 feet on the true curve, that is, a radius through B' passes through B , a radius through D' passes through D , &c. The chord $A'B'$ is, therefore, parallel to AB , and the angle $C'A'B' = CAB$; that is, the deflection angle of the auxiliary curve is equal to that of the true curve. It remains to find the length of the auxiliary chords $A'B'$, $B'D'$, &c. Call the distance $AA' = b$. Then the similar triangles ABO and $A'B'O$ give $AO : A'O = AB : A'B'$, or $R : R - b = 100 : A'B'$. Therefore, $A'B' = \frac{100(R-b)}{R} = 100 - \frac{100b}{R}$. If the auxiliary curve were on the outside of the true curve, we should find in the same way $A'B' = 100 + \frac{100b}{R}$. It is well to make b an aliquot part of R ; for the auxiliary chord is then more easily found. Thus, if n is any whole number, and we make $b = \frac{R}{n}$, we have $A'B' = 100 \pm \frac{100b}{R} = 100 \pm \frac{100}{n}$. If, for example, $b = \frac{R}{100}$, we have $n = 100$, and $A'B' = 100 \pm 1 = 101$ or 99 . When the auxiliary curve has been run, the corresponding stations on the true curve are found, by laying off in the proper direction the distances BB' , DD' , &c., each equal to b .

77. Problem. Having run a curve AB (fig. 28), to change the tangent point from A to C , in such a way that a curve of the same radius may strike a given point D .

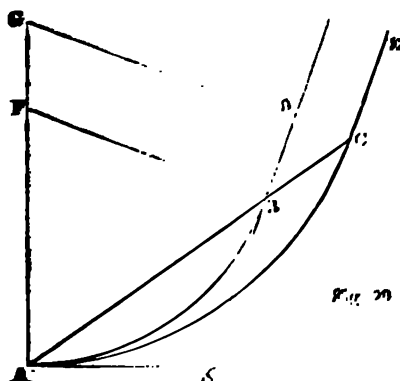
Solution. Measure the distance BD from the curve to D in a direction parallel to the tangent CE . This direction may be sometimes judged of by the eye, or found by the compass. A still more accurate way is to make the angle DBE equal to the intersection angle at E , or to twice BAE , the total deflection angle from A to B ; or if A can be seen from B , the angle DBA may be made equal to BAE .

Measure on the tangent (backward or forward, as the case may be) a distance $AC = BD$, and C will be the new tangent point required. For, if CH be drawn equal and parallel to AF , we have FH equal and par-

allel to AC , and therefore equal and parallel to BD . Hence $DH = BF = AF = CH$ and DH being equal to CH a circle of radius CH from the tangent point C must pass through D .



72. Problem. Having in a curve AC of radius R on deflection angle D , terminating in a tangent CE , to find the radius R' on deflection angle D' of a curve AB that shall terminate in a given parallel tangent CE .



Solution. Since the radii BF and CE are perpendicular to the parallel tangents CE and BD , they are parallel, and the angle $ACG = AFB$. Therefore, ACG the half-supplement of ACB is equal to

This problem serves to find the radius of a curve on a track already laid. For if from any point C on the curve we measure two equal chords AC and BC , and also the perpendicular CD from C upon the whole chord AB , we have the data of this problem.

80. Problem. To draw a tangent FG (fig. 30) to a given curve from a given point F .

Solution. On any straight line FA , which cuts the curve in two points, measure FC and FA , the distances to the curve. Then, by Geometry,



$$FG = \sqrt{FC \times FA}.$$

This length being measured from F , will give the point G . When FG exceeds the length of the chain, the direction in which to measure it, so that it will just touch the curve, may be found by one or two trials.

81. **Problem.** Having found the radius $AO = R$ of a curve (fig. 31), to substitute for it two radii $AE = R_1$ and $DF = R_2$, the longer of which AE or BE' is to be used for a certain distance only at each end of the curve.

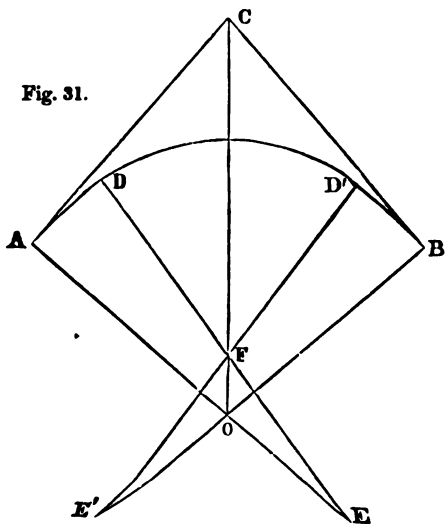


Fig. 31.

Solution. A

of any length which may be thou

proper, and find (§ 9) the corresponding deflection angle D_1 . Suppose that each of the curves AD and BD' is 100 feet long. Then drawing CO , we have, in the triangle FOE , $OE:FE = \sin. OFE : \sin. FOE$. But the side $OE = AE - AO = R_1 - R$, $FE = DE - DF = R_1 - R_2$, the angle $FOE = 180^\circ - AOC = 180^\circ - \frac{1}{2}I$, and the angle $OFE = AOF - OEF = \frac{1}{2}I - 2D_1$, since $OEF = 2D_1$, (§ 7). Substituting these values, and recollecting that $\sin. (180^\circ - \frac{1}{2}I) = \sin. \frac{1}{2}I$, we have $R_1 - R : R_1 - R_2 = \sin. (\frac{1}{2}I - 2D_1) : \sin. \frac{1}{2}I$. Hence

$$\text{☞} \quad R_1 - R_2 = \frac{(R_1 - R) \sin. \frac{1}{2}I}{\sin. (\frac{1}{2}I - 2D_1)}.$$

R_2 is then easily found, and this will be the radius from D to D' , or until the central angle $DFD' = I - 4D_1$.

The object of this problem is to furnish a method of flattening the extremities of a sharp curve. It is not necessary that the first curve should be just 100 feet long; in a long curve it may be longer, and in a short curve shorter. The value of the angle at E will of course change with the length of AD , and this angle must take the place of $2D_1$ in the formula. The longer the first curve is made, the shorter the second radius will be. It must also be borne in mind, in choosing the first radius, that the longer the first radius is taken, the shorter will be the second radius.

Example. Given $R = 1146.28$ and $I = 45^\circ$, to find R_2 , if R_1 is assumed = 1910.08, and AD and BD' each 100. Here, by Table L, $D_1 = 1^\circ 30'$. Then

$R_1 - R = 763.8$	2.882980
$\frac{1}{2}I = 22^\circ 30'$	sin. 9.582840
	2.465820
$\frac{1}{2}I - 2D_1 = 19^\circ 30'$	sin. 9.523495
	2.942325
$\therefore R_2 = R_1 - 875.64 = 1034.44$	

82. Problem. To locate the second branch of a compound or reversed curve from a station on the first branch.

Solution. Let AB (fig. 32) be the first branch of a compound curve, and D its deflection angle, and let it be required to locate the second branch AB' , whose deflection angle is D' , from some station B on AB .

Let n be the number of stations from A to B , and n' the number of stations from A to any station B' on the second branch. Represent by V the angle $A B B'$, which it is necessary to lay off from the chord $B A$ to strike B' . Let the corresponding angle $A B' B$ on the other curve be repre-

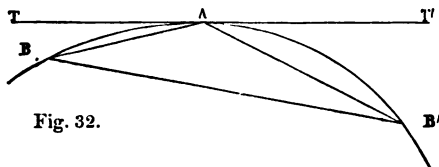


Fig. 32.

sented by V' . Then we have $V + V' = 180^\circ - B A B'$. But if $T T'$ be the common tangent at A , we have $T A B + T' A B' = n D + n' D' = 180^\circ - B A B'$. Therefore, $V + V' = n D + n' D'$. Next in the triangle $A B B'$ we have $\sin. V' : \sin. V = A B : A B'$. But $A B : A B' = n : n'$, nearly, and $\sin. V' : \sin. V = V' : V$, nearly. Therefore we have approximately $V' : V = n : n'$, or $V' = \frac{n}{n'} V$. Substituting this value of V' in the equation for $V + V'$, we have $V + \frac{n}{n'} V = n D + n' D'$. Therefore, $n' V + n V = n' (n D + n' D')$, or

$$V = \frac{n' (n D + n' D')}{n + n'}.$$

The same reasoning will apply to *reversed curves*, the only change being that in this case $V + V' = n D - n' D'$, and consequently

$$V = \frac{n' (n D - n' D')}{n + n'}.$$

When in this formula $n' D'$ becomes greater than $n D$, V becomes *minus*, which signifies that the angle V is to be laid off above $B A$ instead of below.

This problem is particularly useful, when the tangent point of a curve is so situated, that the instrument cannot be set over it. The same method is applicable, when the curve $A B'$ starts from a straight line; for then we may consider $A B'$ as the second branch of a compound curve, of which the straight line is the first branch, having its radius equal to infinity, and its deflection angle $D = 0$. Making $D = 0$, the formula for V becomes

$$V = \frac{n'^2 D'}{n + n'}.$$

When n and n' are each 1, the formula for V is in all cases exact, for then the supposition that $V' : V = n : n'$ is strictly true, since AB will equal AB' , and V and V' , being angles at the base of an isosceles triangle, will also be equal. Making n and n' equal to 1, we have

$$V = \frac{1}{2} (D + D').$$

When the curve starts from a straight line, this formula becomes, by making $D = 0$,

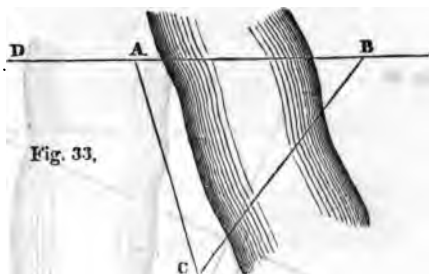
$$V = \frac{1}{2} D'.$$

We have seen that when n or n' is more than 1, the value of V is only approximate. It is, however, so near the truth, that when neither n nor n' exceeds 3, the error in curves up to 5° or 6° varies from a fraction of a second to less than half a minute. The exact value of V might of course be obtained by solving the triangle ABB' , in which the sides AB and AB' may be found from Table II., and the included angle at A is known. The extent to which these formulae may be safely used may be seen by the following table, which gives the approximate values of V for several different values of n , n' , D , and D' , and also the error in each case.

Compound Curves.						Reversed Curves.					
n .	D .	n' .	D' .	V .	Error.	n .	D .	n' .	D' .	V .	Error.
1	0	5	1	4 10	0.9	1	3	4	3	7 12	27.2
1	0	5	3	12 30	25.3	2	3	4	3	4 0	23.5
2	0	3	3	5 24	22.1	3	3	4	3	1 42 ⁵ ₇	8.3
3	0	3	3	4 30	29.7	3	$\frac{1}{2}$	3	3	3 45	24.0
1	1	5	3	13 20	18.6	2	1	1	4	0 40	0.1
2	$\frac{1}{2}$	1	3	1 20	0.7	2	1	4	2	4 0	11.0
2	2	3	3	7 48	15.0	1	6	2	6	4 0	23.5
2	2	4	3	10 40	24.7	1	5	3	5	7 30	51.8
3	3	3	4	10 30	54.0	2	3	5	3	6 25 ⁵ ₇	52.8

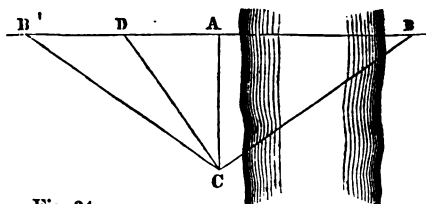
As the given quantities are here arranged, the approximate values of V are all too great; but if the columns n and n' and the columns D and D' were interchanged, and V calculated, the approximate values of V would be just as much too small, the column of errors remaining the same.

83. Problem. *To measure the distance across a river on a given straight line.*



Solution. First Method. Let AB (fig. 33) be the required distance. Measure a line AC along the bank, and take the angles BAC and ACB . Then in the triangle ABC we have one side and two angles to find AB .

If AC is of such a length that an angle $ACB = \frac{1}{2} DAC$ can be laid off to a point on the farther side, we have $ABC = \frac{1}{2} DAC = ACB$. Therefore, without calculation, $AB = AC$.



Second Method. Lay off AC (fig. 34) perpendicular to AB . Measure AC , and at C lay off CD perpendicular to the direction CB , and meeting the line of AB in D . Measure AD . Then the triangles ACD and ABC are similar, and give $AD : AC = AC : AB$.

Therefore, $AB = \frac{AC^2}{AD}$.

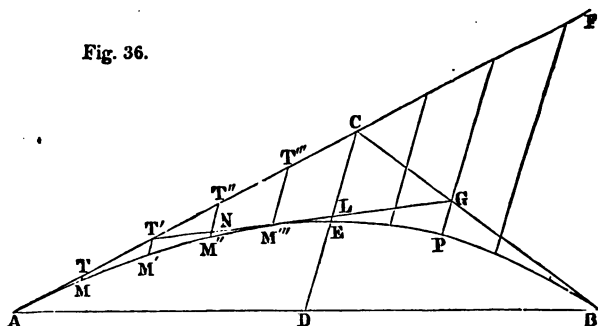
If from C , determined as before, the angle ACB' be laid off equal to ACB , we have, without calculation, $AB = AB'$.

Third Method. Measure a line AD (fig. 35) in an oblique direction from the bank, and fix its middle point C . From any convenient point E in the line of AB , measure the distance EC , and product

V. If any two tangents, as AC and BC , be bisected in M and F the line MF , joining the points of bisection, will be a new tangent, its middle point E being the point of tangency.

85. Problem. Given the tangents AC and BC , equal or unequal (fig. 36,) and the chord AB , to lay out a parabola by tangent deflections.

Fig. 36.



Solution. Bisect AB in D , and measure CD and the angle ACD ; or calculate CD^* and ACD from the original data. Divide the tangent AC into any number n of equal parts, and call the deflection $T'M$ for the first point a . Then (§ 84, II.) the deflection for the second point will be $T'M' = 4a$, for the third point $T''M'' = 9a$, and so on to the n th point or C , where it will be n^2a . But the deflection at this last point is $CE = \frac{1}{2}CD$ (§ 84, I.). Therefore, $n^2a = CE$, and

$$a = \frac{CE}{n^2}.$$

Having thus found a , we have also the succeeding deflections $4a$, $9a$, $16a$, &c. Then laying off at T , T' , &c. the angles ATM , $AT'M'$, &c. each equal to ACD , and measuring down the proper deflections, just found, the points M , M' , &c. of the curve will be determined.

The curve may be finished by laying off on AC produced n parts equal to those on AC , and the proper deflections will be, as before, a multiplied by the square of the number of parts from A . But an

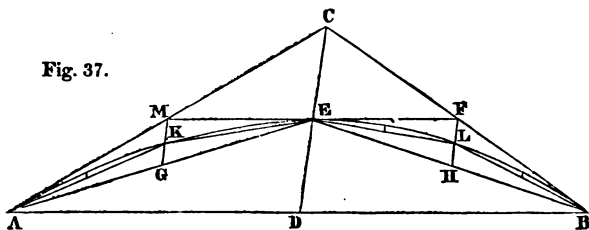
* Since CD is drawn to the middle of the base of the triangle ABC , we have, by geometry, $CD^2 = \frac{1}{4}(AC^2 + BC^2) - AD^2$.

easier way generally of finding points beyond E is to divide the second tangent BC into equal parts, and proceed as in the case of AC . If the number of parts on BC be made the same as on AC , it is obvious that the deflections from both tangents will be of the same length for corresponding points. The angles to be laid off from BC must, of course, be equal to BCD .

The points or stations thus found, though corresponding to equal distances on the tangents, are not themselves equidistant. The length of the curve is obtained by actual measurement.

86. Problem. *Given the tangents AC and BC , equal or unequal, (fig. 37,) and the chord AB , to lay out a parabola by middle ordinates.*

Fig. 37.



Solution. Bisect AB in D , draw CD , and its middle point E will be a point on the curve (§ 84, I.). DE is the first middle ordinate, and its length may be measured or calculated. To the point E draw the chords AE and BE , lay off the second middle ordinates GK and HL , each equal to $\frac{1}{4} DE$ (§ 84, III.), and K and L are points on the curve. Draw the chords AK , KE , EL , and LB , and lay off third middle ordinates, each equal to one fourth the second middle ordinates, and four additional points on the curve will be determined. Continue this process, until a sufficient number of points is obtained.

87. Problem. *To draw a tangent to a parabola at any station.*

Solution. I. If the curve has been laid out by tangent deflections (§ 85), let M''' (fig. 36) be the station, at which the tangent is to be drawn. From the preceding or succeeding station, lay off, parallel to CD , a distance $M''N$ or EL equal to a , the first tangent deflection (§ 85), and $M'''N$ or $M'''L$ will be the required tangent. The same thing may be done by laying off from the second station a distance $M'T' = 4a$, or at the third station a distance $GP = 9a$; for the

required tangent will then pass through T' or G . It will be seen, also, that the tangent at M''' passes through a point on the tangent at A corresponding to half the number of stations from A to M''' ; that is, M''' is *four* stations from A , and the tangent passes through T' , the *second* point on the tangent AC . In like manner, M''' is *six* stations from B , and the tangent passes through G , the *third* point on the tangent BC .

II. If the curve has been laid out by middle ordinates (§ 86), the tangent deflection for one station is equal to the last middle ordinate made use of in laying out the curve. For if the tangent AC (fig. 37) were divided into four equal parts corresponding to the number of stations from A to E , the method of tangent deflections would give the same points on the curve, as were obtained by the method of § 86. In this case, the tangent deflection for one station would be $a = \frac{1}{16} CE = \frac{1}{16} DE$; but the last middle ordinate was made equal to $\frac{1}{4} GK$ or $\frac{1}{16} DE$. Therefore, a is equal to the last middle ordinate, and a tangent may be drawn at any station by the first method of this section.

A tangent may also be drawn at the extremity of any middle ordinate, by drawing a line through this extremity, parallel to the chord of that ordinate (§ 84, IV.).

88. In laying out a parabola by the method in § 85, it may sometimes be impossible or inconvenient to lay off all the points from the original tangents. A new tangent may then be drawn by § 87 to any station already found, as at M''' (fig. 36), and the tangent deflections $a, 4a, 9a$, &c. may be laid off from this tangent, precisely as from the first tangent. These deflections must be parallel to CD , and the distances on the new tangent must be equal to $T'N$ or NM''' , which may be measured.

89. Problem. *Given the tangents AC and BC , equal or unequal, (fig 38,) to lay out a parabola by bisecting tangents.*

Solution. Bisect AC and BC in D and F , join DF , and find E , the middle point of DF . E will be a point on the curve (§ 84, V.). We have now two pairs of what may be called second tangents, AD and DE , and EF and FB . Bisect AD in G and DE in H , join GH , and its middle point M will be a point on the curve. Bisect EF and FB in K and L , join KL , and its middle point N will be a point on the curve. We have now four pairs of third tangents, AG and GM , MH and HE , EK and KN , and NL and LB . Bisect each pair in rn , join the points of bisection, and the middle points of the joining

lines will be four new points, M' , M'' , N'' , and N' . The same method may be continued, until a sufficient number of points is obtained.

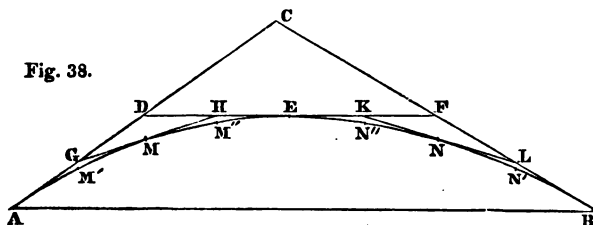


Fig. 38.

90. Problem. Given the tangents AC and BC , equal or unequal, (fig. 39,) and the chord AB , to lay out a parabola by intersections.

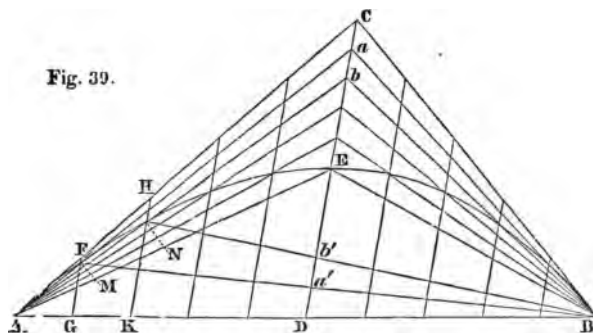


Fig. 39.

Solution. Bisect AB in D , draw CD , and bisect it in E . Divide the tangents AC and BC , the half-chords AD and DB , and the line CE , into the same number of equal parts; five, for example. Then the intersection M of Aa and FG will be a point on the curve. For $FM = \frac{1}{5} Ca$, and $Ca = \frac{1}{5} CE$. Therefore, $FM = \frac{1}{25} CE$, which is the proper deflection from the tangent at F to the curve (§ 85). In like manner, the intersection N of Ab and HK may be shown to be a point on the curve, and the same is true of all the similar intersections indicated in the figure.

If the line DE were also divided into five equal parts, the line Aa would be intersected in M on the curve by a line drawn from B through a' , the line Ab would be intersected in N on the curve by a line drawn

same tangent deflections as were laid off from AK , namely, $M5 = 22.5$, $N6 = 10$, and $O7 = 2.5$. To lay off the second half of the curve by middle ordinates (§ 86), measure $EB = 784.49$. Bisect EB in P , and lay off the middle ordinate $PR = \frac{1}{4}DE = 40$. Measure $ER = 386.08$, and $BR = 402.31$, and lay off the middle ordinates ST and VW , each equal to $\frac{1}{4}PR = 10$. By measuring the chords ET , TR , RW , and WB , and laying off an ordinate from each, equal to 2.5, four additional points might be found.

ARTICLE II. — RADIUS OF CURVATURE.

92. THE curvature of circular arcs is always the same for the same arc, and in different arcs varies inversely as the radii of the arcs. Thus, the curvature of an arc of 1,000 feet radius is double that of an arc of 2,000 feet radius. The curvature of a parabola is continually changing. In fig. 39, for example, it is least at the tangent point A , the extremity of the longest tangent, and increases by a fixed law, until it becomes greatest at a point, called the vertex, where a tangent to the curve would be perpendicular to the diameter. From this point to B it decreases again by the same law. We may, therefore, consider a parabola to be made up of a succession of infinitely small circular arcs, the radii of which continually increase in going from the vertex to the extremities. The radius of the circular arc, corresponding to any part of a parabola, is called the *radius of curvature* at that point.

If a parabola forms part of the line of a railroad, it will be necessary, in order that the rails may be properly curved (§ 28), to know how the radius of curvature may be found. It will, in general, be necessary to find the radius of curvature at a few points only. In short curves it may be found at the two tangent points and at the middle station, and in longer curves at two or more intermediate points besides. The rails curved according to the radius at any point should be sufficient in number to reach, on each side of that point, half-way to the next point.

93. Problem. *To find the radius of curvature at certain stations on a parabola.*

Solution. Let AEB (fig. 41) be any parabola, and let it be required to find the radii of curvature at a certain number of stations

101. The field notes are kept in the following form. The *first* column in the table contains the *stations*, and also the benches marked *B.*, and the turning points marked *t. p.*, except when coincident with a station. The *second* column contains the *plus sights*; the *third* column shows the *height of the instrument*; the *fourth* contains the *minus sights*; and the *fifth* contains the *heights* of the points in the first column.

Station	+ S.	H. I.	- S.	H.
B.	3.125			25.000
0		28.125	3.413	24.712
1			3.102	25.023
2			3.827	24.298
3			4.816	23.309
4			6.952	21.173
5	2.740		9.016	19.109
6		21.849	3.311	18.538
7			4.027	17.822
8			3.824	18.025
9			2.516	19.333
B.			2.635	19.214
10			0.314	21.535

The height of the bench is set down as assumed above, namely, 25 feet; the first plus sight is set opposite *B.*, on which point it was taken, and, being added to the height in the same line, gives the height of the instrument, which is set opposite 0; the minus sights are set opposite the points on which they are taken, and, being subtracted from the height of the instrument, give the heights of these points, as set down in the fifth column. The minus sights are subtracted from the same height of the instrument, as far as the turning point at station 5, inclusive. The plus sight on station 5 is set opposite this station, and a new height obtained for the instrument by adding the plus sight to the height of the turning point. This new height of the instrument is set opposite station 6, where the minus sights to be subtracted from it commence. These sights are again set opposite the points on which they were taken, and, being subtracted from the new height of the instrument, give the heights in the last column.

102. **Problem.** To set slope stakes for excavations and embankments.

Solution. Let $ABHKC$ (fig. 45) be a cross-section of a proposed caration, and let the centre cut $AM = c$, and the width of the road-

an embankment. What was before a *fall* to *B*, becomes now a *rise*, and what was before a *rise* to *C*, becomes now a *fall*.

When the section is partly in excavation and partly in embankment, the method above applies directly only to the side which is in excavation at the same time that the centre of the road-bed is in excavation, or in embankment at the same time that the centre is in embankment. On the opposite side, however, it is only necessary to make *c* in the expressions above *minus*, because its effect here is to diminish the distance out. The formula for this distance out will, therefore, become $\frac{1}{2}b - \frac{3}{2}c + \frac{3}{2}g$.

ARTICLE II.—CORRECTION FOR THE EARTH'S CURVATURE AND FOR REFRACTION.

103. LET *AC* (fig. 46) represent a portion of the earth's surface. Then, if a level be set at *A*, the *line of sight* of the level will be the tangent *AD*, while the *true level* will be *AC*. The difference *DC* between the line of sight and the true level is the correction for the earth's curvature for the distance *AD*.

104. A correction in the opposite direction arises from refraction. Refraction is the change of direction which light undergoes in passing from one medium into another of different density. As the atmosphere increases in density the nearer it lies to the earth's surface, light, passing from a point *B* to a lower point *A*, enters continually air of greater and greater density, and its path is in consequence a curve concave towards the earth. Near the earth's surface this path may be taken as the arc of a circle whose radius is seven times the radius of the earth.* Now a level at *A*, having its line of sight in the direction *AD*, tangent to the curve *AB*, is in the proper position to receive the light from an object at *B*; so that this object appears to the observer to be at *D*. The effect of refraction, therefore, is to make an object appear higher than its true position. Then, since the correction for the earth's curvature *DC* and the correction for refraction *DB* are in opposite directions, the correction for both will be $BC = DC - DB$.

* Peirce's *Spherical Astronomy*, Chap. X., § 125. It should be observed, however, that the effect of refraction is very uncertain, varying with the state of the atmosphere. Sometimes the path of a ray is even made convex towards the earth and sometimes the rays are refracted horizontally as well as vertically.

This correction must be *added* to the height of any object as determined by the level.

105. Problem. *Given the distance $AD = D$ (fig. 46), the radius of the earth $AE = R$, and the radius of the arc of refracted light $= 7R$, to find the correction $BC = d$ for the earth's curvature and for refraction.*

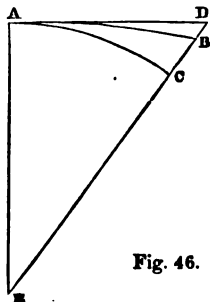


Fig. 46.

Solution. To find the correction for the earth's curvature DC , we have, by Geometry, $DC(DC + 2EC) = AD^2$, or $DC(DC + 2R) = D^2$. But as DC is always very small compared with the diameter of the earth, it may be dropped from the parenthesis, and we have $DC \times 2R = D^2$, or $DC = \frac{D^2}{2R}$. The correction for refraction DB may be found by the method just used for finding DC , merely changing R into $7R$. Hence $DB = \frac{D^2}{14R}$. We have then $d = BC = DC - DB = \frac{D^2}{2R} - \frac{D^2}{14R}$, or



$$d = \frac{3D^2}{7R}.$$

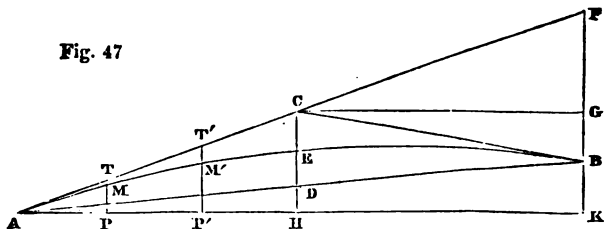
By this formula Table III. is calculated, taking $R = 20,911,790$ ft., as given by Bowditch. The necessity for this correction may be avoided, whenever it is possible to set the level midway between the points whose height is required. In this case, as the distance on each side of the level is the same, the corrections will be equal, and will destroy each other.

ARTICLE III.—VERTICAL CURVES.

106. VERTICAL curves are used to round off the angles formed by the meeting of two grades. Let AC and CB (fig. 47) be two grades, meeting at C . These grades are supposed to be given by the rise per station in going in some particular direction. Thus, starting from A , the grades of AC and CB may be denoted respectively by g and g' ; that is, g denotes what is added to the height at every station on AC , and g' denotes what is added to the height at every station on CB ; but since CB is a descending grade, the quantity added is a minus quantity, and g' will therefore be negative. The parabola furnishes a very simple method of putting in a vertical curve.

107. **Problem.** Given the grade g of AC (fig. 47), the grade g' of CB , and the number of stations n on each side of C to the tangent points A and B , to unite these points by a parabolic vertical curve.

Fig. 47



Solution. Let AEB be the required parabola. Through B and C draw the vertical lines FK and CH , and produce AC to meet FK in F . Through A draw the horizontal line AK , and join AB , cutting CH in D . Then, since the distance from C to A and B is measured horizontally, we have $AH = HK$, and consequently $AD = DB$. The vertical line CD is, therefore, a diameter of the parabola (§ 84, I.), and the distances of the curve in a vertical direction from the stations on the tangent AF are to each other as the squares of the number of stations from A (§ 84, II.). Thus, if a represent this distance at the first station from A , the distance at the second station would be $4a$, at the third station $9a$, and at B , which is $2n$ stations

from A , it would be $4n^2a$; that is, $FB = 4n^2a$, or $a = \frac{FB}{4n^2}$. To find a , it will then be necessary to find FB first. Through C draw the horizontal line CG and we have, from the equal triangles CFG and

ACH , $FG = CH$. But CH is the rise of the first grade g in the n stations from A to C ; that is, $CH = ng$, or $FG = ng$. GB is also the rise of the second grade g' in n stations, but since g' is negative (§ 106), we must put $GB = -ng'$. Therefore, $FB = FG + GB = ng - ng'$. Substituting this value of FB in the equation for a we have $a = \frac{ng - ng'}{4n^2}$, or

$$a = \frac{g - g'}{4n}$$

The value of a being thus determined, all the distances of the curve from the tangent AF , viz. $a, 4a, 9a, 16a$, &c., are known. Now if T and T' be the first and second stations on the tangent, and vertical lines TP and $T'P'$ be drawn to the horizontal line AK , the height TP of the first station above A will be g , the height $T'P'$ of the second station above A will be $2g$, and in like manner for succeeding stations we should find the heights $3g, 4g$, &c. As we have already found $TM = a$, $T'M' = 4a$, &c., we shall have for the heights of the curve above the level of A , $MP = TP - TM = g - a$, $M'P' = T'P' - T'M' = 2g - 4a$, and in like manner for the succeeding heights $3g - 9a, 4g - 16a$, &c. Then to find the grades for the curve at the successive stations from A , that is, the rise of each height over the preceding height, we must subtract each height from the next following height, thus: $(g - a) - 0 = g - a$, $(2g - 4a) - (g - a) = g - 3a$, $(3g - 9a) - (2g - 4a) = g - 5a$, $(4g - 16a) - (3g - 9a) = g - 7a$, &c. The successive grades for the vertical curve are, therefore,

$$g - a, g - 3a, g - 5a, g - 7a, \&c.$$

In finding these grades, strict regard must be paid to the algebraic signs. The results are then general; though the figure represents but one of the six cases that may arise from various combinations of ascending and descending grades. If proper figures were drawn to represent the remaining cases, the above solution, with due attention to the signs, would apply to them all, and lead to precisely the same formulæ.

108. *Examples.* Let the number of stations on each side of C be 3, and let AC ascend .9 per station, and CB descend .6 per station. Here $n = 3$, $g = .9$, and $g' = -.6$. Then, $a = \frac{g - g'}{4n} = \frac{.9 - (-.6)}{4 \times 3} = \frac{1.5}{12} = .125$, and the grades from A to B will be

$$\begin{aligned}
 g - a &= .9 - .125 = .775, \\
 g - 3a &= .9 - .375 = .525, \\
 g - 5a &= .9 - .625 = .275, \\
 g - 7a &= .9 - .875 = .025, \\
 g - 9a &= .9 - 1.125 = -.225, \\
 g - 11a &= .9 - 1.375 = -.475.
 \end{aligned}$$

As a second example, let the first of two grades descend .8 per station, and the second ascend .4 per station, and assume two stations on each side of C as the extent of the curve. Here $g = -.8$, $g' = .4$, and $n = 2$. Then $a = \frac{-.8 - .4}{4 \times 2} = \frac{-1.2}{8} = -.15$, and the four grades required will be

$$\begin{aligned}
 g - a &= -.8 - (-.15) = -.8 + .15 = -.65, \\
 g - 3a &= -.8 - (-.45) = -.8 + .45 = -.35, \\
 g - 5a &= -.8 - (-.75) = -.8 + .75 = -.05, \\
 g - 7a &= -.8 - (-1.05) = -.8 + 1.05 = +.25.
 \end{aligned}$$

It will be seen, that, after finding the first grade, the remaining grades may be found by the continual subtraction of $2a$. Thus, in the first example, each grade after the first is .25 less than the preceding grade, and in the second example, a being here negative, each grade after the first is .3 greater than the preceding grade.

109. The grades calculated for the whole stations, as in the foregoing examples, are sufficient for all purposes except for laying the track. The grade stakes being then usually only 20 feet apart, it will be necessary to ascertain the proper grades on a vertical curve for these sub-stations. To do this, nothing more is necessary than to let g and g' represent the given grades for a sub-station of 20 feet, and n the number of sub-stations on each side of the intersection, and to apply the preceding formulæ. In the last example, for instance, the first grade descends .8 per station, or .16 every 20 feet, the second grade ascends .4 per station, or .08 every 20 feet, and the number of sub-stations in 200 feet is 10. We have then $g = -.16$, $g' = .08$, and $n = 10$

Hence $a = \frac{-.16 - .08}{4 \times 10} = \frac{-.24}{40} = -.006$. The first grade is, therefore, $g - a = -.16 + .006 = -.154$, and as each subsequent grade increases .012 (§ 108), the whole may be written down without farther trouble, thus: $-.154, -.142, -.130, -.118, -.106, -.094, -.082, -.070, -.058, -.046, -.034, -.022, -.010, +.002, +.014, +.026, +.038, +.050, +.062, +.074$.

ARTICLE IV.—ELEVATION OF THE OUTER RAIL ON CURVES.

110. Problem. *Given the radius of a curve R , the gauge of the track g , and the velocity of a car per second v , to determine the proper elevation e of the outer rail of the curve.*

Solution. A car moving on a curve of radius R , with a velocity per second $= v$, has, by Mechanics, a centrifugal force $= \frac{v^2}{R}$. To counteract this force, the outer rail on a curve is raised above the level of the inner rail, so that the car may rest on an inclined plane. This elevation must be such, that the action of gravity in forcing the car down the inclined plane shall be just equal to the centrifugal force, which impels it in the opposite direction. Now the action of gravity on a body resting on an inclined plane is equal to 32.2 multiplied by the ratio of the height to the length of the plane. But the height of the plane is the elevation e , and its length the gauge of the track g . This action of gravity, which is to counteract the centrifugal force, is, therefore, $= \frac{32.2 e}{g}$. Putting this equal to the centrifugal force, we have $\frac{32.2 e}{g} = \frac{v^2}{R}$. Hence



$$e = \frac{g v^2}{32.2 R}$$

If we substitute for R its value (§ 10) $R = \frac{50}{\sin D}$, we have $e = \frac{g v^2 \sin D}{50 \times 32.2} = .00062112 g v^2 \sin D$. If the velocity is given in miles per hour, represent this velocity by M , and we have $v = \frac{M \times 5280}{60 \times 60}$. Substituting this value of v , we find $e = .0013361 g M^2 \sin D$. When $g = 4.7$, this becomes $e = .00627966 M^2 \sin D$. By this formula Table IV. is calculated. In determining the proper elevation in any given case, the usual practice is to adopt the highest customary speed of passenger trains as the value of M .

111. Still the outer rail of a curve, though elevated according to the preceding formula, is generally found to be much more worn than the inner rail. On this account some are led to distrust the formula, and to give an increased elevation to the rail. So far, however, as the centrifugal force is concerned, the formula is undoubtedly correct, and the evil in question must arise from other causes,—causes which are not counteracted by an additional elevation of the outer rail. The principal of these causes is probably improper “coning” of the wheels. Two wheels, immovable on an axle, and of the same radius, must,

no slip is allowed, pass over equal spaces in a given number of revolutions. Now as the outer rail of a curve is longer than the inner rail, the outer wheel of such a pair must on a curve fall behind the inner wheel. The first effect of this is to bring the flange of the outer wheel against the rail, and to keep it there. The second is a strain on the axle consequent upon a slip of the wheels equal in amount to the difference in length of the two rails of the curve. To remedy this, coning of the wheels was introduced, by means of which the radius of the outer wheel is in effect increased, the nearer its flange approaches the rail, and this wheel is thus enabled to traverse a greater distance than the inner wheel.

To find the amount of coning for a play of the wheels of one inch, let r and r' represent the proper radii of the inner and outer wheels respectively, when the flange of the outer wheel touches the rail. Then $r' - r$ will be the coning for one inch in breadth of the tire. To enable the wheels to keep pace with each other in traversing a curve, their radii must be proportional to the lengths of the two rails of the curve, or, which is the same thing, proportional to the radii of these rails. If R be taken as the radius of the inner rail, the radius of the outer rail will be $R + g$, and we shall have $r : r' = R : R + g$. Therefore, $rR + rg = r'R$, or

$$r' - r = \frac{rg}{R}.$$

As an example, let $R = 600$, $r = 1.4$, and $g = 4.7$. Then we have $r' - r = \frac{1.4 \times 4.7}{600} = .011$ ft. For a tire 3.5 in. wide, the coning would be $3.5 \times .011 = .0385$ ft., or nearly half an inch. Wheels coned to this amount would accommodate themselves to any curves of not less than 600 feet radius. On a straight line the flanges of the two wheels would be equally distant from the rails, making both wheels of the same diameter. On a curve of say 2400 feet radius, the flange of the outer wheel would assume a position one fourth of an inch nearer to the rail than the flange of the inner wheel, which would increase the radius of the outer wheel just one fourth of the necessary increase on a curve of 600 feet. Should the flange of the outer wheel get too near the rail, the disproportionate increase of the radius of this wheel would make it get the start of the inner wheel, and cause the flange to recede from the rail again. If the shortest radius were taken 900 feet, r and g remaining the same, we should have $r' - r = \frac{1.4 \times 4.7}{900}$

= .0073, and for the coning of the whole tire $3.5 \times .0073 = .0256$ ft., or about three tenths of an inch. Wheels coned to this amount would accommodate themselves to any curve of not less than 900 feet radius. If the wheels are larger, the coning must be greater, or if the gauge of the track is wider, the coning must be greater. If the play of the wheels is greater, the coning may be diminished. Hence it might be advisable to increase the play of the wheels on short curves, by a slight increase of the gauge of the track.

Two distinct things, therefore, claim attention in regard to the motion of cars on a curve. The first is the centrifugal force, which is generated in all cases, when a body is constrained to move in a curvilinear path, and which may be effectually counteracted for any given velocity by elevating the outer rail. The second is the unequal length of the two rails of a curve, in consequence of which two wheels fixed on an axle cannot traverse a curve properly, unless some provision is made for increasing the diameter of the outer wheel. Coning of the wheels seems to be the only thing yet devised for obtaining this increase of diameter. At present, however, there is little regularity either in the coning itself, or in the distance between the flanges of wheels for tracks of the same gauge. The tendency has been to diminish the coning,* without substituting any thing in its place. If the wheels could be made to turn independently of each other, the whole difficulty would vanish; but if this is thought to be impracticable, the present method ought at least to be reduced to some system.

* Bush and Lobdell, extensive wheel-makers, say, in a note published in Appletons' *Mechanic's Magazine* for August, 1852, that wheels made by them for the New York and Erie road have a coning of but one sixteenth of an inch. This coning on a track of six feet gauge with the other data as given above, would suit no curve of less than a mile radius.

CHAPTER IV.

EARTH-WORK.

ARTICLE I.—PRISMOIDAL FORMULA.

112. EARTH-WORK includes the regular excavation and embankment on the line of a road, borrow-pits, or such additional excavations as are made necessary when the embankment exceeds the regular excavation, and, in general, any transfers of earth that require calculation. We begin with the prismoidal formula, as this formula is frequently used in calculating cubical contents both of earth and masonry.

A *prismoid* is a solid having two parallel faces, and composed of prisms, wedges, and pyramids, whose common altitude is the perpendicular distance between the parallel faces.

113. **Problem.** *Given the areas of the parallel faces B and B' , the middle area M , and the altitude a of a prismoid, to find its solidity S .*

Solution. The middle area of a prismoid is the area of a section midway between the parallel faces and parallel to them, and the altitude is the perpendicular distance between the parallel faces. If now b represents the base of any prism of altitude a , its solidity is ab . If b represents the base of a regular wedge or half-parallelpipedon of altitude a , its solidity is $\frac{1}{2}ab$. If b represents the base of a pyramid of altitude a , its solidity is $\frac{1}{3}ab$. The solidity of these three bodies admits of a common expression, which may be found thus. Let m represent the middle area of either of these bodies, that is, the area of a section parallel to the base and midway between the base and top. In the prism, $m = b$, in the regular wedge, $m = \frac{1}{2}b$, and in the pyramid, $m = \frac{1}{3}b$. Moreover, the upper base of the prism $= b$, and the upper base of the wedge or pyramid $= 0$. Then the expressions ab , $\frac{1}{2}ab$, and $\frac{1}{3}ab$ may be thus transformed. Solidity of

$$\text{prism} = ab = \frac{a}{6} \times 6b = \frac{a}{6} (b + b + 4b) = \frac{a}{6} (b + b + 4m),$$

$$\text{wedge} = \frac{1}{2}ab = \frac{a}{6} \times 3b = \frac{a}{6} (0 + b + 2b) = \frac{a}{6} (0 + b + 4m),$$

$$\text{pyramid} = \frac{1}{3}ab = \frac{a}{6} \times 2b = \frac{a}{6} (0 + b + b) = \frac{a}{6} (0 + b + 4m).$$

Hence, the solidity of either of these bodies is found by adding together the area of the upper base, the area of the lower base, and four times the middle area, and multiplying the sum by one sixth of the altitude. Irregular wedges, or those not half-parallelopipedons, may be measured by the same rule, since they are the sum or difference of a regular wedge and a pyramid of common altitude, and as the rule applies to both these bodies, it applies to their sum or difference.

Now a prismoid, being made up of prisms, wedges, and pyramids of common altitude with itself, will have for its solidity the sum of the solidities of the combined solids. But the sum of the areas of the upper and lower bases of the combined solids is equal to $B + B'$, the sum of the areas of the parallel faces of the prismoid; and the sum of the middle areas of the combined solids is equal to M , the middle area of the prismoid. Therefore

$$S = \frac{a}{6} (B + B' + 4 M).$$

ARTICLE II. — BORROW-PITS.

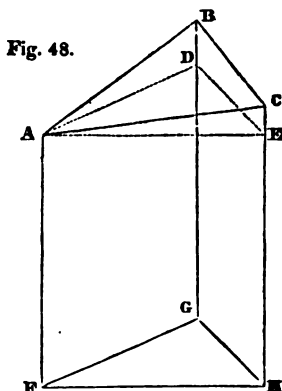
114. FOR the measurement of small excavations, such as borrow-pits, &c., the usual method of preparing the ground is to divide the surface into parallelograms* or triangles, small enough to be considered planes, laid off from a base line, that will remain untouched by the excavation. A convenient bench-mark is then selected, and levels taken at all the angles of the subdivisions. After the excavation is made, the same subdivisions are laid off from the base line upon the bottom of the excavation, and levels referred to the same bench-mark are taken at all the angles.

This method divides the excavation into a series of vertical prisms, generally truncated at top and bottom. The vertical edges of these prisms are known, since they are the differences of the levels at the top and bottom of the excavation. The horizontal section of the prisms is also known, because the parallelograms or triangles, into which the surface is divided, are always measured horizontally.

115. **Problem.** *Given the edges h , h_1 , and h_2 , to find the solidity*

* If the ground is divided into rectangles, as is generally done, and one side be made 27 feet, or some multiple of 27 feet, the contents may be obtained at once in cubic yards, by merely omitting the factor 27 in the calculation.

S of a vertical prism, whether truncated or not, whose horizontal section is a triangle of given area A.



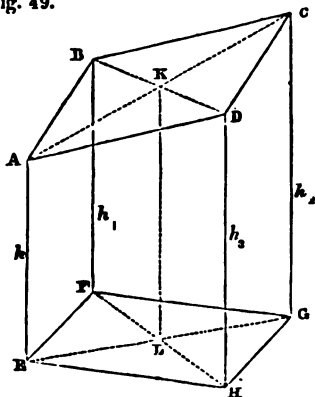
Solution. When the prism is not truncated, we have $h = h_1 = h_2$. The ordinary rule for the solidity of a prism gives, therefore, $S = Ah = A \times \frac{1}{2} (h + h_1 + h_2)$. When the prism is truncated, let $ABCF-GH$ (fig. 48) represent such a prism, truncated at the top. Through the lowest point A of the upper face draw a horizontal plane ADE cutting off a pyramid, of which the base is the trapezoid $BDEC$, and the altitude a perpendicular let fall from A on DE . Represent this perpendicular by p , and we have (Tab. X. 52) the solidity of the pyramid $= \frac{1}{3} p \times BDEC = \frac{1}{3} p \times DE \times \frac{1}{2} (BD + CE) = \frac{1}{3} p \times DE \times \frac{1}{2} (BD + CE) = A \times \frac{1}{2} (BD + CE)$, since $\frac{1}{3} p \times DE = ADE = A$. But $\frac{1}{2} (BD + CE)$ is the mean height of the vertical edges of the truncated portion, the height at A being 0. Hence the formula already found for a prism not truncated, will apply to the portion above the plane ADE , as well as to that below. The same reasoning would apply, if the lower end also were truncated. Hence, for the solidity of the whole prism, whether truncated or not, we have

$$\boxed{\text{E}} \quad S = A \times \frac{1}{2} (h + h_1 + h_2).$$

116. Problem. Given the edges h, h_1, h_2 , and h_3 , to find the solidity S of a vertical prism, whether truncated or not, whose horizontal section is a parallelogram of given area A .

Solution. Let BH (fig. 49) represent such a prism, whether truncated or not, and let the plane $BFHD$ divide it into two triangular

Fig. 49.



prisms AFH and CFH . The horizontal section of each of these prisms will be $\frac{1}{2} A$, and if h , h_1 , h_2 , and h_3 represent the edges to which they are attached in the figure, we have for their solidity (§ 115) $AFH = \frac{1}{2} A \times \frac{1}{2} (h + h_1 + h_3)$, and $CFH = \frac{1}{2} A \times \frac{1}{2} (h_1 + h_2 + h_3)$. Therefore, the whole prism will have for its solidity $S = \frac{1}{2} A \times \frac{1}{2} (h + 2h_1 + h_2 + 2h_3)$. Let the whole prism be again divided by the plane $AEGC$ into two triangular prisms BEG and DEG . Then we have for these prisms, $BEG = \frac{1}{2} A \times \frac{1}{2} (h + h_1 + h_2)$, and $DEG = \frac{1}{2} A \times \frac{1}{2} (h + h_2 + h_3)$, and for the whole prism, $S = \frac{1}{2} A \times \frac{1}{2} (2h + h_1 + 2h_2 + h_3)$. Adding the two expressions found for S , we have $2S = \frac{1}{2} A (h + h_1 + h_2 + h_3)$, or

$$S = A \times \frac{1}{4} (h + h_1 + h_2 + h_3).$$

It will be seen by the figure, that $\frac{1}{2} (h + h_2) = KL = \frac{1}{2} (h_1 + h_3)$, or $h + h_2 = h_1 + h_3$. The expression for S might, therefore, be reduced to $S = A \times \frac{1}{2} (h + h_2)$, or $S = A \times \frac{1}{2} (h_1 + h_3)$. But as the ground surfaces $ABCD$ and $EFGH$ are seldom perfect planes, it is considered better to use the mean of the four heights, instead of the mean of two diagonally opposite.

117. Corollary. When all the prisms of an excavation have the same horizontal section A , the calculation of any number of the

may be performed by one operation. Let figure 50 be a plan of an excavation, the heights at the angles being denoted by $a, a_1, a_2,$

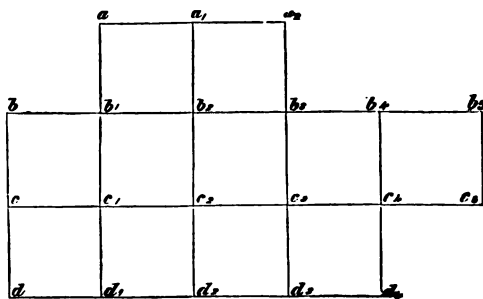


Fig. 50.

b_1 , &c. Then the solidity of the whole will be equal to $\frac{1}{4} A$ multiplied by the sum of the heights of the several prisms (§ 116). In this sum the corner heights $a, a_2, b, b_5, c_5, d,$ and d_4 will enter *once*, each being found in but one prism; the heights a_1, b_4, c, d_1, a and d_5 will enter *twice*, each being common to two prisms; the heights b_1, b_3 , and c_4 will enter *three* times, each being common to three prisms; and the heights b_2, c_1, c_2 , and c_3 will enter *four* times, each being common to four prisms. If, therefore, the sum of the first set heights is represented by s_1 , the sum of the second by s_2 , of the third by s_3 , and of the fourth by s_4 , we shall have for the solidity of all the prisms



$$S = \frac{1}{4} A (s_1 + 2s_2 + 3s_3 + 4s_4).$$

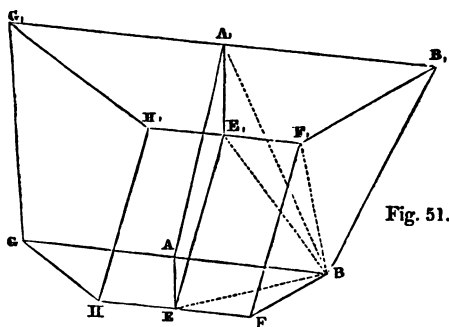
ARTICLE III. — EXCAVATION AND EMBANKMENT.

118. As embankments have the same general shape as excavations it will be necessary to consider excavations only. The simplest case is when the ground is considered level on each side of the centre line. Figure 51 represents the mass of earth between two stations in an excavation of this kind. The trapezoid $GBFH$ is a section of the mass at the first station, and $G_1B_1F_1H_1$ a section at the second station; AE is the centre height at the first station, and A_1E_1 the centre height at the second station; $H_1H_1F_1F$ is the road-bed, G, G_1, B_1, F_1

surface of the ground, and GG_1H_1H and BB_1F_1F the planes forming the side slopes. This solid is a prismoid, and might be calculated by the prismoidal formula (§ 113). The following method gives the same result.

A. Centre Heights alone given.

119. Problem. Given the centre heights c and c_1 , the width of the road-bed b , the slope of the sides s , and the length of the section l , to find the solidity S of the excavation.



Solution. Let c be the centre height at A (Fig. 51) and c_1 the height at A_1 . The slope s is the ratio of the base of the slope to its perpendicular height (§ 102). We have then the distance out $AB = \frac{1}{2}b + sc$, and the distance out $A_1B_1 = \frac{1}{2}b + sc_1$ (§ 102). Divide the whole mass into two equal parts by a vertical plane AA_1E_1E drawn through the centre line, and let us find first the solidity of the right-hand half. Through B draw the planes BE_1E , BA_1E_1 , and BE_1F_1 , dividing the half-section into three quadrangular pyramids, having for their common vertex the point B , and for their bases the planes AA_1E_1E , EE_1F_1F , and $A_1B_1F_1E_1$. For the areas of these bases we have

$$\begin{aligned} \text{Area of } AA_1E_1E &= \frac{1}{2}EE_1 \times (AE + A_1E_1) = \frac{1}{2}l(c + c_1), \\ \text{" } EE_1F_1F &= EF \times EE_1 = \frac{1}{2}bl, \\ \text{" } A_1B_1F_1E_1 &= \frac{1}{2}A_1E_1 \times (E_1F_1 + A_1B_1) = \frac{1}{2}(bc_1 + sc_1^2); \end{aligned}$$

and for the perpendiculars from the vertex B on these bases, produced when necessary,

$$\text{Perpendicular on } A A_1 E_1 E = AB = \frac{1}{2}b + sc,$$

$$\text{" " } E E_1 F_1 F = AE = c,$$

$$\text{" " } A_1 B_1 F_1 E_1 = EE_1 = l.$$

Then (Tab. X. 52) the solidities of the three pyramids are

$$B - A A_1 E_1 E = \frac{1}{3} \left(\frac{1}{2}b + sc \right) \times \frac{1}{2}l (c + c_1) = \frac{1}{6}l \left(\frac{1}{2}bc + \frac{1}{2}bc_1 + sc^2 + sc c_1 \right)$$

$$B - E E_1 F_1 F = \frac{1}{3}c \times \frac{1}{2}bl = \frac{1}{6}l b c,$$

$$B - A_1 B_1 F_1 E_1 = \frac{1}{3}l \times \frac{1}{2}(bc_1 + sc_1^2) = \frac{1}{6}l (bc_1 + sc_1^2).$$

Their sum, or the solidity of the half-section, is

$$\frac{1}{2}S = \frac{1}{6}l \left[\frac{1}{2}b(c + c_1) + s(c^2 + c_1^2 + cc_1) \right].$$

Therefore the solidity of the whole section is

$$S = \frac{1}{3}l \left[\frac{1}{2}b(c + c_1) + s(c^2 + c_1^2 + cc_1) \right],$$

or

$$\text{☞ } S = \frac{1}{3}l [b(c + c_1) + \frac{2}{3}s(c^2 + c_1^2 + cc_1)]$$

When the slope is $1\frac{1}{2}$ to 1, $s = \frac{2}{3}$, and the factor $\frac{2}{3}s = 1$ may be dropped.

120. Problem. To find the solidity S of any number n of successive sections of equal length.

Solution. Let $c, c_1, c_2, c_3, \&c.$ denote the centre heights at the successive stations. Then we have (§ 119)

$$\text{Solidity of first section} = \frac{1}{3}l [b(c + c_1) + \frac{2}{3}s(c^2 + c_1^2 + cc_1)],$$

$$\text{" " second section} = \frac{1}{3}l [b(c_1 + c_2) + \frac{2}{3}s(c_1^2 + c_2^2 + c_1c_2)],$$

$$\text{" " third section} = \frac{1}{3}l [b(c_2 + c_3) + \frac{2}{3}s(c_2^2 + c_3^2 + c_2c_3)],$$

&c.

&c.

For the solidity of any number n of sections, we should have $\frac{1}{2}l$ multiplied by the sum of the quantities in n parentheses formed as those just given. The last centre height, according to the notation adopted, will be represented by c_n , and the next to the last by c_{n-1} . Collecting the terms multiplied by b into one line, the squares multiplied by $\frac{2}{3}s$ into a second line, and the remaining terms into a third line, we have for the solidity of n sections

$$\text{☞ } S = \frac{1}{3}l \left\{ \begin{array}{l} b(c + 2c_1 + 2c_2 + 2c_3 \dots + 2c_{n-1} + c_n) \\ + \frac{2}{3}s(c^2 + 2c_1^2 + 2c_2^2 + 2c_3^2 \dots + 2c_{n-1}^2 + c_n^2) \\ + \frac{2}{3}s(cc_1 + c_1c_2 + c_2c_3 + c_3c_4 \dots + c_{n-1}c_n). \end{array} \right.$$

When $s = \frac{2}{3}$, the factor $\frac{2}{3}s = 1$ may be dropped.

Example. Given $l = 100$, $b = 28$, $s = \frac{3}{2}$, and the stations and centre heights as set down in the first and second columns of the annexed table. The calculation is thus performed. Square the heights, and set the squares in the third column. Form the successive products cc_1 , c_1c_2 , &c., and place them in the fourth column. Add up the last three columns. To the sum of the second column add the sum itself, minus the first and the last height, and to the sum of the third column add the sum itself, minus the first and the last square. Then 86 is the multiplier of b in the first line of the formula, 592 is the second line, since $\frac{3}{2}s$ is here 1, and 274 is the third line. The product of 86 by $b = 28$ is 2408, and the sum of 274, 592, and 2408 is 3274. This multiplied by $\frac{1}{2}l = 50$ gives for the solidity 163,700 cubic feet.

Station.	c.	c ² .	c c ₁ .
0	2	4	
1	4	16	8
2	7	49	28
3	6	36	42
4	10	100	60
5	7	49	70
6	6	36	42
7	4	16	24
	46	306	274
	40	286	592
	86	592	2408
	28		2)3274
	2408		163700.

B. Centre and Side Heights given.

121. When greater accuracy is required than can be attained by the preceding method, the side heights and the distances out (§ 102) are introduced. Let figure 52 represent the right-hand side of an excavation between two stations. AA_1B_1B is the ground surface; $AE = c$ and $A_1E_1 = c_1$ are the centre heights; $BG = h$ and $B_1G_1 = h_1$, the side heights; and d and d_1 , the distances out, or the horizontal distances of B and B_1 from the centre line. The whole ground surface may sometimes be taken as a plane, and sometimes the part on each side of the centre line may be so taken; * but neither of these suppo-

* It is easy in any given case to ascertain whether a surface like AA_1B_1B is

sitions is sufficiently accurate to serve as the basis of a general method. In most cases, however, we may consider the surface on each side of the centre line to be divided into two triangular planes by a diagonal passing from one of the centre heights to one of the side heights. A ridge or depression will, in general, determine which diagonal ought to be taken as the dividing line, and this diagonal must be noted in the field. Thus, in the figure a ridge is supposed to run from B to A_1 , from which the ground slopes downward on each side to A and B_1 . Instead of this, a depression might run from A to B_1 , and the ground rise each way to A_1 and B . If the ridge or depression is very marked, and does not cross the centre or side lines at the regular stations, intermediate stations must be introduced to make the triangular planes conform better to the nature of the ground. If the surface happens to be a plane, or nearly so, the diagonal may be taken in either direction. It will be seen, therefore, that the following method is applicable to all ordinary ground. When, however, the ground is very irregular, the method of § 127 is to be used.

122. Problem. *Given the centre heights c and c_1 , the side heights on the right h and h_1 , on the left h' and h'_1 , the distances out on the right d and d_1 , on the left d' and d'_1 , the width of the road-bed b , the length of the section l , and the direction of the diagonals, to find the solidity S of the excavation.*

Solution. Let figure 52 represent the right-hand side of the excavation, and let us suppose first, that the diagonal runs, as shown in the figure, from B to A_1 . Through B draw the planes $BE E_1$, $BA_1 E_1$, and $BE_1 F_1$, dividing the half-section into three quadrangular pyramids, having for their common vertex the point B , and for their bases the planes $AA_1 E_1 E$, $EE_1 F_1 F$, and $A_1 B_1 F_1 E_1$. For the areas of these bases we have

$$\text{Area of } AA_1 E_1 E = \frac{1}{2} EE_1 \times (AE + A_1 E_1) = \frac{1}{2} l (c + c_1),$$

$$\text{" " } EE_1 F_1 F = EF \times EE_1 = \frac{1}{2} b l,$$

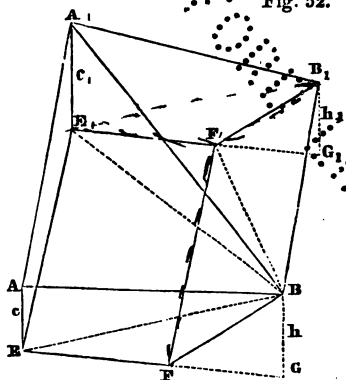
$$\text{" " } A_1 B_1 F_1 E_1 = \frac{1}{2} A_1 E_1 \times d_1 + \frac{1}{2} E_1 F_1 \times h_1 = \frac{1}{2} d_1 c_1 + \frac{1}{2} b h_1,$$

and for the perpendiculars from the vertex B on these bases, produced when necessary,

plane; for if it is a plane, the descent from A to B will be to the descent from A_1 to B_1 , as the distance out at the first station is to the distance out at the second station, that is, $c - h : c_1 - h_1 = d : d_1$. If we had $c = 9$, $h = 6$, $c_1 = 12$, $h_1 = 8$, $d = 24$, and $d_1 = 27$, the formula would give $3 : 4 = 24 : 27$ which shows that the surface is not a plane.

Perpendicular on $A A_1 E_1 E = EG = d$,
 " $E E_1 F_1 F = BG = h$,
 " $A_1 B_1 F_1 E_1 = E E_1 = l$.

Fig. 52.



Then (Tab. X. 52) the solidities of the three pyramids are

$$\begin{aligned} B - A A_1 E_1 E &= \frac{1}{3} d \times \frac{1}{2} l (c + c_1) &= \frac{1}{6} l (dc + dc_1), \\ B - E E_1 F_1 F &= \frac{1}{3} h \times \frac{1}{2} bl &= \frac{1}{6} l bh, \\ B - A_1 B_1 F_1 E_1 &= \frac{1}{3} l \times \frac{1}{2} (d_1 c_1 + \frac{1}{2} b h_1) &= \frac{1}{6} l (d_1 c_1 + \frac{1}{2} b h_1). \end{aligned}$$

Their sum, or the solidity of the half-section, is

$$\frac{1}{6} l (dc + d_1 c_1 + dc_1 + bh + \frac{1}{2} b h_1). \quad (1)$$

Next, suppose that the diagonal runs from A to B_1 . In this case, through B_1 draw the planes $B_1 E_1 E$, $B_1 A E$, and $B_1 E F$ (not represented in the figure), dividing the half-section again into three quadrangular pyramids, having for their common vertex the point B_1 , and for their bases the planes $A A_1 E_1 E$, $E E_1 F_1 F$, and $A B F E$. For the areas of these bases we have

$$\begin{aligned} \text{Area of } A A_1 E_1 E &= \frac{1}{2} E E_1 \times (AE + A_1 E_1) = \frac{1}{2} l (c + c_1), \\ \text{" " } E E_1 F_1 F &= EF \times E E_1 = \frac{1}{2} bl, \\ \text{" " } A B F E &= \frac{1}{2} AE \times d + \frac{1}{2} EF \times h = \frac{1}{2} dc + \frac{1}{4} bh; \end{aligned}$$

and for the perpendiculars from B_1 on these bases, produced when necessary,

Perpendicular on $AA_1E_1E = E_1G_1 = d_1$,

" " $EE_1F_1F = B_1G_1 = h_1$,

" " $ABFE = EE_1 = l$.

Then (Tab. X. 52) the solidities of the three pyramids are

$$B_1 - AA_1E_1E = \frac{1}{3} d_1 \times \frac{1}{2} l (c + c_1) = \frac{1}{6} l (d_1 c + d_1 c_1),$$

$$B_1 - EE_1F_1F = \frac{1}{3} h_1 \times \frac{1}{2} b l = \frac{1}{6} l b h_1,$$

$$B_1 - ABFE = \frac{1}{3} l \times \frac{1}{2} (dc + \frac{1}{2} b h) = \frac{1}{6} l (dc + \frac{1}{2} b h).$$

Their sum, or the solidity of the half-section, is.

$$\frac{1}{6} l (dc + d_1 c_1 + d_1 c + b h_1 + \frac{1}{2} b h). \quad (2)$$

We have thus found the solidity of the half-section for both directions of the diagonal. Let us now compare the results (1) and (2), and express them, if possible, by one formula. For this purpose let (1) be put under the form

$$\frac{1}{6} l [dc + d_1 c_1 + d c_1 + \frac{1}{2} b (h + h_1 + h)],$$

and (2) under the form

$$\frac{1}{6} l [dc + d_1 c_1 + d_1 c + \frac{1}{2} b (h + h_1 + h_1)].$$

The only difference in these two expressions is, that dc_1 and the last h in the first, become $d_1 c$ and h_1 in the second. But in the first case, c_1 and h are the heights at the extremities of the diagonal, and d is the distance out corresponding to h ; and in the second case, c and h_1 are the heights at the extremities of the diagonal, and d_1 is the distance out corresponding to h_1 . Denote the centre height touched by the diagonal by C , the side height touched by the diagonal by H , and the distance out corresponding to the side height H by D . We may then express both dc_1 and $d_1 c$ by DC , and both h and h_1 by H ; so that the solidity of the half-section on the right of the centre line, whichever way the diagonal runs, may be expressed by

$$\frac{1}{6} l [dc + d_1 c_1 + DC + \frac{1}{2} b (h + h_1 + H)]. \quad (3)$$

To obtain the contents of the portion on the left of the centre line, we designate the quantities on the left by the same letters used for corresponding quantities on the right, merely attaching a (*'*) to them to distinguish them. Thus the side heights are h' and h'_1 , and the distances out d' and d'_1 , while D , C , and H become D' , C' , and H' . The solidity of the half-section on the left may therefore be taken directly from (3), which will become

$$\frac{1}{6} l [d' c + d'_1 c_1 + D' C' + \frac{1}{2} b (h' + h'_1 + H')]. \quad (4)$$

Finally, by uniting (3) and (4), we obtain the following formula for the solidity of the whole section between two stations

$$S = \frac{1}{6} l [(d + d') c + (d_1 + d'_1) c_1 + D C + D' C' + \frac{1}{2} b (h + h_1 + H + h' + h'_1 + H')].$$

Example. Given $l = 100$, $b = 18$, and the remaining data, as arranged in the first six columns of the following table. The first column gives the stations; the fourth gives the centre heights, namely, $c = 13.6$ and $c_1 = 8$; the two columns on the left of the centre heights give the side heights and distances out on the left of the centre line of the road, and the two columns on the right of the centre heights give the side heights and distances out on the right. The direction of the diagonals is marked by the oblique lines drawn from $h' = 8$ to $c_1 = 8$ and from $c = 13.6$ to $h_1 = 12$.

Sta.	d' .	h' .	c .	h .	d .	$d + d'$.	$(d + d') c$.	$D' C'$.	$D C$.
0	21	8	13.6	10	24	45	612		
1	15	4	8.0	12	27	42	336	168	367.2
		12					168		
					20		367.2		
					$54 \times 9 =$		486		
							6)1969.20		
							32820.		

To apply the formula, the distances out at each station are added together, and their sum placed in the seventh column; these sums, multiplied by the respective centre heights, are placed in the eighth column; the product of $d' = 21$ (which is the distance out corresponding to the side height touched by the left-hand diagonal) by $c_1 = 8$ (which is the centre height touched by the same diagonal) is placed in the ninth column, and the similar product of $d_1 = 27$ by $c = 13.6$ is placed in the last column. The terms in the formula multiplied by $\frac{1}{2} b$ are all the side heights, and in addition all the side heights touched by diagonals, or $8 + 4 + 10 + 12 + 8 + 12 = 54$. Then by substitution in the formula, we have $S = \frac{1}{6} \times 100 (612 + 336 + 168 + 367.2 + 9 \times 54) = 32,820$ cubic feet.*

* The example here given is the same as that calculated in Mr. Borden's "Sya-

By applying the rule given in the note to § 121, we see that the surface on the left of the centre line in the preceding example is a plane; since $13.6 - 8 : 8 - 4 = 21 : 15$. The diagonal on that side might, therefore, be taken either way, and the same solidity would be obtained. This may be easily seen by reversing the diagonal in this example, and calculating the solidity anew. The only parts of the formula affected by the change are $D' C'$ and $\frac{1}{2} b H'$. In the one case the sum of these terms is $21 \times 8 + 9 \times 8$, and in the other $15 \times 13.6 + 9 \times 4$, both of which are equal to 240.

123. Problem. *To find the solidity S of any number n of successive sections of equal length.*

Solution. Let c, c_1, c_2, c_3 , &c. be the centre heights at the successive stations; h, h_1, h_2, h_3 , &c. the right-hand side heights; h', h'_1, h'_2, h'_3 , &c. the left-hand side heights; d, d_1, d_2, d_3 , &c. the distances out on the right; and d', d'_1, d'_2, d'_3 , &c. the distances out on the left. Then the formula for the solidity of one section (§ 122) gives for the solidities of the successive sections

$$\frac{1}{6} l [(d + d') c + (d_1 + d'_1) c_1 + D C + D' C' + \frac{1}{2} b (h + h_1 + H + h' + h'_1 + H')],$$

$$\frac{1}{6} l [(d_1 + d'_1) c_1 + (d_2 + d'_2) c_2 + D_1 C_1 + D'_1 C'_1 + \frac{1}{2} b (h_1 + h_2 + H_1 + h'_1 + h'_2 + H'_1)],$$

$$\frac{1}{6} l [(d_2 + d'_2) c_2 + (d_3 + d'_3) c_3 + D_2 C_2 + D'_2 C'_2 + \frac{1}{2} b (h_2 + h_3 + H_2 + h'_2 + h'_3 + H'_2)],$$

and so on, for any number of sections. For the solidity of any number n of sections, we should have $\frac{1}{6} l$ multiplied by the sum of n parentheses formed as those just given. Hence

$$S = \frac{1}{6} l \left\{ (d + d') c + 2(d_1 + d'_1) c_1 + 2(d_2 + d'_2) c_2 \dots + (d_n + d'_n) c_n \right. \\ \left. + D C + D' C' + D_1 C_1 + D'_1 C'_1 + D_2 C_2 + D'_2 C'_2 + \&c. \right. \\ \left. + \frac{1}{2} b \left[h + 2h_1 + 2h_2 \dots + h_n + H + H_1 + H_2 + \&c. \right. \right. \\ \left. \left. + h' + 2h'_1 + 2h'_2 \dots + h'_n + H' + H'_1 + H'_2 + \&c. \right] \right\}$$

tem of Useful Formulæ, &c.,” page 187. It will be seen, that his calculation makes the solidity 32,460 cubic feet, which is 360 cubic feet less than the result above. This difference is owing to the omission, by Mr. Borden’s method, of a pyramid inclosed by the four pyramids, into which the upper portion of the right-hand half section is by that method divided.

Example. Given $l = 100$, $b = 28$, and the remaining data as given in the first six columns of the following table.

Sta.	d'	h'	c	h	d	$d + d'$	$(d + d') c$	$D' C'$	$D C$
0	17	2	2	2	17	34	68		
1	18.5	3	> 4	5	21.5	40	160	68	43
2	20	4	5	6	23	43	215	80	92
3	23	6	6	8	26	49	294	115	130
4	21.5	5	6	> 7	24.5	46	276	129	147
5	20	4	6	4	20	40	240	120	147
6	15.5	1	4	3	18.5	34	136	93	80
		25		35			1389	605	639
		22		30			1185		
		22		37			605		
		69		102			639		
		102					2394		
		171					6) 6212		
							103533		

$171 \times 14 = 2394$ 103533 cubic feet.

The data in this table are arranged precisely as in the example for calculating one section (§ 122), and the remaining columns are calculated as there shown. Then, to obtain the first line of the formula, add all the numbers in the column headed $(d + d') c$, making 1389, and afterwards all the numbers except the first and the last, making 1185. The next line of the formula is the sum of the columns $D' C'$ and $D C$, which give respectively 605 and 639. To obtain the first line of the quantities multiplied by $\frac{1}{2} b$, add all the numbers in column h , making 35, next all the numbers except the first and the last, making 30, and lastly all the numbers touched by diagonals (doubling any one touched by two diagonals), making 37. The second line of the quantities multiplied by $\frac{1}{2} b$ is obtained in the same way from the column marked h' . The sum of these numbers is 171, and this multiplied by $\frac{1}{2} b = 14$ gives 2394. We have now for the first line of the formula $1389 + 1185$, for the second $605 + 639$, and for the remainder 2394.

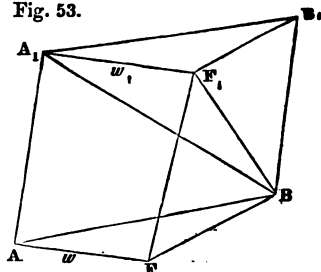
By adding these together, and multiplying the sum by $\frac{1}{6} l = \frac{100}{6}$, we get the contents of the six sections in feet.

124. When the section is partly in excavation and partly in embankment, the preceding formulæ are still applicable; but as this application introduces minus quantities into the calculation, the following method, *similar in principle, is preferable.*

125. **Problem.** Given the widths of an excavation at the road-bed

$A F = w$ and $A_1 F_1 = w_1$ (fig. 53), the side heights h and h_1 , the length of the section l , and the direction of the diagonal, to find the solidity S of the excavation, when the section is partly in excavation and partly in embankment.

Fig. 53.



Solution. Suppose, first, that the surface is divided into two triangles by the diagonal BA_1 . Through B draw the plane BA_1F_1 , dividing that part of the section which is in excavation into two pyramids $B-A A_1 F_1 F$ and $B-A_1 B_1 F_1$, the solidities of which are

$$B-A A_1 F_1 F = \frac{1}{2} h \times \frac{1}{2} l (w + w_1) = \frac{1}{6} l (w h + w_1 h),$$

$$B-A_1 B_1 F_1 = \frac{1}{2} l \times \frac{1}{2} w_1 h_1 = \frac{1}{6} l w_1 h_1.$$

The whole solidity is, therefore,

$$S = \frac{1}{6} l (w h + w_1 h_1 + w_1 h).$$

Next, suppose the dividing diagonal to run from A to B_1 . Through B_1 draw a plane $B_1 A F$ (not represented in the figure), dividing the excavation again into two pyramids, of which the solidities are

$$B_1-A A_1 F_1 F = \frac{1}{2} h_1 \times \frac{1}{2} l (w + w_1) = \frac{1}{6} l (w h_1 + w_1 h_1),$$

$$B_1-A B F = \frac{1}{2} l \times \frac{1}{2} w h = \frac{1}{6} l w h.$$

The whole solidity is, therefore,

$$S = \frac{1}{6} l (w h + w_1 h_1 + w h_1).$$

The only difference in these two expressions is, that $w_1 h$ in the first becomes $w h_1$ in the second. But in the first case the diagonal touches w_1 and h , and in the second case it touches w and h_1 . If, then, we designate the width touched by the diagonal by W , and the height touched by the diagonal by H , we may express both $w_1 h$ and $w h_1$ by WH ; so that the solidity in either case may be expressed by

$$S = \frac{1}{3} l (w h + w_1 h_1 + W H).$$

Corollary. When several sections of equal length succeed one another, the whole may be calculated together. For this purpose, the preceding formula gives for the solidities of the successive sections

$$\frac{1}{3} l (w h + w_1 h_1 + W H),$$

$$\frac{1}{3} l (w_1 h_1 + w_2 h_2 + W_1 H_1),$$

$$\frac{1}{3} l (w_2 h_2 + w_3 h_3 + W_2 H_2),$$

and so on for any number of sections. Hence for the solidity of any number n of sections we should have

$$S = \frac{1}{3} l (w h + 2 w_1 h_1 + 2 w_2 h_2 \dots + w_n h_n + W H + W_1 H_1 + W_2 H_2 + \&c.)$$

Example. Given $l = 100$, and the remaining data as given in the first three columns of the following table.

Station.	w .	h .	wh .	WH .
0	2	1	2	
1	8	6	48	8
2	10	7	70	56
3	13	7	91	70
4	9	4	36	52
			247	186
			209	
			186	
			6)642	
			10700.	

The fourth column contains the products of the several widths by the corresponding heights, and the next column the products of those widths and heights touched by diagonals. The sum of the products in the fourth column is 247, the sum of all but the first and the last is 209, and the sum of the products in the fifth column is 186. These three sums are added together, multiplied by 100, and divided by 6, according to the formula. This gives the solidity of the four sections = 10700 cubic feet.

126. When the excavation does not begin on a line at right angles to the centre line, intermediate stations are taken where the excavation begins on each side of the road-bed, and the section may be

lated as a pyramid, having its vertex at the first of these points, and for its base the cross-section at the second. The preceding method gives the same result, since w and h in this case become 0, and reduce the formula to $S = \frac{1}{2} l w_1 h_1$. The same remarks apply to the end of an excavation.

C. Ground very Irregular.

127. Problem. *To find the solidity of a section, when the ground is very irregular.*

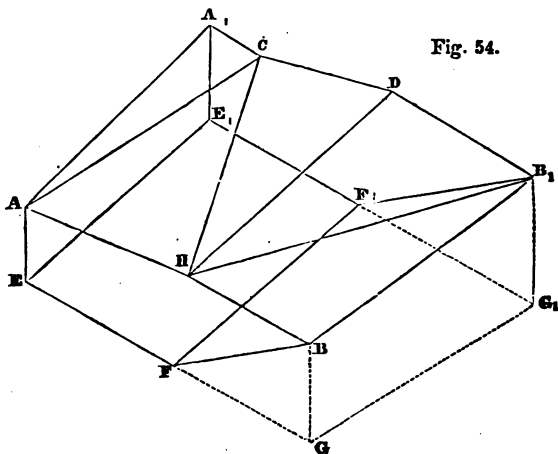


Fig. 54.

Solution. Let $A H B F E - A_1 C D B_1 F_1 E_1$ (fig. 54) represent one side of a section, the surface of which is too irregular to be divided into two planes. Suppose, for instance, that the ground changes at H , C , and D , making it necessary to divide the surface into five triangles running from station to station.* Let heights be taken at H , C , and D , and let the distances out of these points be measured. If now we suppose the earth to be excavated vertically downward through the side line $B B_1$ to the plane of the road-bed, we may form as many vertical triangular prisms as there are triangles on the surface. This will be made evident by drawing vertical planes through the sides

* It will often be necessary to introduce intermediate stations, in order to make the subdivision into triangles more conveniently and accurately.

$A C, H C, H D$, and $H B_1$. Then the solidity of the half-section will be equal to the sum of these prisms, minus the triangular mass $B F G - B_1 F_1 G_1$.

The horizontal section of the prisms may be found from the distances out and the length of the section, and the vertical edges or heights are all known. Hence the solidities of these prisms may be calculated by § 115.

To find the solidity of the portion $B F G - B_1 F_1 G_1$, which is to be deducted, represent the slope of the sides by s (§ 102), the heights at B and B_1 by h and h_1 , and the length of the section by l . Then we have $F G = s h$, and $F_1 G_1 = s h_1$. Moreover, the area of $B F G = \frac{1}{2} s h^2$, and that of $B_1 F_1 G_1 = \frac{1}{2} s h_1^2$. Now as the triangles $B F G$ and $B_1 F_1 G_1$ are similar, the mass required is the frustum of a pyramid, and the mean area is $\sqrt{\frac{1}{2} s h^2 \times \frac{1}{2} s h_1^2} = \frac{1}{2} s h h_1$. Then (Tab. X. 53) the solidity is $B F G - B_1 F_1 G_1 = \frac{1}{6} l s (h^2 + h_1^2 + h h_1)$.

Example. Given $l = 50, b = 18, s = \frac{3}{2}$, the heights at A, H , and B respectively 4, 7, and 6, the distances $A H = 9$ and $H B = 9$, the heights at A_1, C, D , and B_1 respectively 6, 7, 9, and 8, and the distances $A_1 C = 4, C D = 5$, and $D B_1 = 12$. Then the horizontal section of the first prism adjoining the centre line is $\frac{1}{2} l \times A_1 C$, since the distance $A_1 C$ is measured horizontally; and the mean of the three heights is $\frac{1}{3} (4 + 6 + 7) = \frac{1}{3} \times 17$. The solidity of this prism is therefore $\frac{1}{2} l \times A_1 C \times \frac{1}{3} \times 17 = \frac{1}{6} l \times 4 \times 17$, that is, equal to $\frac{1}{6} l$ multiplied by the base of the triangle and by the sum of the heights. In this way we should find for the solidity of the five prisms

$$\frac{1}{6} l (4 \times 17 + 9 \times 18 + 5 \times 23 + 12 \times 24 + 9 \times 21) = \frac{1}{6} l \times 822.$$

For the frustum to be deducted, we have

$$\frac{1}{6} l \times \frac{3}{2} (6^2 + 8^2 + 6 \times 8) = \frac{1}{6} l \times 222.$$

Hence the solidity of the half-section is

$$\frac{1}{6} l (822 - 222) = \frac{1}{6} \times 50 \times 600 = 5000 \text{ cubic feet.}$$

128. Let us now examine the usual method of calculating excavation, when the cross-section of the ground is not level. This method consists, first, in finding the area of a cross-section at each end of the mass; secondly, in finding the height of a section, level at the top, equivalent in area to each of these end sections; thirdly, in finding from the average of these two heights the middle $\therefore$ the mass;

and, lastly, in applying the prismoidal formula to find the contents. The heights of the equivalent sections level at the top may be found approximately by Trautwine's Diagrams,* or exactly by the following method. Let A represent the area of an irregular cross-section, b the width of the road-bed, and s the slope of the sides. Let x be the required height of an equivalent section level at the top. The bottom of the equivalent section will be b , the top $b + 2sx$, and the area will be the sum of the top and bottom lines multiplied by half the height or $\frac{1}{2}x(2b + 2sx) = sx^2 + bx$. But this area is to be equal to A . Therefore, $sx^2 + bx = A$, and from this equation the value of x may be found in any given case.

According to this method, the contents of the section already calculated in § 122 will be found thus. Calculating the end areas, we find the first end area to be 387 and the second to be 240. Then as s is here $\frac{3}{4}$ and $b = 18$, the equations for finding the heights of the equivalent end sections will be $\frac{3}{4}x^2 + 18x = 387$, and $\frac{3}{4}x^2 + 18x = 240$. Solving these equations, we have for the height at the first station $x = 11.146$, and at the second, $x = 8$. The middle area will, therefore, have the height $\frac{1}{2}(11.146 + 8) = 9.573$, and from this height the middle area is found to be 309.78. Then by the prismoidal formula (§ 113) the solidity will be $S = \frac{1}{3} \times 100 (387 + 240 + 4 \times 309.78) = 31102$ cubic feet.

But the true solidity of this section was found to be 32820 cubic feet, a difference of 1718 feet. The error, of course, is not in the prismoidal formula, but in assuming that, if the earth were levelled at the ends to the height of the equivalent end sections, the intervening earth might be so disposed as to form a plane between these level ends, thus reducing the mass to a prismoid. This supposition, however, may sometimes be very far from correct, as has just been shown. If the diagonal on the right-hand side in this example were reversed, that is, if the dividing line were formed by a depression, the true solidity found by § 122 would be 29600 feet; whereas the method by equivalent sections would give the same contents as before, or 1502 feet too much.

D. *Correction in Excavation on Curves*

129. In excavations on curves the ends of a section are not parallel

* *A New Method of Calculating the Cubic Contents of Excavations and Embankments by the aid of Diagrams.* By John C. Trautwine.

to each other, but converge towards the centre of the curve. A section between two stations 100 feet apart on the centre line will, therefore, measure less than 100 feet on the side nearest to the centre of the curve, and more than 100 feet on the side farthest from that centre. Now in calculating the contents of an excavation, it is assumed that the ends of a section are parallel, both being perpendicular to the chord of the curve. Thus, let figure 55 represent the plan of two sections of

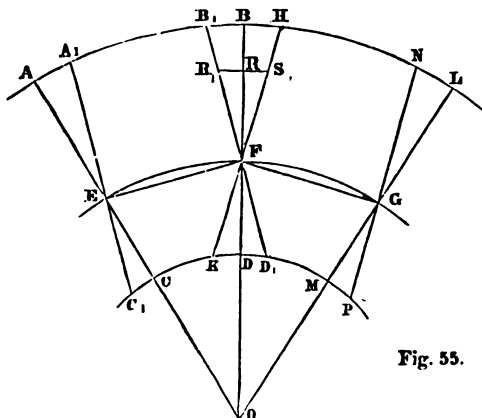
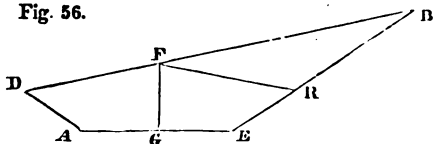


Fig. 55.

an excavation, $EF G$ being the centre line, AL and CM the extreme side lines, and O the centre of the curve. Then the calculation of the first section would include all between the lines $A_1 C_1$ and $B_1 D_1$; while the true section lies between AC and BD . In like manner, the calculation of the second section would include all between HK and NP , while the true section lies between BD and LM . It is evident, therefore, that at each station on the curve, as at F , the calculation is too great by the wedge-shaped mass represented by KFD_1 , and too

Fig. 56.



small by the mass represented by $B_1 F H$. These masses balance

each other, when the distances out on each side of the centre line are equal, that is, when the cross-section may be represented by $ADFRE$ (fig. 56). But if the excavation is on the side of a hill, so that the distances out differ very much, and the cross-section is of the shape $ADFBE$, the difference of the wedge-shaped masses may require consideration.

130. Problem. *Given the centre height c , the greatest side height h , the least side height h' , the greatest distance out d , the least distance out d' , and the width of the road-bed b , to find the correction in excavation C , at any station on a curve of radius R or deflection angle D .*

Solution. The correction, from what has been said above, is a triangular prism of which BFR (fig. 56) is a cross-section. The height of this prism at B (fig. 55) is B_1H , the height at R is R_1S , and the height at F is 0. B_1H and R_1S , being very short, are here considered straight lines. Now we have the cross-section $BFR = FBE G - FREG = (\frac{1}{2}cd + \frac{1}{4}bh) - (\frac{1}{2}cd' + \frac{1}{4}bh') = \frac{1}{2}c(d - d') + \frac{1}{4}b(h - h')$. To find the height B_1H , we have the angle $BFH = BFB_1 = D$, and therefore $B_1H = 2HF \sin. D = 2d \sin. D$. In like manner, $R_1S = KD_1 = 2KF \sin. D = 2d' \sin. D$. Then since the height at F is 0, one third of the sum of the heights of the prism will be $\frac{2}{3}(d + d') \sin. D$, and the correction, or the solidity of the prism, will be (§ 115)

$$\text{C} = [\frac{1}{2}c(d - d') + \frac{1}{4}b(h - h')] \times \frac{2}{3}(d + d') \sin. D.$$

When R is given, and not D , substitute for $\sin. D$ its value (§ 9) $\sin. D = \frac{50}{R}$. The correction then becomes

$$\text{C} = [\frac{1}{2}c(d - d') + \frac{1}{4}b(h - h')] \times \frac{100(d + d')}{3R}.$$

This correction is to be *added*, when the highest ground is on the convex side of the curve, and *subtracted*, when the highest ground is on the concave side. At a tangent point, it is evident, from figure 55, that the correction will be just half of that given above.

Example. Given $c = 28$, $h = 40$, $h' = 16$, $d = 74$, $d' = 38$, $b = 28$, and $R = 1400$, to find C . Here the area of the cross-section $BFR = \frac{28}{2}(74 - 38) + \frac{28}{4}(40 - 16) = 672$, and one third of the sum of the heights of the prism is $\frac{100(74 + 38)}{3 \times 1400} = \frac{8}{3}$. Hence $C = 672 \times \frac{8}{3} = 1792$ cubic feet.

131. When the section is partly in excavation and partly in embankment, the cross-section of the excavation is a triangle lying wholly on one side of the centre line, or partly on one side and partly on the other. The surface of the ground, instead of extending from B to D (fig. 56), will extend from B to a point between G and E , or to a point between A and G . In the first case, the correction will be a triangular prism lying between the lines $B_1 F$ and $H F$ (fig. 55), but not extending below the point F . In the second case, the excavation extends below F , and the correction, as in § 129, is the difference between the masses above and below F . This difference may be obtained in a very simple manner, by regarding the mass on both sides of F as one triangular prism the bases of which intersect on the line $G F$ (fig. 56), in which case the height of the prism at the edge below F must be considered to be *minus*, since the direction of this edge, referred to either of the bases, is contrary to that of the two others. The solidity of this prism will then be the difference required.

132. **Problem.** *Given the width of the excavation at the road-bed w , the width of the road-bed b , the distance out d , and the side height h , to find the correction in excavation C , at any station on a curve of radius R or deflection angle D , when the section is partly in excavation and partly in embankment.*

Solution. When the excavation lies wholly on one side of the centre line, the correction is a triangular prism having for its cross-section the cross-section of the excavation. Its area is, therefore, $\frac{1}{2} w h$. The height of this prism at B (fig. 56) is (§ 130) $B_1 H = 2 H F \sin. D = 2 d \sin. D$. In a similar manner, the height at E will be $2 G E \sin. D = b \sin. D$, and at the point intermediate between G and E , the distance of which from the centre line is $\frac{1}{2} b - w$, the height will be $2 (\frac{1}{2} b - w) \sin. D = (b - 2 w) \sin. D$. Hence, the correction, or the solidity of the prism, will be (§ 115) $C = \frac{1}{2} w h \times \frac{1}{3} (2 d + b + b - 2 w) \sin. D = \frac{1}{2} w h \times \frac{2}{3} (d + b - w) \sin. D$.

When the excavation lies on both sides of the centre line, the correction, from what has been said above, is a triangular prism having also for its cross-section the cross-section of the excavation. Its area will, therefore, be $\frac{1}{2} w h$. The height of this prism at B is also $2 d \sin. D$, and the height at E , $b \sin. D$; but at the point intermediate between A and G , the distance of which from the centre line is $w - \frac{1}{2} b$, the height will be $2 (w - \frac{1}{2} b) \sin. D = (2 w - b) \sin. D$. As this height is to be considered *minus*, it must be subtracted from the others, and the correction required will be $C = \frac{1}{2} w h \times \frac{1}{3} (2 d + b - 2 w + b) \sin. D$.

$= \frac{1}{2} w h \times \frac{2}{3} (d + b - w) \sin. D$. Hence, in all cases, when the section is partly in excavation and partly in embankment, we have the formula

$$\text{Fig. 54} \quad C = \frac{1}{2} w h \times \frac{2}{3} (d + b - w) \sin. D.$$

When R is given, and not D , substitute for $\sin. D$ its value (§ 9) $\sin. D = \frac{50}{R}$. The correction then becomes

$$\text{Fig. 55} \quad C = \frac{1}{2} w h \times \frac{100 (d + b - w)}{3 R}.$$

This correction is to be *added*, when the highest ground is on the convex side of the curve, and *subtracted* when the highest ground is on the concave side. At a tangent point the correction will be just half of that given above.

Example. Given $w = 17$, $b = 30$, $d = 51$, $h = 24$, and $R = 160$ to find C . Here the area of the cross-section is $\frac{1}{2} w h = 17 \times 12 = 204$, and one third of the sum of the heights of the prism is $\frac{100(d+b-w)}{3R} = \frac{100(51+30-17)}{3 \times 160} = \frac{4}{3}$. Hence $C = 204 \times \frac{4}{3} = 272$ cubic feet.

133. The preceding corrections (§ 130 and § 132) suppose the length of the sections to be 100 feet. If the sections are shorter, the angle BFH (fig. 55) may be regarded as the same part of D that FG is 100 feet, and B_1FB as the same part of D that EF is of 100 feet. The true correction may then be taken as the same part of C that the sum of the lengths of the two adjoining sections is of 200 feet.

TABLE I.

RADII, ORDINATES, DEFLECTIONS,

AND

ORDINATES FOR CURVING RAILS.

*Formula for Radii, § 10 ; for Ordinates, § 25 ; for Deflections, § 19
for Curving Rails, § 29.*

Degree.	Radii.	Ordinates.				Tangent Deflection.	Chord Deflection.	Ordinates for Rails.	
		12 $\frac{1}{2}$.	25.	37 $\frac{1}{2}$.	50.			18.	20.
0	5	63754.94	.008	.014	.017	.018	.073	.145	.001
10		34377.48	.016	.027	.034	.036	.145	.291	.001
15		22918.33	.024	.041	.051	.055	.218	.436	.002
20		17188.76	.032	.055	.068	.073	.291	.582	.002
25		13751.02	.040	.068	.085	.091	.364	.727	.003
30		11459.19	.048	.082	.102	.109	.436	.873	.004
35		9322.18	.056	.095	.119	.127	.509	1.018	.004
40		8594.41	.064	.109	.136	.145	.582	1.164	.005
45		7639.49	.072	.123	.153	.164	.654	1.309	.005
50		6875.55	.080	.136	.170	.182	.727	1.454	.006
55		6250.51	.087	.150	.187	.200	.800	1.600	.006
1	0	5729.65	.095	.164	.205	.218	.873	1.745	.007
5		5258.92	.103	.177	.222	.236	.945	1.891	.008
10		4911.15	.111	.191	.239	.255	1.018	2.036	.008
15		4583.75	.119	.205	.256	.273	1.091	2.182	.009
20		4297.28	.127	.218	.273	.291	1.164	2.327	.009
25		4044.51	.135	.232	.290	.309	1.236	2.472	.010
30		3819.83	.143	.245	.307	.327	1.309	2.618	.011
35		3618.80	.151	.259	.324	.345	1.382	2.763	.011
40		3437.87	.159	.273	.341	.364	1.454	2.909	.012
45		3274.17	.167	.286	.358	.382	1.527	3.054	.012
50		3125.36	.175	.300	.375	.400	1.600	3.200	.013
55		2989.48	.183	.314	.392	.418	1.673	3.345	.014
2	0	2864.93	.191	.327	.409	.436	1.745	3.490	.014
5		2750.35	.199	.341	.426	.455	1.818	3.636	.015
10		2644.58	.207	.355	.443	.473	1.891	3.781	.015
15		2546.64	.215	.363	.460	.491	1.963	3.927	.016
20		2455.70	.223	.382	.477	.509	2.036	4.072	.016
25		2371.04	.231	.395	.494	.527	2.109	4.218	.017
30		2292.01	.239	.409	.511	.545	2.181	4.363	.018
35		2218.09	.247	.423	.528	.564	2.254	4.508	.018
40		2148.79	.255	.436	.545	.582	2.327	4.654	.019
45		2083.68	.263	.450	.562	.600	2.400	4.799	.019
50		2022.41	.270	.464	.580	.618	2.472	4.945	.020
55		1964.64	.278	.477	.597	.636	2.545	5.090	.021
3	0	1910.08	.286	.491	.614	.655	2.618	5.235	.021
5		1858.47	.294	.505	.631	.673	2.690	5.381	.022
10		1809.57	.302	.518	.648	.691	2.763	5.526	.022
15		1763.18	.310	.532	.665	.709	2.836	5.672	.023
20		1719.12	.318	.545	.682	.727	2.908	5.817	.024
25		1677.20	.326	.559	.699	.745	2.981	5.963	.024
30		1637.28	.334	.573	.716	.764	3.054	6.108	.025
35		1599.21	.342	.586	.733	.782	3.127	6.253	.025
40		1562.58	.350	.600	.750	.800	3.199	6.398	.026
45		1528.16	.358	.614	.767	.818	3.272	6.544	.027
50		1494.95	.366	.627	.784	.836	3.345	6.689	.027
55		1463.16	.374	.641	.801	.855	3.417	6.835	.028
4	0	1432.69	.382	.655	.818	.873	3.490	6.980	.028
5		1403.46	.390	.668	.835	.891	3.563	7.125	.029
10		1375.40	.398	.682	.852	.909	3.635	7.271	.029
15		1348.45	.406	.695	.869	.927	3.708	7.416	.030
20		1322.53	.414	.709	.886	.945	3.781	7.561	.031
25		1297.58	.422	.723	.903	.964	3.853	7.707	.031
30		1273.57	.430	.736	.921	.982	3.926	7.852	.032
35		1250.42	.438	.750	.938	1.000	3.999	7.997	.032
40		1228.11	.446	.764	.955	1.018	4.071	8.143	.033
45		1206.57	.454	.777	.972	1.036	4.144	8.288	.034
50		1185.78	.462	.791	.989	1.055	4.217	8.433	.034
55		1165.70	.469	.805	1.006	1.073	4.289	8.579	.035
5	0	1146.28	.477	.818	1.023	1.091	4.362	8.724	.035

Degree.	Radii.	Ordinates.				Tangent Deflection.	Chord Deflection.	Ordinates for Rails.	
		12 $\frac{1}{2}$.	25.	37 $\frac{1}{2}$.	50.			18.	20.
5 5	1127.50	.485	.832	1.040	1.109	4.435	8.869	.036	.041
10	1109.33	.493	.846	1.057	1.127	4.507	9.014	.037	.045
15	1091.73	.501	.859	1.074	1.146	4.580	9.160	.037	.046
20	1074.68	.509	.873	1.091	1.164	4.653	9.305	.038	.047
25	1058.16	.517	.887	1.108	1.182	4.725	9.450	.038	.047
30	1042.14	.525	.900	1.125	1.200	4.798	9.596	.039	.048
35	1026.60	.533	.914	1.142	1.218	4.870	9.741	.039	.049
40	1011.51	.541	.928	1.159	1.237	4.943	9.886	.040	.049
45	996.87	.549	.941	1.176	1.255	5.016	10.031	.041	.050
50	982.64	.557	.955	1.193	1.273	5.088	10.177	.041	.051
55	968.81	.565	.968	1.210	1.291	5.161	10.322	.042	.052
6 0	955.37	.573	.982	1.228	1.309	5.234	10.467	.042	.052
5	942.29	.581	.996	1.245	1.327	5.306	10.612	.043	.053
10	929.57	.589	1.009	1.262	1.346	5.379	10.758	.044	.054
15	917.19	.597	1.023	1.279	1.364	5.451	10.903	.044	.055
20	905.13	.605	1.037	1.296	1.382	5.524	11.048	.045	.055
25	893.39	.613	1.050	1.313	1.400	5.597	11.193	.045	.056
30	881.95	.621	1.064	1.330	1.418	5.669	11.339	.046	.057
35	870.79	.629	1.078	1.347	1.437	5.742	11.484	.047	.057
40	859.92	.637	1.091	1.364	1.455	5.814	11.629	.047	.058
45	849.32	.645	1.105	1.381	1.473	5.887	11.774	.048	.059
50	838.97	.653	1.118	1.398	1.491	5.960	11.919	.048	.060
55	828.88	.661	1.132	1.415	1.510	6.032	12.065	.049	.060
7 0	819.02	.669	1.146	1.432	1.528	6.105	12.210	.049	.061
5	809.40	.677	1.159	1.449	1.546	6.177	12.355	.050	.062
10	800.00	.685	1.173	1.466	1.564	6.250	12.500	.051	.063
15	790.81	.693	1.187	1.483	1.582	6.323	12.645	.051	.063
20	781.84	.701	1.200	1.501	1.600	6.395	12.790	.052	.064
25	773.07	.709	1.214	1.517	1.619	6.468	12.936	.052	.065
30	764.49	.717	1.228	1.535	1.637	6.540	13.081	.053	.065
35	756.10	.725	1.242	1.552	1.655	6.613	13.226	.054	.066
40	747.89	.733	1.255	1.569	1.673	6.685	13.371	.054	.067
45	739.86	.740	1.269	1.586	1.691	6.758	13.516	.055	.068
50	732.01	.748	1.283	1.603	1.710	6.831	13.661	.055	.068
55	724.31	.756	1.296	1.620	1.728	6.903	13.806	.056	.069
8 0	716.78	.764	1.310	1.637	1.746	6.976	13.951	.057	.070
5	709.40	.772	1.324	1.654	1.764	7.048	14.096	.057	.070
10	702.18	.780	1.337	1.671	1.782	7.121	14.241	.058	.071
15	695.09	.788	1.351	1.688	1.801	7.193	14.387	.058	.072
20	688.16	.796	1.365	1.705	1.819	7.266	14.532	.059	.073
25	681.35	.804	1.378	1.722	1.837	7.338	14.677	.059	.073
30	674.69	.812	1.392	1.739	1.855	7.411	14.822	.060	.074
35	668.15	.820	1.406	1.757	1.873	7.483	14.967	.061	.075
40	661.74	.828	1.419	1.774	1.892	7.556	15.112	.061	.076
45	655.45	.836	1.433	1.791	1.910	7.628	15.257	.062	.076
50	649.27	.844	1.447	1.808	1.928	7.701	15.402	.062	.077
55	643.22	.852	1.460	1.825	1.945	7.773	15.547	.063	.078
9 0	637.27	.860	1.474	1.842	1.965	7.846	15.692	.064	.078
5	631.44	.868	1.488	1.859	1.983	7.918	15.837	.064	.079
10	625.71	.876	1.501	1.876	2.001	7.991	15.982	.065	.080
15	620.09	.884	1.515	1.893	2.019	8.063	16.127	.065	.081
20	614.56	.892	1.529	1.910	2.037	8.136	16.272	.066	.081
25	609.14	.900	1.542	1.927	2.056	8.208	16.417	.066	.082
30	603.80	.908	1.556	1.944	2.074	8.281	16.562	.067	.083
35	598.57	.916	1.570	1.961	2.092	8.353	16.707	.068	.084
40	593.42	.924	1.583	1.979	2.110	8.426	16.852	.068	.084
45	588.36	.932	1.597	1.996	2.128	8.498	16.996	.069	.085
50	583.38	.940	1.611	2.013	2.147	8.571	17.141	.069	.086
55	578.49	.948	1.624	2.030	2.165	8.643	17.286	.070	.086
0	573.69	.956	1.638	2.047	2.183	8.716	17.431	.071	.087

118 TABLE I. RADII, ORDINATES, DEFLECTIONS, &C.

Degree.	Radii.	Ordinates.				Tangent Deflection.	Chord Deflection.	Ordinates for Rails.	
		12 $\frac{1}{2}$.	25.	37 $\frac{1}{2}$.	50.			18.	20.
10	564.31	.972	1.665	2.081	2.219	8.860	17.721	.072	.089
20	555.23	.988	1.693	2.115	2.256	9.005	18.011	.073	.090
30	546.44	1.004	1.720	2.149	2.292	9.150	18.300	.074	.092
40	537.92	1.020	1.748	2.184	2.329	9.295	18.590	.075	.093
50	529.67	1.036	1.775	2.218	2.365	9.440	18.880	.076	.094
11	521.67	1.052	1.802	2.252	2.402	9.585	19.169	.078	.096
10	513.91	1.068	1.830	2.286	2.438	9.729	19.459	.079	.097
20	505.38	1.084	1.857	2.320	2.475	9.874	19.748	.080	.099
30	499.06	1.100	1.884	2.354	2.511	10.019	20.038	.081	.100
40	491.96	1.116	1.912	2.389	2.547	10.164	20.327	.082	.102
50	485.05	1.132	1.938	2.423	2.584	10.308	20.616	.084	.103
12	478.34	1.148	1.967	2.457	2.620	10.453	20.906	.085	.105
10	471.81	1.164	1.994	2.491	2.657	10.597	21.195	.086	.106
20	465.46	1.180	2.021	2.525	2.693	10.742	21.484	.087	.107
30	459.28	1.196	2.049	2.560	2.730	10.887	21.773	.088	.109
40	453.26	1.212	2.076	2.594	2.766	11.031	22.063	.089	.110
50	447.40	1.228	2.104	2.628	2.803	11.176	22.352	.091	.112
13	441.68	1.244	2.131	2.662	2.839	11.320	22.641	.092	.113
10	436.12	1.260	2.159	2.697	2.876	11.465	22.930	.093	.115
20	430.69	1.277	2.186	2.731	2.912	11.609	23.219	.094	.116
30	425.40	1.293	2.213	2.765	2.949	11.754	23.507	.095	.118
40	420.23	1.309	2.241	2.799	2.985	11.898	23.796	.096	.119
50	415.19	1.325	2.268	2.833	3.022	12.043	24.085	.098	.120
14	410.28	1.341	2.296	2.868	3.058	12.187	24.374	.099	.122
10	405.47	1.357	2.323	2.902	3.095	12.331	24.663	.100	.123
20	400.78	1.373	2.351	2.936	3.131	12.476	24.951	.101	.125
30	396.20	1.389	2.378	2.970	3.168	12.620	25.240	.102	.126
40	391.72	1.405	2.406	3.005	3.204	12.764	25.528	.103	.128
50	387.34	1.421	2.433	3.039	3.241	12.908	25.817	.105	.129
15	383.06	1.437	2.461	3.073	3.277	13.053	26.105	.106	.131
10	378.88	1.453	2.488	3.107	3.314	13.197	26.394	.107	.132
20	374.79	1.469	2.515	3.142	3.350	13.341	26.682	.108	.133
30	370.78	1.486	2.543	3.176	3.387	13.485	26.970	.109	.135
40	366.86	1.502	2.570	3.210	3.423	13.629	27.258	.110	.136
50	363.02	1.518	2.598	3.245	3.460	13.773	27.547	.112	.138
16	359.26	1.534	2.625	3.279	3.496	13.917	27.835	.113	.139
10	355.59	1.550	2.653	3.313	3.533	14.061	28.123	.114	.141
20	351.98	1.566	2.680	3.347	3.569	14.205	28.411	.115	.142
30	348.45	1.582	2.708	3.382	3.606	14.349	28.699	.116	.143
40	344.99	1.598	2.736	3.416	3.643	14.493	28.986	.117	.145
50	341.60	1.615	2.763	3.450	3.679	14.637	29.274	.119	.146
17	338.27	1.631	2.791	3.485	3.716	14.781	29.562	.120	.148
10	335.01	1.647	2.818	3.519	3.752	14.925	29.850	.121	.149
20	331.82	1.663	2.846	3.553	3.789	15.069	30.137	.122	.151
30	328.68	1.679	2.873	3.588	3.825	15.212	30.425	.123	.152
40	325.60	1.695	2.901	3.622	3.862	15.356	30.712	.124	.154
50	322.59	1.711	2.928	3.656	3.898	15.500	31.000	.126	.155
18	319.62	1.728	2.956	3.691	3.935	15.643	31.287	.127	.156
10	316.71	1.744	2.983	3.725	3.972	15.787	31.574	.128	.158
20	313.86	1.760	3.011	3.759	4.008	15.931	31.861	.129	.159
30	311.06	1.776	3.039	3.794	4.045	16.074	32.149	.130	.161
40	308.30	1.792	3.066	3.828	4.081	16.218	32.436	.131	.162
50	305.60	1.809	3.094	3.862	4.118	16.361	32.723	.133	.164
19	302.94	1.825	3.121	3.897	4.155	16.505	33.010	.134	.165
10	300.33	1.841	3.149	3.931	4.191	16.648	33.296	.135	.166
20	297.77	1.857	3.177	3.965	4.228	16.792	33.583	.136	.168
30	295.25	1.873	3.204	4.000	4.265	16.935	33.870	.137	.169
40	292.77	1.890	3.232	4.034	4.301	17.078	34.157	.138	.171
50	290.33	1.906	3.259	4.069	4.338	17.222	34.443	.140	.172
20	287.91	1.922	3.287	4.103	4.374	17.365	34.730	.141	.174

TABLE II.
LONG CHORDS. § 69.

Degree of Curve.	2 Stations.	3 Stations.	4 Stations.	5 Stations.	6 Stations.
0 1	200.000	299.999	399.998	499.996	599.993
0 10	199.999	.997	.992	.983	.970
20	.998	.992	.981	.962	.933
30	.997	.986	.966	.932	.882
40	.995	.979	.947	.894	.815
50					
1 0	199.992	299.970	399.924	499.848	599.733
10	.990	.959	.896	.793	.637
20	.986	.946	.865	.729	.526
30	.983	.932	.829	.657	.401
40	.979	.915	.789	.577	.260
50	.974	.898	.744	.488	.105
2 0	199.970	299.878	399.695	499.391	598.934
10	.964	.857	.643	.285	.750
20	.959	.834	.586	.171	.550
30	.952	.810	.524	.049	.336
40	.946	.783	.459	498.918	.108
50	.939	.756	.389	.778	597.862
3 0	199.931	299.726	399.315	498.630	597.604
10	.924	.695	.237	.474	.331
20	.915	.662	.154	.309	.043
30	.907	.627	.068	.136	596.740
40	.898	.591	398.977	497.955	.423
50	.888	.553	.882	.765	.091
4 0	199.878	299.513	398.782	497.566	595.744
10	.868	.471	.679	.360	.383
20	.857	.423	.571	.145	.007
30	.846	.338	.459	496.921	594.617
40	.834	.337	.343	.689	.212
50	.822	.289	.223	.449	593.792
5 0	199.810	299.239	398.099	496.200	593.358
10	.797	.187	397.970	495.944	592.909
20	.783	.134	.837	.678	.446
30	.770	.079	.700	.405	591.968
40	.756	.023	.559	.123	.476
50	.741	298.964	.413	494.832	590.970
6 0	199.726	298.904	397.264	494.534	590.449
10	.710	.843	.110	.227	589.913
20	.695	.779	396.952	493.912	.361
30	.678	.714	.790	.588	588.800
40	.662	.648	.623	.257	.221
50	.644	.579	.453	492.917	587.628
7 0	199.627	298.509	396.278	492.568	587.021
10	.609	.438	.099	.212	586.400
20	.591	.364	395.916	491.847	585.765
30	.572	.289	.729	.474	.115
40	.553	.212	.538	.093	584.451
50	.533	.134	.342	490.704	583.773
8 0	.513	298.054	395.142	490.306	583.081

TABLE III.

CORRECTION FOR THE EARTH'S CURVATURE AND
FOR REFRACTION. § 105.

<i>D.</i>	<i>d.</i>	<i>D.</i>	<i>d.</i>	<i>D.</i>	<i>d.</i>	<i>D.</i>	<i>d.</i>
300	.002	1800	.066	3300	.223	4800	.472
400	.003	1900	.074	3400	.237	4900	.492
500	.005	2000	.082	3500	.251	5000	.512
600	.007	2100	.090	3600	.266	5100	.533
700	.010	2200	.099	3700	.281	5200	.554
800	.013	2300	.108	3800	.296	1 mile	.571
900	.017	2400	.118	3900	.312	2 "	2.285
1000	.020	2500	.128	4000	.328	3 "	5.142
1100	.025	2600	.139	4100	.345	4 "	9.142
1200	.030	2700	.149	4200	.362	5 "	14.284
1300	.035	2800	.161	4300	.379	6 "	20.568
1400	.040	2900	.172	4400	.397	7 "	27.996
1500	.046	3000	.184	4500	.415	8 "	36.566
1600	.052	3100	.197	4600	.434	9 "	46.279
1700	.059	3200	.210	4700	.453	10 "	57.135

TABLE IV.

ELEVATION OF THE OUTER RAIL ON CURVES.
§ 110.

Degree.	<i>M</i> = 15.	<i>M</i> = 20.	<i>M</i> = 25.	<i>M</i> = 30.	<i>M</i> = 40.	<i>M</i> = 50.
0						
1	.012	.022	.034	.049	.088	.137
2	.025	.044	.068	.099	.175	.274
3	.037	.066	.103	.148	.263	.411
4	.049	.088	.137	.197	.351	.548
5	.062	.110	.171	.247	.438	.685
6	.074	.131	.205	.296	.526	.822
7	.086	.153	.240	.345	.613	.958
8	.099	.175	.274	.394	.701	1.095
9	.111	.197	.308	.443	.788	1.232
10	.123	.219	.342	.493	.876	1.368

TABLE V.

FROG ANGLES, CHORDS, AND ORDINATES FOR
TURNOUTS.

This table is calculated for $g = 4.7$, $d = .42$, and $S = 1^\circ 20'$. For
mula for frog angle F , and chord BF , § 50; for m , the middle or-
dinate of BF , § 25; for m' , the middle ordinate for curving an 18 ft
rail, § 29.

R.	F.	BF.	m.	m'.	R.	F.	BF.	m.	m'.
1000	$\begin{smallmatrix} 0 \\ 5\ 27\ 44 \end{smallmatrix}$	72.22	.651	.041	600	$\begin{smallmatrix} 6\ 57\ 48 \end{smallmatrix}$	59.17	.727	.068
975	$\begin{smallmatrix} 5\ 31\ 39 \end{smallmatrix}$	71.53	.655	.042	575	$\begin{smallmatrix} 7\ 6\ 26 \end{smallmatrix}$	58.16	.733	.070
950	$\begin{smallmatrix} 5\ 35\ 44 \end{smallmatrix}$	70.83	.659	.043	550	$\begin{smallmatrix} 7\ 15\ 40 \end{smallmatrix}$	57.12	.739	.074
925	$\begin{smallmatrix} 5\ 39\ 59 \end{smallmatrix}$	70.11	.663	.044	525	$\begin{smallmatrix} 7\ 25\ 33 \end{smallmatrix}$	56.05	.745	.077
900	$\begin{smallmatrix} 5\ 44\ 24 \end{smallmatrix}$	69.38	.667	.045	500	$\begin{smallmatrix} 7\ 36\ 10 \end{smallmatrix}$	54.94	.752	.081
875	$\begin{smallmatrix} 5\ 49\ 1 \end{smallmatrix}$	68.64	.671	.046	475	$\begin{smallmatrix} 7\ 47\ 37 \end{smallmatrix}$	53.79	.758	.085
850	$\begin{smallmatrix} 5\ 53\ 50 \end{smallmatrix}$	67.88	.676	.048	450	$\begin{smallmatrix} 8\ 0\ 1 \end{smallmatrix}$	52.61	.765	.090
825	$\begin{smallmatrix} 5\ 58\ 52 \end{smallmatrix}$	67.10	.680	.049	425	$\begin{smallmatrix} 8\ 13\ 30 \end{smallmatrix}$	51.37	.773	.095
800	$\begin{smallmatrix} 6\ 4\ 9 \end{smallmatrix}$	66.30	.685	.051	400	$\begin{smallmatrix} 8\ 28\ 14 \end{smallmatrix}$	50.09	.780	.101
775	$\begin{smallmatrix} 6\ 9\ 41 \end{smallmatrix}$	65.49	.690	.052	375	$\begin{smallmatrix} 8\ 44\ 26 \end{smallmatrix}$	48.75	.788	.106
750	$\begin{smallmatrix} 6\ 15\ 30 \end{smallmatrix}$	64.65	.695	.054	350	$\begin{smallmatrix} 9\ 2\ 20 \end{smallmatrix}$	47.35	.796	.116
725	$\begin{smallmatrix} 6\ 21\ 37 \end{smallmatrix}$	63.80	.700	.056	325	$\begin{smallmatrix} 9\ 22\ 16 \end{smallmatrix}$	45.88	.805	.125
700	$\begin{smallmatrix} 6\ 28\ 4 \end{smallmatrix}$	62.92	.705	.058	300	$\begin{smallmatrix} 9\ 44\ 39 \end{smallmatrix}$	44.34	.814	.135
675	$\begin{smallmatrix} 6\ 34\ 52 \end{smallmatrix}$	62.02	.710	.060	275	$\begin{smallmatrix} 10\ 10\ 1 \end{smallmatrix}$	42.72	.824	.147
650	$\begin{smallmatrix} 6\ 42\ 4 \end{smallmatrix}$	61.09	.716	.062	250	$\begin{smallmatrix} 10\ 39\ 6 \end{smallmatrix}$	41.00	.834	.162
625	$\begin{smallmatrix} 6\ 49\ 42 \end{smallmatrix}$	60.14	.721	.065	225	$\begin{smallmatrix} 11\ 12\ 55 \end{smallmatrix}$	39.16	.845	.180

TABLE VI.

LENGTH OF CIRCULAR ARCS IN PARTS OF RADIUS.

0				1				1			
1	.01745	32925	19943	1	.00029	08882	08666	1	.00000	48181	36811
2	.03490	65850	39887	2	.00053	17764	17331	2	.00000	96962	73622
3	.05235	98775	59830	3	.00087	26616	25997	3	.00001	45444	10433
4	.06981	31700	79773	4	.00116	35528	34663	4	.00001	93925	47244
5	.08726	61625	99716	5	.00145	44410	43329	5	.00002	42106	84055
6	.10471	97551	19660	6	.00174	53292	51994	6	.00002	90888	20867
7	.12217	30476	39603	7	.00203	62174	60660	7	.00003	39369	57678
8	.13962	63401	59546	8	.00232	71056	69326	8	.00003	87850	94489
9	.15707	96326	79490	9	.00261	79938	77991	9	.00004	36332	31300

TABLE VII.
EXPANSION BY HEAT.

Bodies.	32° to 212°.	1°.	Authority.
Platina,	.0008842	.000004912	Hassler
Gold,	.001466	.000008141	"
Silver,	.001909	.000010605	"
Mercury,	.018018	.0001001	"
Brass,	.00189163	.000010509	"
Iron,	.00125344	.000006964	"
Water,	.0466	not uniform.	"
Granite,	.00036850	.000004825	Prof. Barlett.
Marble,	.00102024	.000005668	"
Sandstone,	.00171576	.000009532	"

TABLE VIII.

PROPERTIES OF MATERIALS.

The authorities referred to by the capital letters in the table are:—

B. Barlow, <i>On the Strength of Materials.</i>	L. Lamé.
Be. Bevan.	M. Musschenbroek, <i>Int. to Nat Phil.</i>
Br. Lieut. Brown.	R. Rennie, <i>Phil. Trans.</i>
C. Couch.	Ro. Rondelet, <i>L'Art de Batur.</i>
F. Franklin Institute, <i>Report on Steam Boilers.</i>	T. Telford.
G. Gordon, <i>Eng. Translation of Weisbach.</i>	Ta. Taylor, <i>Statistics of Coal.</i>
H. Hodgkinson, <i>Reports to Brit. Association.</i>	W. Weisbach, <i>Mech. of Machinery and Engineering.</i>
Ha. Hassler, <i>Tables.</i>	The numbers without letters are taken from Prof. Moseley's <i>Engineering and Architecture</i>

In finding the weights, a cubic foot of water has, for convenience, been taken at 62.5 lbs.

The numbers for compression taken from Hodgkinson were obtained by him from prisms high enough to allow the wedge of rupture to slide freely off. He shows that this is essential in experiments on compression.

The modulus of rupture S is the breaking weight of a prism 1 in broad, 1 in. deep, and 1 in. between the supports, the weight being applied in the middle. To find the corresponding breaking weight W of a rectangular beam of any other size, let l = its length, b = its breadth, and d = its depth, all in inches. Then $W = \frac{2bd^2}{3l} \times S$.

The numbers in the last three columns express absolute strength. For safety, a certain proportion only of these numbers is taken. The divisors for wood may be from 6 to 10, for metal from 3 to 6, for stone 10, and for ropes 3.

When double numbers are used in the column headed "Crushing Force per Square Inch in lbs.," the first applies to specimens moderately dry, the second to specimens turned and kept dry in a warm place two months longer. In the case of American Birch, Elm, and Oak, the numbers apply to seasoned speci-

Materials.	Specific Gravity.	Weight per Cubic Foot in lbs.	Tensile Strength per Square Inch in lbs.	Crushing Force per Square Inch in lbs.	Modulus of Rupture in lbs.
<i>Metals.</i>					
Free cast, ...	8.399	524.94	17968 R.		
Copper, cast, ...	8.607	537.94	19072		
" rolled, ...	8.864 F.	554.00	32326 F.		
" wire-drawn, ...	8.878	554.87	61228		
Gold, ...	19.258 Ha.	1203.62			
	19.361 Ha.	1210.06			
Iron, cast, ...					
Caron No. 2, cold blast,	7.066 H.	441.62	16683 H.	106375 H.	38556 H
" " hot	7.046 H.	440.37	13505 H.	108540 H.	37503 H
Devon No. 3, cold	7.295 H.	455.94			36288 H
" " hot	7.229 H.	451.81	21907 H.	145435 H.	43497 H
Buffery No. 1, cold	7.079 H.	442.44	17466 H.	93336 H.	37503 H
" " hot	6.998 H.	437.37	13434 H.	86397 H.	35316 H
Iron, wrought, ...					
English bar, ...	7.700	481.25	57120 L.	56000 ? G.	54000 G
Welsh "			61960 T.		
Swedish "			61960 T.		
" "	7.478 F.	467.37	58184 F.		
Lancaster Co., 2, ...	7.740 F.	483.75	58661 F.		
Tennessee "	7.805 F.	487.81	52099 F.		
Missouri "	7.722 F.	482.62	47909 F.		
Iron wire, ...					
English, 3/4"	7.727 F.	482.94	80214 T.		
Phillips' 3/4"			84186 F.		
" 1/2"			73888 F.		
" 1/4"			89162 F.		
Lead, cast, ...	11.446 M.	715.37	1824 R.		
Lead wire, ...	11.217	707.31	2581 M.		
Mercury, ...	13.598 W.	849.87			
Platina, ...	22.669 Ha.	1416.81			
Silver, ...	10.474 Ha.	654.62	40902 M.		
Steel, soft, ...	7.780	480.25	120000		
razor-tempera, ...	7.840	490.00	160000		
Tin, cast, ...	7.291	456.62	6322 M.		
Zinc, fused, ...	7.050 W.	440.32			
" rolled, ...	7.540 W	471.25			
<i>Woods.</i>					
Ash, English, ...	.760 B	47.50	15000 B.	{ 8683 H. 9363 H. 3297 H. 6402 H. }	12156 I
Birch, English, ...	.792 B.	49.50	15000 B.	{ 1663 H. 8771 H. 5074 H. 5074 H. }	10920 I
" American, ...	.648 B.	40.50	20000 B.		9621 I
Box, ...	.960 B.	60.00	11400 B.		
Cedar, Canadian, ...	.909 C.	56.81	13300 R.		
Chestnut, ...	.657 R.	41.06	12400		
Deal, Christiania mill, ...	.698 B.	43.62			9864 F
" Memel "	.590 B.	36.87			10386 F
" Norway Spruce, ...	.340	21.25	17600		
" English, ...	.470	29.37	7000		
Elm, seasoned, ...	.553 B	34.53	13489 M.	10331 B.	8778 F
Fir, New England, ...	.553 B.	34.56		{ 5740 F. 6586 H. }	8412 F
" Riga, ...	.753 B.	47.06	12000 B.		8412 F
Lignum-vitæ, ...	1.220	76.25	11800 M.		
Mahogany, Spanish, ...	.800	50.00	16500	{ 8198 F. 8193 H. }	

Materials.	Specific Gravity.	Weight per Cubic Foot in lbs.	Tensile Strength per Square Inch in lbs.	Crushing Force per Square Inch in lbs.	Modulus of Rupture <i>S</i> in lbs.
<i>Woods.</i>					
Oak, English, ...	.934 B.	58.37	10000 B.	{ 6184 H. 10058 H. 4231 H. 5932 H. 6790 H. 6790 H. 5395 H. 7518 H. }	10032 B.
" Canadian, ...	.872 B.	54.50	10253		10596 B.
Pine, pitch, ...	.660 B.	41.25	7818 M.		9792 B.
" red, ...	.657 B.	41.06			8046 B.
" American, white, ...	.455 Br.	23.44			7829 Br.
" " Southern, ...	.372 Br.	54.50			13937 Br.
Poplar, ...	.333 M.	23.94	7200 Be.	{ 3107 H. 5124 H.	
Teak, ...	.745 B.	46.56	15000 B.	12101 H.	14772 B.
<i>Other Materials.</i>					
Brick, red, ...	2.163 R.	135.50	230	803 R.	340 W.
" pale red, ...	2.035 R.	130.31	300	562 R.	130 W.
Chalk, ...	2.784 1.869	174.00 116.81		501 R.	
Coal, Penn. anthracite, ...	1.327 Ta. 1.700 Ta.	82.94 106.25			
" " semi-bituminous, ...	1.453 Ta.	90.81			
" Md. "	1.552 Ta.	97.00			
" Penn. bituminous, ...	1.312 Ta.	82.00			
" Ohio "	1.270 Ta.	79.37			
" English "	1.259 Ta.	73.69			
Earth, ...					
loamy hard-stamped, fresh, ...	2.060 W.	123.75			
" " dry, ...	1.930 W.	121.62			
garden, fresh, ...	2.051 W.	123.12			
" dry, ...	1.630 W.	101.87			
dry, poor, ...	1.340 W.	83.75			
Glass, plate, ...	2.453	153.31	9420		
Gravel, ...	1.920	120.00			
Granite, Aberdeen, ...	2.625 R.	164.06		10914 R.	
Ivory, ...	1.826	114.12	16626		
Limestone, ...	2.400 W.	150.00		1500 W.	700 W.
" "	2.860 W.	178.75		6000 W.	1700 W.
Marble, white Italian, ...	2.638 H.	164.87		9583 G.	1062
" black Galloway, ...	2.695 H.	163.44			2664
Masonry, quarry stone, dry, ...	2.400 W.	150.00			
" sandstone, ...	2.050 W.	123.12			
" brick, dry, ...	1.470 W. 1.590 W.	91.87 99.37			
Ropes, ...					
hemp, under 1 inch diam., ...			9230 W.		
" from 1 to 3 in. "			7218 W.		
" over 3 inches "			5156 W.		
Sand, river, ...	1.886	117.87			
Sandstone, ...	1.900 W. 2.700 W.	118.75 163.75		1400 W. 13000 W.	600 W. 800 W.
" Dundee, ...	2.530 R.	158.12		6630 R.	
" Derby, red and friable, ...	2.316 R.	144.75		3142 R.	
Slate, Welsh, ...	2.833	180.50	12300		
" Scotch, ...			9600		

TABLE IX.

MAGNETIC VARIATION.

THE following table has been made up from various sources, principally, however, from the results of the United States Coast Survey, kindly furnished in manuscript by the Superintendent, Prof. A. D. Bache. "These results," he remarks in an accompanying note, "are from preliminary computations, and may be somewhat changed by the final ones." Among the other sources may be mentioned the Smithsonian Contributions for 1852, Trans. Am. Phil. Soc. for 1846, Lond. Phil. Trans. for 1849, Silliman's Journal for 1838, 1840, 1846, and 1852, and the various American, British, and Russian Government Observations. The latitudes and longitudes here given are not always to be relied on as minutely correct. Many of them, for places in the Western States, were confessedly taken from maps and other uncertain sources. Those of the Coast Survey Stations, however, as well as those of American and foreign Government Observatories and Stations, are presumed to be accurate.

It will be seen that the variation of the magnetic needle in the United States is in some places west and in others east. *The line of no variation* begins in the northwest part of Lake Huron, and runs through the middle of Lake Erie, the southwest corner of Pennsylvania, the central parts of Virginia, and through North Carolina to the coast. All places on the east of this line have the variation of the needle west,—all places on the west of this line have the variation of the needle east; and, as a general rule, the farther a place lies from this line, the greater is the variation. The position of the line of no variation given above is the position assigned to it by Professor Loomis for the year 1840. But this line has for many years been moving slowly westward, and this motion still continues. Hence places whose variation is west are every year farther and farther from this line, so that the variation west is constantly increasing. On the contrary, places whose variation is east are every year nearer and nearer to this line, so that the variation east is constantly decreasing. The rate of this increase or decrease, as the case may be, is said to average about 2' for the Southern States, 4' for the Middle and Western States, and 6' for the New England States.* The increase in Washington in 1840–2 was 3' 44.2"; in Toronto in 1841–2 it was 4' 46 2". The changes in

* Prof Loomis in Silliman's Journal. Vol. XXXIX., 1840.

Cambridge, Mass. may be seen from the following determinations of the variation, taken from the Memoirs of the American Academy for 1846.

Cambridge, 1708,	9 0	Cambridge, 1788,	6 38
" 1742,	8 0	Boston, 1793,	6 30
" 1757,	7 20	Salem, 1805,	5 57
" 1761,	7 14	" 1808,	5 20
" 1763,	7 0	" 1810,	6 22
" 1780,	7 2	Cambridge, 1810,	7 30
" 1782,	6 46	" 1835,	8 51
" 1783,	6 52	" 1840,	9 18

But besides this change in the variation, which may be called secular, there is an annual and a diurnal change, and very frequently there are irregular changes of considerable amount. With respect to the annual change, the variation west in the Northern hemisphere is generally found to be somewhat greater, and the variation east somewhat less, in the summer than in the winter months. The amount of this change is different in different places, but it is ordinarily too small to be of any practical importance. The diurnal change is well determined. At Washington in 1840-2, the mean diurnal change in the variation was,* —

Summer, 10 4.1 Autumn, 6 21.2 Winter, 5 9.1 Spring, 8 10.7
At Toronto the means were, † —

	1841.	1843.	1845.	1847.	1849.	1850.	1851.
Winter,	6.67	5.64	5.73	7.28	8.25	8.01	7.01
Spring and Autumn,	9.46	9.36	9.15	10.08	12.25	10.90	10.82
Summer,	12.38	11.70	13.36	13.84	14.80	13.74	12.61

The diurnal change in the variation is such that the north end of the needle in the Northern hemisphere attains its extreme westerly position about 2 o'clock, P. M., and its extreme easterly position about 8 o'clock, A. M. In places, therefore, whose variation is west, the maximum variation occurs about 2 P. M., while in places whose variation is east, the maximum variation occurs about 8 A. M. In Washington, according to the report of Lieutenant Gilliss, the maximum variation, taking the mean of two years' observations, occurs at 1^h 33^m. P. M., the minimum at 8^h 6^m. A. M.

The determinations of the Coast Survey are distinguished by the letters C. S. attached to the name of the observer. In some instances the name of the nearest town has been added to the name of the Coast Survey station.

* Lieut. Gilliss's Report, Senate Document 172, 1845

† London Philosophical Transactions, 1852

Place.	Latitude.	Longitude.	Authority.	Date.	Variation.
<i>Maine.</i>					
Agamenticus,	43 13.4	70 41.2	T. J. Lee, C. S.	Sept., 1817	10 10.0 W.
Bethel,	44 28.0	70 51.0	J. Locke,	June, 1845	11 50.0 "
Bowdoin Hill, Portland,	43 33.8	70 16.2	J. E. Hilgard, C. S.	Aug., 1851	11 41.1 "
Cape Neddick, York	43 11.6	70 36.1	J. E. Hilgard, C. S.	Aug., 1851	11 9.0 "
Cape Small,	43 46.7	69 50.4	G. W. Dean, C. S.	Oct., 1851	12 5.5 "
Kennebunkport,	43 21.4	70 27.8	J. E. Hilgard, C. S.	Aug., 1851	11 23.6 "
Kittery Point,	43 4.8	70 43.3	J. E. Hilgard, C. S.	Sept., 1850	10 30.2 "
Mt. Pleasant,	44 1.6	70 49.0	G. W. Dean, C. S.	Aug., 1851	14 32.0 "
Portland,	43 41.0	70 20.5	J. Locke,	June, 1845	11 23.3 "
Richmond Island,	43 32.4	70 14.0	J. E. Hilgard, C. S.	Sept., 1850	12 17.9 "
<i>New Hampshire.</i>					
Fabyan's Hotel,	44 16.0	71 29.0	J. Locke,	June, 1845	11 32.0 W.
Hanover,	43 42.0	72 10.0	Prof. Young,	1839	9 15.0 "
Isle of Shoals,	42 59.2	70 36.5	T. J. Lee, C. S.	Aug, 1847	10 3.4 "
Patuccawa,	43 7.2	71 11.5	G. W. Dean, C. S.	Aug., 1849	10 42.9 "
Unkonoosuc,	42 59.0	71 35.0	J. S. Ruth, C. S.	Oct., 1848	9 5.6 "
<i>Vermont.</i>					
Burlington,	44 27.0	73 10.0	J. Locke,	June, 1845	9 22.0 W.
<i>Massachusetts.</i>					
Annis-squam,	42 39.4	70 40.3	G. W. Keely, C. S.	Aug., 1849	11 36.7 W.
Baker's Island,	42 32.2	70 46.8	G. W. Keely, C. S.	Sept., 1849	12 17.0 "
Blue Hill, Milton,	42 12.7	71 6.5	T. J. Lee, C. S.	Sept. and } Oct., 1845 }	9 13.8 "
Cambridge,	42 22.9	71 7.2	W. C. Bond,	1852	10 8.0 "
Chappaquidick, Edgartown,	41 22.7	70 28.7	T. J. Lee, C. S.	July, 1846	8 47.7 "
Coddon's Hill, Marblehead,	42 31.0	70 50.9	G. W. Keely, C. S.	Sept., 1849	11 49.8 "
Copecut Hill,	41 43.3	71 3.3	T. J. Lee, C. S.	Sept. and } Oct., 1844 }	9 12.1 "
Dorchester,	42 19.0	71 4.0	W. C. Bond,	1839	9 2.0 "
Fort Lee, Salem,	42 31.9	70 52.1	G. W. Keely, C. S.	Aug., 1849	10 14.5 "
Hyannis,	41 33.0	70 18.0	T. J. Lee, C. S.	Aug., 1846	9 22.0 "
Indian Hill,	41 25.7	70 40.3	T. J. Lee, C. S.	Aug., 1846	8 49.3 "
Little Nahant,	42 26.2	70 55.5	G. W. Keely, C. S.	Aug., 1849	9 40.9 "
Nantasket,	42 18.2	70 54.0	T. J. Lee, C. S.	Sept., 1847	9 33.5 "
Nantucket,	41 17.0	70 6.0	T. J. Lee, C. S.	July, 1846	9 14.0 "
New Bedford,	41 38.0	70 54.0	T. J. Lee, C. S.	Oct., 1845	8 54.6 "
Shootflying Hill,	41 41.1	70 20.5	T. J. Lee, C. S.	Aug., 1846	9 40.1 "
Tarpaulin Cove,	41 28.1	70 45.1	T. J. Lee, C. S.	Aug., 1846	9 10.1 "
<i>Rhode Island.</i>					
Beacon-pole Hill,	41 59.7	71 26.7	T. J. Lee, C. S.	Oct. and } Nov., 1844 }	9 29.8 W.
McSparran Hill,	41 29.7	71 27.1	T. J. Lee, C. S.	July, 1844	8 53.3 "
Point Judith,	41 21.9	71 28.9	R. H. Fauntleroy, C. S.	Sept., 1847	8 59.4 "
Spencer Hill,	41 40.7	71 29.3	T. J. Lee, C. S.	July and } Aug. 1844 }	9 11.9 "
<i>Connecticut.</i>					
Black Rock, Fairfield,	41 8.6	73 12.6	J. Renwick, C. S.	Sept., 1845	6 53.5 W.
Bridgeport,	41 10.0	73 11.0	J. Renwick, C. S.	Sept., 1845	6 19.3 "
Fort Wooster,	41 16.9	72 53.2	J. S. Ruth, C. S.	Aug., 1848	7 26.4 "
Groton Point, New London,	41 18.0	72 0.0	J. Renwick, C. S.	Aug., 1845	7 23.5 "

TABLE IX. MAGNETIC VARIATION.

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Place.	Latitude.	Longitude.	Authority.	Date.	Variation.
Milford,	41° 16.0	73° 1.0	J. Renwick, C. S.	Sept., 1845	0 38.3 W.
New Haven, Pavilion,	41 18.5	72 55.4	J. S. Ruth, C. S.	Aug., 1848	6 37.5 "
New Haven, Yale College,	41 18.5	72 55.4	J. Renwick, C. S.	Sept., 1845	6 17.3 "
Norwalk,	41 7.1	73 24.2	J. Renwick, C. S.	Sept., 1844	6 46.3 "
Oyster Point, New Haven,	41 17.0	72 55.4	J. S. Ruth, C. S.	Aug., 1848	6 32.3 "
Eachem's Head,	41 17.0	72 43.0	J. Renwick, C. S.	Aug., 1845	6 15.2 "
Gulford,	40 59.5	73 39.4	J. Renwick, C. S.	Sept., 1844	6 1.6 "
Sawpits,	41 16.0	72 20.0	J. Renwick, C. S.	Aug., 1845	6 49.9 "
Saybrook,	41 3.5	73 32.0	J. Renwick, C. S.	Sept., 1844	6 40.4 "
Stamford,	41 20.0	71 54.0	J. Renwick, C. S.	Aug., 1845	7 38.2 "
Stonington,					
<i>New York.</i>					
Albany,	42 39.0	73 44.0	Regents' Report,	1836	6 47.0 W.
Bloomingtondale Asylum,	40 48.8	73 57.4	J. Locke, C. S.	April, 1846	5 10.9 "
Cole, Staten Island,	40 31.8	74 13.8	J. Locke, C. S.	April, 1846	5 33.8 "
Drowned Meadow, L. I.,	40 56.1	73 3.5	J. Renwick, C. S.	Sept., 1845	6 3.6 "
Flatbush, L. I.,	40 40.2	73 57.7	J. Locke, C. S.	April, 1846	5 54.6 "
Greenport, L. I.,	41 6.0	72 21.0	J. Renwick, C. S.	Aug., 1845	7 14.6 "
Leggett,	40 48.9	73 53.0	R. H. Fauntleroy, C. S.	Oct., 1847	5 40.6 "
Lloyd's Harbor, L. I.,	40 55.6	73 21.8	J. Renwick, C. S.	Sept., 1844	6 12.5 "
New Rochelle,	40 52.5	73 47.0	J. Renwick, C. S.	Sept., 1844	5 31.5 "
New York,	40 42.7	74 0.1	J. Renwick, C. S.	Sept., 1845	6 25.3 "
Oyster Bay, L. I.,	40 52.3	73 31.3	J. Renwick, C. S.	Sept., 1844	6 53.6 "
Rouse's Point,	45 0.0	73 21.0	Boundary Survey,	Oct., 1845	11 23.0 "
Sands Lighthouse, L. I.,	40 51.9	73 43.5	R. H. Fauntleroy, C. S.	Oct., 1847	6 9.7 "
Sands Point, L. I.,	40 52.0	73 43.0	J. Renwick, C. S.	Sept., 1845	7 14.6 "
Watchhill, Fire Island,	40 41.4	72 58.9	R. H. Fauntleroy, C. S.	Oct., 1847	7 33.5 "
West Point,	41 25.0	73 56.0	Prof. Davies,	Sept., 1835	6 32.0 "
<i>New Jersey.</i>					
Cape May Light-house,	33 55.8	74 57.6	J. Locke, C. S.	June, 1846	3 3.2 W.
Chew,	39 48.2	75 9.7	J. Locke, C. S.	July, 1846	3 20.4 "
Church Landing,	39 40.9	75 30.3	J. Locke, C. S.	June, 1846	5 45.8 "
Egg Island,	39 10.4	75 7.8	J. Locke, C. S.	June, 1846	3 18.2 "
Hawkins,	39 25.5	75 17.1	J. Locke, C. S.	June, 1846	2 58.7 "
Mt. Rose, Princeton,	40 22.2	74 42.9	J. E. Hilgard, C. S.	Aug., 1852	5 31.8 "
Newark,	40 44.8	74 7.0	J. Locke, C. S.	April, 1846	5 32.7 "
Pine Mountain,	39 25.0	75 19.9	J. Locke, C. S.	June, 1846	2 52.0 "
Port Norris,	39 14.5	75 1.0	J. Locke, C. S.	June, 1846	3 6.5 "
Sandy Hook,	40 28.0	73 59.8	J. Renwick, C. S.	Aug., 1844	5 54.0 "
Town Bank, Cape May,	33 58.6	74 57.4	J. Locke, C. S.	June, 1846	3 3.2 "
Tucker's Island,	39 30.8	74 16.9	T. J. Lee, C. S.	Nov., 1846	4 23.8 "
White Hill, Bordentown,	40 8.3	74 43.8	J. Locke, C. S.	April, 1846	4 22.5 "
<i>Pennsylvania.</i>					
Girard College,					
Philadelphia,	39 58.4	75 9.9	J. Locke, C. S.	May, 1846	3 50.7 W.
Pittsburg,	40 26.0	79 58.0	J. Locke,	May, 1845	0 33.1 "
Vanuxem, Bristol,	40 5.9	74 52.7	J. Locke, C. S.	July, 1846	4 20.5 "

* Local attraction exists here, according to Prof. Locke.

Place.	Latitude.	Longitude.	Authority.	Date.	Variation.
<i>Delaware.</i>					
Bombay Hook Lighthouse,	39 21.8	75 30.3	J. Locke, C. S.	June, 1846	3 17.9 W.
Fort Delaware, Delaware River,	39 35.3	75 33.8	J. Locke, C. S.	June, 1846	3 16.0 "
Lewes Landing,	38 43.8	75 11.5	J. Locke, C. S.	July, 1846	2 47.7 "
Pilot Town,	38 47.1	75 9.2	J. Locke, C. S.	July, 1846	2 42.2 "
Sawyer,	39 42.0	75 33.5	J. Locke, C. S.	June, 1846	2 47.8 "
Wilmington,	39 44.9	75 33.6	J. Locke, C. S.	May, 1846	2 31.8 "
<i>Maryland.</i>					
Annapolis,	38 56.0	76 35.0	T. J. Lee, C. S.	June, 1845	2 14.0 W.
Bodkin,	39 8.0	76 25.2	T. J. Lee, C. S.	April, 1847	2 2.6 "
Finlay,	39 24.4	76 31.2	J. Locke, C. S.	April, 1846	2 19.5 "
Fort McHenry, Baltimore,	39 15.7	76 34.5	T. J. Lee, C. S.	April, 1847	2 13.0 "
Hill,	38 53.9	76 52.5	G. W. Dean, C. S.	Sept., 1850	2 15.4 "
Kent Island,	39 1.8	76 18.8	J. Heuston, C. S.	July, 1849	2 30.5 "
Marriott's,	38 52.4	76 36.3	T. J. Lee, C. S.	June, 1849	2 5.2 "
North Point,	39 11.7	76 26.3	T. J. Lee, C. S.	July, 1846	1 42.1 "
Osborne's Ruin,	39 27.9	76 16.6	T. J. Lee, C. S.	June, 1845	2 32.4 "
Poole's Island,	39 17.1	76 15.5	T. J. Lee, C. S.	June, 1847	2 28.5 "
Rosanne,	39 17.5	76 42.8	T. J. Lee, C. S.	June, 1845	2 12.0 "
Soper,	39 5.1	76 56.7	G. W. Dean, C. S.	July, 1850	2 7.0 "
South Base, Kent Island,	38 53.8	76 21.7	T. J. Lee, C. S.	June, 1845	2 26.2 "
Susquehanna Light-house, Havre de Grace,	39 32.4	76 4.8	T. J. Lee, C. S.	July, 1847	2 51.1 "
Taylor,	38 59.8	76 27.6	T. J. Lee, C. S.	May, 1847	2 18.4 "
Webb,	39 5.4	76 40.2	G. W. Dean, C. S.	Nov., 1850	2 7.9 "
<i>District of Columbia.</i>					
Causten, Georgetown,	38 55.5	77 4.1	G. W. Dean, C. S.	June, 1851	2 11.3 W.
Washington,	38 53.7	77 2.8	J. M. Gilliss,	June, 1842	1 26.0 "
<i>Virginia.</i>					
Charlottesville,	38 2.0	78 31.0	Prof. Patterson,	1835	0 0.0
Roslyn, Petersburg,	37 14.4	77 23.5	G. W. Dean, C. S.	Aug., 1852	0 26.4 W.
Wheeling,	40 8.0	80 47.0	J. Locke,	April, 1845	2 4.0 E.
<i>North Carolina.</i>					
Bodie's Island,	35 47.5	75 31.6	C. O. Boutelle, C. S.	Dec., 1846	1 13.4 W.
Shellbank,	36 3.3	75 44.1	C. O. Boutelle, C. S.	Mar., 1847	1 44.8 "
Stevenson's Point,	36 6.3	76 11.0	C. O. Boutelle, C. S.	Feb., 1847	1 39.7 "
<i>South Carolina.</i>					
Broach Inlet,	32 46.3	79 48.7	C. O. Boutelle, C. S.	April, 1849	2 16.5 E.
Charleston,	32 41.0	79 53.0	Capt. Barnett,	May, 1841	2 24.0 "
East Base, Edisto,	32 33.3	80 10.0	G. Davidson, C. S.	April, 1850	2 53.6 "
<i>Georgia.</i>					
Athens,	34 0.0	83 20.0	Prof. McCay,	1837	4 31.0 E.
Columbus,	32 23.0	85 10.0	Geol. Survey,	1839	5 30.0 "
Milledgeville,	33 7.0	83 20.0	Geol. Survey,	1838	5 51.0 "
Savannah,	32 5.0	81 5.2	J. R. Illgard, C. S.	April, 1852	3 45.0 "

Place.	Latitude.	Longitude.	Authority.	Date.	Variation.
<i>Florida.</i>					
Cape Florida,	25 39.9	80 9.4	J. E. Hilgard, C. S.	Feb., 1850	4 25.2 E.
Cedar Keys,	29 7.5	83 2.8	J. E. Hilgard, C. S.	Mar., 1852	5 20.5 "
St. Marks Light,	30 4.5	84 12.5	J. E. Hilgard, C. S.	April, 1852	5 29.2 "
Sand Key,	24 27.2	81 52.0	J. E. Hilgard, C. S.	Aug., 1849	5 29.0 "
<i>Alabama.</i>					
Fort Morgan, Mobile Bay,	30 13.8	88 0.4	R. H. Fauntleroy, C. S.	May, 1817	7 3.8 E.
Tuscaloosa,	33 12.0	87 42.0	Prof. Barnard,	1839	7 23.0 "
<i>Mississippi.</i>					
East Pascagoula,	30 20.7	88 31.4	R. H. Fauntleroy, C. S.	June, 1847	7 12.4 E.
<i>Texas.</i>					
Dollar Point, Galveston,	29 26.0	94 53.0	R. H. Fauntleroy, C. S.	April, 1848	8 57.2 E.
Mouth of Sabine,	29 43.9	93 51.5	J. D. Graham,	Feb., 1840	8 40.2 "
<i>Ohio.</i>					
Carrolton,	39 33.0	84 9.0	J. Locke,	Sept., 1845	4 45.4 E.
Cincinnati,	39 6.0	84 22.0	J. Locke,	April, 1845	4 4.0 "
Columbus,	39 57.0	83 3.0	J. Locke,	July, 1845	2 29.3 "
Hudson,	41 15.0	81 26.0	E. Loomis,	April, 1840	0 52.0 "
Marietta,	39 26.0	81 29.0	J. Locke,	April, 1845	2 25.0 "
Oxford,	39 30.0	81 33.0	J. Locke,	Aug., 1845	4 50.0 "
St. Mary's,	40 32.0	84 19.0	J. Locke,	Sept., 1845	3 4.0 "
<i>Tennessee.</i>					
Nashville,	36 10.0	86 49.0	Prof. Hamilton,	1835	7 7.0 E.
<i>Michigan.</i>					
Detroit,	42 24.0	82 58.0	Geol. Report,	1840	2 0.0 E.
<i>Indiana.</i>					
Richmond,	39 49.0	84 47.0	J. Locke,	Sept., 1845	4 52.0 E.
South Hanover,	38 45.0	85 23.0	Prof. Dunn,	1837	4 35.0 "
<i>Illinois.</i>					
Alton,	38 52.0	90 12.0	H. Loomis,	1840	7 45.0 E.
<i>Missouri.</i>					
St. Louis,	38 36.0	89 36.0	Col. Nicolls,	1835	8 49.0 E.
<i>Wisconsin.</i>					
Madison,	43 5.0	89 41.0	U. S. Surveyors,	Nov., 1839	7 30.0 E.
Prairie du Chien,	43 1.0	91 8.0	U. S. Surveyors,	Oct., 1839	9 5.0 "
<i>Iowa.</i>					
Brown's Settlement,	42 2.0	91 18.0	J. Locke,	Sept., 1839	9 4.0 E.
Davenport,	41 30.0	90 34.0	U. S. Surveyors,	Sept., 1839	7 50.0 "
Farmer's Creek,	42 13.0	90 39.0	J. Locke,	Oct., 1839	9 11.0 "
Wapsipinnicon River,	41 44.0	90 39.0	J. Locke,	Sept., 1839	8 25.0 "
<i>California.</i>					
Point Conception,	34 26.9	120 26.0	G. Davidson, C. S.	Sept., 1850	13 49.5 E.

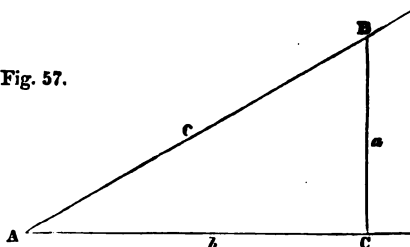
Place.	Latitude.	Longitude.	Authority.	Date.	Variation.
Point Pinos, Monterey,	36 38.0	121 54.0	G. Davidson, C. S.	Feb., 1851	14 58.0 E.
Presidio, San Francisco,	37 47.8	122 27.0	G. Davidson, C. S.	Feb., 1852	15 26.9 "
San Diego,	32 42.0	117 14.0	G. Davidson, C. S.	May, 1851	12 29.0 "
<i>Oregon.</i>					
Cape Disap- pointment,	46 16.6	124 2.0	G. Davidson, C. S.	July, 1851	20 45.0 E.
Ewing Harbor,	42 44.4	124 21.0	G. Davidson, C. S.	Nov., 1851	18 29.2 "
<i>Washington Territory.</i>					
Scarboro' Har- bor,	43 21.8	124 37.2	G. Davidson, C. S.	Aug., 1852	21 30.2 E.
BRITISH AMER- ICA.					
Montreal,	45 30.0	73 35.0	Capt. Lefroy,	1842	8 58.0 W.
Quebec,	46 49.0	71 16.0	Capt. Lefroy,	1842	14 12.0 "
St. Johns, C. E.	45 19.0	73 18.0	Capt. Lefroy,	1842	11 22.0 "
Stanstead,	45 0.0	72 13.0	Boundary Survey,	Nov., 1845	11 33.0 "
Toronto,	43 39.6	79 21.5	British Govern.,	Sept., 1844	1 27.2 "
NEW GRENADA					
Panama,	8 57.2	79 29.4	W. H. Emory,	Mar., 1849	6 54.6 E.
EASTERN HEMI- SPHERE.					
Greenwich, Eng- land,	51 23.0	0 0.0	Prof. Airy,	1841	23 16.0 W.
Makerstoun, Scotland,	55 35.0	2 31.0 W.	J. A. Broun,	1842	25 23.6 "
Paris, France,	48 50.0	2 20.0 E.	Paris Observatory	Nov., 1851	20 25.0 "
Munich, Bava- ria,	48 9.0	11 37.0 "		1842	16 48.0 "
St. Petersburg, Russia,	59 56.0	30 19.0 "	Russian Govern.,	1812	6 21.1 "
Catherinenburg Siberia,	56 51.0	60 34.0 "	Russian Govern.,	1842	6 33.9 E.
Nertchinsk, Si- beria,	51 56.0	116 31.0 "	Russian Govern.,	1842	3 46.9 W.
St. Helena,	15 56.7 S.	5 40.5 W.	British Govern.,	Dec., 1845	23 36.6 "
Cape of Good Hope,	33 56.0 "	18 23.7 E.	British Govern.,	July, 1846	29 8.0 "
Hobarton, Van Diemen's Id.,	42 52.5	147 27.5 "	British Govern.,	Dec., 1848	10 8.0 E.

TABLE X.

TRIGONOMETRICAL AND MISCELLANEOUS FORMULÆ

LET A (fig. 57) be any acute angle, and let a perpendicular BC be drawn from any point in one side to the other side. Then, if the sides

Fig. 57.



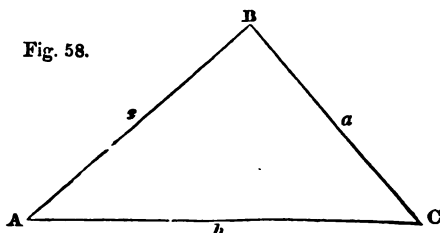
of the right triangle thus formed are denoted by letters, as in the figure, we shall have these six formulæ:—

- | | |
|-----------------------------|---|
| 1. $\sin. A = \frac{a}{c}.$ | 4. $\operatorname{cosec}. A = \frac{c}{a}.$ |
| 2. $\cos. A = \frac{b}{c}.$ | 5. $\sec. A = \frac{c}{b}.$ |
| 3. $\tan. A = \frac{a}{b}.$ | 6. $\cot. A = \frac{b}{a}.$ |

Solution of Right Triangles (fig. 57).

	Given.	Sought.	Formulæ.
7	a, c	A, B, b	$\sin. A = \frac{a}{c}, \cos. B = \frac{a}{c}, b = \sqrt{(c+a)(c-a)}$
8	a, b	A, B, c	$\tan. A = \frac{a}{b}, \cot. B = \frac{a}{b}, c = \sqrt{a^2 + b^2}.$
9	A, a	B, b, c	$B = 90^\circ - A, b = a \cot. A, c = \frac{a}{\sin. A}.$
10	A, b	B, a, c	$B = 90^\circ - A, a = b \tan. A, c =$
11	A, c		$a = c \sin. A, b =$

Solution of Oblique Triangles (fig. 58).



	Given.	Sought.	Formulae.
12	A, B, a	b	$b = \frac{a \sin. B}{\sin. A}.$
13	A, a, b	B	$\sin. B = \frac{b \sin. A}{a}.$
14	a, b, C	$A - B$	$\tan. \frac{1}{2} (A - B) = \frac{(a - b) \tan. \frac{1}{2} (A + B)}{a + b}$
15	a, b, c	A	$\left\{ \begin{array}{l} \text{If } s = \frac{1}{2} (a + b + c), \quad \sin. \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}} \\ \cos. \frac{1}{2} A = \sqrt{\frac{s(s - a)}{bc}}, \quad \tan. \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}} \\ \sin. A = \frac{2 \sqrt{s(s - a)(s - b)(s - c)}}{bc}. \end{array} \right.$
16	A, B, C, a	area	$\text{area} = \frac{a^2 \sin. B \sin. C}{2 \sin. A}.$
17	A, b, c	area	$\text{area} = \frac{1}{2} bc \sin. A.$
18	a, b, c	area	$s = \frac{1}{2} (a + b + c), \quad \text{area} = \sqrt{s(s - a)(s - b)(s - c)}.$

General Trigonometrical Formulae.

- 19 $\sin.^2 A + \cos.^2 A = 1.$
- 20 $\sin. (A \pm B) = \sin. A \cos. B \pm \sin. B \cos. A.$
- 21 $\cos. (A \pm B) = \cos. A \cos. B \mp \sin. A \sin. B.$
- 22 $\sin. 2 A = 2 \sin. A \cos. A.$
- 23 $\cos. 2 A = \cos.^2 A - \sin.^2 A = 1 - 2 \sin.^2 A = 2 \cos.^2 A - 1.$
- 24 $\sin.^2 A = \frac{1}{2} - \frac{1}{2} \cos. 2 A.$
- 25 $\cos.^2 A = \frac{1}{2} + \frac{1}{2} \cos. 2 A.$
- 26 $\sin. A + \sin. B = 2 \sin. \frac{1}{2} (A + B) \cos. \frac{1}{2} (A - B).$
- 27 $\sin. A - \sin. B = 2 \cos. \frac{1}{2} (A + B) \sin. \frac{1}{2} (A - B).$
- 28 $\cos. A + \cos. B = 2 \cos. \frac{1}{2} (A + B) \cos. \frac{1}{2} (A - B).$
- 29 $\cos. B - \cos. A = 2 \sin. \frac{1}{2} (A + B) \sin. \frac{1}{2} (A - B).$
- 30 $\sin.^2 A - \sin.^2 B = \cos.^2 B - \cos.^2 A = \sin. (A + B) \sin. (A - B).$
- $\cos.^2 A - \sin.^2 B = \cos. (A + B) \cos. (A - B).$

$$\begin{aligned}
32 \quad \tan. A &= \frac{\sin. A}{\cos. A} \\
33 \quad \cot. A &= \frac{\cos. A}{\sin. A} \\
34 \quad \tan. (A \pm B) &= \frac{\tan. A \pm \tan. B}{1 \mp \tan. A \tan. B} \\
35 \quad \tan. A \pm \tan. B &= \frac{\sin. (A \pm B)}{\cos. A \cos. B} \\
36 \quad \cot. A \pm \cot. B &= \pm \frac{\sin. (A \pm B)}{\sin. A \sin. B} \\
37 \quad \frac{\sin. A + \sin. B}{\sin. A - \sin. B} &= \frac{\tan. \frac{1}{2} (A + B)}{\tan. \frac{1}{2} (A - B)} \\
38 \quad \frac{\sin. A + \sin. B}{\cos. A + \cos. B} &= \tan. \frac{1}{2} (A + B) \\
39 \quad \frac{\sin. A + \sin. B}{\cos. B - \cos. A} &= \cot. \frac{1}{2} (A - B) \\
40 \quad \frac{\sin. A - \sin. B}{\cos. A + \cos. B} &= \tan. \frac{1}{2} (A - B) \\
41 \quad \frac{\sin. A - \sin. B}{\cos. B - \cos. A} &= \cot. \frac{1}{2} (A + B) \\
42 \quad \tan. \frac{1}{2} A &= \frac{\sin. A}{1 + \cos. A} \\
43 \quad \cot. \frac{1}{2} A &= \frac{\sin. A}{1 - \cos. A}
\end{aligned}$$

Miscellaneous Formulæ.

	Sought.	Given.	Formula.
44	Area of Circle	Radius = r	πr^2 .
45	Ellipse	Semi-axes = a and b	$\pi a b$.
46	Parabola	Chord = c , height = h	$\frac{2}{3} c h$.*
47	Regular Polygon	{ Side = a , number of sides = n }	$\frac{1}{2} a^2 n \cot. \frac{180^\circ}{n}$.
	Surface of		
48	Sphere	Radius = r	$4 \pi r^2$.
49	Zone	Radius = r , height = h	$2 \pi r h$.
50	Spherical Polygon	{ Radius of sphere = r sum of angles = S number of sides = n }	$\pi r^2 \times \frac{S - (n - 2) 180^\circ}{180^\circ}$
	Solidity of		
51	Prism or Cylinder	Base = b , height = h	$b h$.
52	Pyramid or Cone	Base = b , height = h	$\frac{1}{3} b h$.
53	Frustum of Pyr- amid or Cone }	{ Bases = b and b_1 , height = h }	$\frac{1}{3} h (b + b_1 + \sqrt{b b_1})$

* The area of a circular segment on railroad curves, where the chord is very long proportion to the height, may be found with great accuracy by the above form.

	Sought.	Given.	Formula.
54	Sphere	Radius = r	$\frac{4}{3} \pi r^3$.
55	Spherical Segment	$\left\{ \begin{array}{l} \text{Radii of bases} = r \\ \text{and } r_1, \text{ height} = h \end{array} \right\}$	$\frac{1}{2} \pi h (r^2 + r_1^2 + \frac{1}{3} h^2)$
56	Prolate Spheroid	$\left\{ \begin{array}{l} \text{Semi-transverse axis} \\ \text{of ellipse} = a \end{array} \right\}$	$\frac{4}{3} \pi a b^2$.
57	Oblate Spheroid	$\left\{ \begin{array}{l} \text{Semi-conjugate axis} \\ \text{of ellipse} = b \end{array} \right\}$	$\frac{4}{3} \pi a^2 b$.
58	Paraboloid	$\left\{ \begin{array}{l} \text{Radius of base} = r, \\ \text{height} = h \end{array} \right\}$	$\frac{1}{2} \pi r^2 h$.

$$\pi = 3.14159 \ 26535 \ 89793 \ 23846 \ 26433 \ 83280.$$

$$\text{Log. } \pi = 0.49714 \ 98726 \ 94133 \ 85435 \ 12682 \ 88291$$

$$\text{United States Standard Gallon} = 231 \text{ cub. in.} = 0.133681 \text{ cub. ft.}$$

$$\text{“ “ “ Bushel} = 2150.42 \text{ “} = 1.244456 \text{ “}$$

$$\text{British Imperial Gallon} = 277.27384 \text{ “} = 0.160459 \text{ “}$$

	According to Hassler.	As usually given.
French Metre,	= 3.2817431 ft.,	= 3.280899 ft.
“ Litre,	= 61.0741569 cub. in.,	= 61.02705 cub. in.
“ Kilogram,	= 2.204737 lb. avoird.,	= 2.204597 lb. avoird.

Weight of Cubic Foot of Water,

$$\text{Barom. 30 inches, Therm. Fahr. } 39.83^{\circ}, = 62.379 \text{ lb. avoird.}$$

$$\text{“ “ “ “ } 62^{\circ}, = 62.321 \text{ “}$$

$$\text{Length of Seconds Pendulum at New York} = 39.10120 \text{ inches.}$$

$$\text{“ “ “ “ London} = 39.13908 \text{ “}$$

$$\text{“ “ “ “ Paris} = 39.12843 \text{ “}$$

$$\text{Equatorial Radius of Earth according to Bessel} = 20,923,597.017 \text{ feet}$$

$$\text{Polar “ “ “ “} = 20,853,654.177 \text{ “}$$

TABLE XI.

SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS,

AND

RECIPROCAL OF NUMBERS

FROM 1 TO 1054.

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocal.
1	1	1	1.000000	1.000000	1.00000000
2	4	8	1.4142136	1.2599210	.50000000
3	9	27	1.7320508	1.4122496	.33333333
4	16	64	2.0000000	1.5874011	.25000000
5	25	125	2.2360680	1.7099759	.20000000
6	36	216	2.4494897	1.8171206	.16666667
7	49	343	2.6457513	1.9129312	.14285714
8	64	512	2.8234271	2.0000000	.12500000
9	81	729	3.0000000	2.0800837	.11111111
10	100	1000	3.1622777	2.1544347	.10000000
11	121	1331	3.3166248	2.2239801	.09090909
12	144	1728	3.4641016	2.2894236	.08333333
13	169	2197	3.6055513	2.3513347	.07692307
14	196	2744	3.7416574	2.4101422	.07142857
15	225	3375	3.8729833	2.4662121	.06666667
16	256	4096	4.0000000	2.5193421	.06250000
17	289	4913	4.1231056	2.5712316	.05882352
18	324	5832	4.2126107	2.6207414	.05555556
19	361	6859	4.3588939	2.6634016	.05263157
20	400	8000	4.4721360	2.7144177	.05000000
21	441	9261	4.5825757	2.7589243	.04761904
22	484	10648	4.6904158	2.8020393	.04545454
23	529	12167	4.7958315	2.8438670	.04347826
24	576	13824	4.8989795	2.8844991	.04166667
25	625	15625	5.0000000	2.9240177	.04000000
26	676	17576	5.0990195	2.9624960	.03846153
27	729	19683	5.1961524	3.0000000	.03703703
28	784	21952	5.2915026	3.0365889	.03571428
29	841	24389	5.3851648	3.0723163	.03448275
30	900	27000	5.4772256	3.1072325	.03333333
31	961	29791	5.5677644	3.1413806	.03225806
32	1024	32768	5.6568542	3.1748021	.03125000
33	1089	35937	5.7445626	3.2076343	.03030303
34	1156	39304	5.8309519	3.2396118	.02941176
35	1225	42875	5.9160798	3.2710663	.02857142
36	1296	46656	6.0000000	3.3019272	.02777777
37	1369	50653	6.0827625	3.3322218	.02702702
38	1444	54872	6.1644140	3.3619754	.02631578
39	1521	59319	6.2449930	3.3912114	.02564102
40	1600	64000	6.3245553	3.4199519	.02500000
41	1681	68921	6.4031242	3.4482172	.02439024
42	1764	74038	6.4807407	3.4760266	.02380952
43	1849	79507	6.5574395	3.5033981	.02325581
44	1936	85184	6.6332496	3.5303483	.02272727
45	2025	91125	6.7082039	3.5568933	.02222222
46	2116	97336	6.7823300	3.5830479	.02173913
47	2209	103823	6.8556546	3.6088261	.02127660
48	2304	110592	6.9282032	3.6342411	.02083333
49	2401	117649	7.0000000	3.6593057	.02040816
50	2500	125000	7.0710678	3.6840314	.02000000
51	2601	132651	7.144234	3.7084298	.01960784
52	2704	140608	7.2111026	3.7325111	.01923076
53	2809	148877	7.2801099	3.7562858	.01886792
54	2916	157464	7.3484692	3.7797631	.01851851
55	3025	166375	7.4161995	3.8029525	.01818181
56	3136	175616	7.4833148	3.8258624	.01785714
57	3249	185193	7.5498344	3.8485011	.01754386
58	3364	195112	7.6157731	3.8708766	.01724137
59	3481	205379	7.6811457	3.8929965	.01694915
60	3600	216000	7.7459667	3.9149676	.01666667
61	3721	226981	7.8102497	3.9364972	.01639344
62	3844	238328	7.8740079	3.9578915	.01612903

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
63	3969	250047	7.9372539	3.9790571	.015873016
64	4096	262144	8.0000000	4.0000000	.015625000
65	4225	274625	8.0622577	4.0207256	.015384615
66	4356	287496	8.1240384	4.0112401	.015151515
67	4489	300763	8.1853528	4.0615480	.014925373
68	4624	314482	8.2462113	4.0816551	.014706882
69	4761	328509	8.3066239	4.1015661	.014492754
70	4900	343000	8.3666003	4.1212853	.014285714
71	5041	357911	8.4261498	4.1408178	.014084507
72	5184	373248	8.4852814	4.1601676	.013888889
73	5329	389017	8.5440037	4.1793390	.013696630
74	5476	405224	8.6023253	4.1983364	.013513514
75	5625	421875	8.6602540	4.2171633	.013333333
76	5776	438976	8.7177979	4.2358236	.013157895
77	5929	456533	8.7749644	4.2543210	.012987013
78	6084	474552	8.8317609	4.2726586	.012820513
79	6241	493039	8.8881944	4.2908404	.012658228
80	6400	512000	8.9442719	4.3088695	.012500000
81	6561	531441	9.0000000	4.3267487	.012345679
82	6724	551368	9.0553851	4.3444816	.012195122
83	6889	571787	9.1104336	4.3620707	.012048193
84	7056	592704	9.1651514	4.3795191	.011904762
85	7225	614125	9.2195445	4.3968296	.011764706
86	7396	636056	9.2735185	4.4140049	.011627907
87	7569	658503	9.3273791	4.4310476	.011494253
88	7744	681472	9.3808315	4.4479602	.011363636
89	7921	704969	9.4339811	4.4647451	.011235955
90	8100	729000	9.4868330	4.4814047	.011111111
91	8281	753571	9.5393920	4.4979414	.010989011
92	8464	778698	9.5916630	4.5143574	.010869565
93	8649	804357	9.6436508	4.5306549	.010752688
94	8836	830584	9.6953597	4.5468359	.010638298
95	9025	857375	9.7467943	4.5629026	.010526316
96	9216	884736	9.7979590	4.5788570	.010416667
97	9409	912673	9.8488578	4.5947009	.010309278
98	9604	941192	9.8994949	4.6104363	.010204082
99	9801	970299	9.9498744	4.6260650	.010101010
100	10000	1000000	10.0000000	4.6415888	.010000000
101	10201	1030301	10.0498756	4.6570095	.009900990
102	10404	1061208	10.0995049	4.6723287	.009803922
103	10609	1092727	10.1488916	4.6875482	.009707838
104	10816	1124864	10.1980390	4.7026694	.009613385
105	11025	1157625	10.2469508	4.7176940	.009520310
106	11236	1191016	10.2956301	4.7326235	.009428362
107	11449	1225043	10.3440804	4.7474594	.009337571
108	11664	1259712	10.3923048	4.7622032	.009247925
109	11881	1295029	10.4403065	4.7768562	.009159312
110	12100	1331000	10.4880085	4.7914199	.009071909
111	12321	1367631	10.5356538	4.8058955	.009000000
112	12544	1404928	10.5830052	4.8202845	.008928571
113	12769	1442897	10.6301458	4.8345881	.008857958
114	12996	1481544	10.6770783	4.8488076	.008787193
115	13225	1520875	10.7233053	4.8629442	.008716362
116	13456	1560896	10.7703296	4.8769990	.008645509
117	13689	1601613	10.8166538	4.8909732	.008574709
118	13924	1643032	10.8627805	4.9048681	.008504156
119	14161	1685159	10.9087121	4.9186847	.008433761
120	14400	1728000	10.9544512	4.9324242	.008363333
121	14641	1771561	11.0000000	4.9460874	.008292929
122	14884	1815848	11.0453610	4.9596757	.008222611
123	15129	1860867	11.0905365	4.9731898	.008152381
124	15376	1906624	11.1355237	4.9866310	.008082151

110 TABLE XI. SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocal.
125	15625	1953125	11.1803399	5.0000000	.00909091
126	15876	2000000	11.2219722	5.0132979	.00793658
127	16129	2053833	11.2691277	5.0265257	.00784016
128	16384	2109752	11.3137085	5.0396342	.00774250
129	16641	2167859	11.3575167	5.0527743	.00764398
130	16900	2228100	11.4017543	5.0657970	.00754468
131	17161	2290521	11.4455231	5.0787531	.00744468
132	17424	2355168	11.4899253	5.0916134	.00734398
133	17689	2422067	11.5325626	5.1044637	.00724250
134	17956	2491264	11.5758369	5.1172299	.00714025
135	18225	2562825	11.6189500	5.1299278	.00703725
136	18496	2636816	11.6619033	5.1425632	.00693354
137	18769	2713303	11.7046999	5.1551367	.00682920
138	19044	2792352	11.7473444	5.1676493	.00672437
139	19321	2873919	11.7899261	5.1801015	.00661904
140	19600	2958000	11.8325196	5.1924941	.00651325
141	19881	3044621	11.8749421	5.2049279	.00640700
142	20164	3133808	11.9163753	5.2173103	.00630033
143	20449	3225607	11.9578207	5.2296321	.00619325
144	20736	3319072	12.0000000	5.2419928	.00608576
145	21025	3414265	12.0415916	5.2543879	.00597788
146	21316	3511236	12.0830460	5.2668174	.00586960
147	21609	3610033	12.1243557	5.2792821	.00576093
148	21904	3710712	12.1655251	5.2917825	.00565188
149	22201	3813329	12.2065556	5.3043192	.00554244
150	22500	3375000	12.2474437	5.3168923	.00543261
151	22801	3488821	12.2882057	5.3295040	.00532239
152	23104	3505008	12.3288480	5.3420553	.00521178
153	23409	3522607	12.36933169	5.3545462	.00510078
154	23716	3642064	12.4096736	5.3670767	.00498939
155	24025	3763425	12.4498800	5.3796468	.00487761
156	24336	3786716	12.4899600	5.3921565	.00476544
157	24649	3812003	12.5299111	5.4047057	.00465288
158	24964	3939312	12.5697401	5.4171942	.00454003
159	25281	4068679	12.6094502	5.4297221	.00442688
160	25600	4099000	12.6491106	5.4421895	.00431334
161	25921	4130281	12.6887275	5.4546964	.00420041
162	26244	4262528	12.7282021	5.4672428	.00408708
163	26569	4396747	12.7677153	5.4797287	.00397335
164	26896	4432944	12.8071685	5.4922541	.00385922
165	27225	4570125	12.8465626	5.5047190	.00374469
166	27556	4608306	12.8858987	5.5171234	.00362976
167	27889	4647503	12.9251780	5.5295673	.00351443
168	28224	4787712	12.9644014	5.5419507	.00340070
169	28561	4828949	13.0035680	5.5543735	.00328657
170	28900	4971200	13.0426781	5.5668358	.00317204
171	29241	5014461	13.0817320	5.5792376	.00305811
172	29584	5158736	13.1207307	5.5916789	.00294378
173	29929	5204027	13.1596744	5.6041597	.00282905
174	30276	5249336	13.1985631	5.6166800	.00271492
175	30625	5295665	13.2373968	5.6292397	.00260139
176	30976	5342016	13.2761755	5.6417389	.00248746
177	31329	5388389	13.3149092	5.6542776	.00237313
178	31684	5434784	13.3535979	5.6668559	.00225840
179	32041	5481201	13.3922416	5.6793737	.00214327
180	32400	5527640	13.4308403	5.6919310	.00202774
181	32761	5574101	13.4693940	5.7045277	.00191281
182	33124	5620584	13.5079027	5.7171639	.00179748
183	33489	5667089	13.5463664	5.7298396	.00168175
184	33856	5713616	13.5847851	5.7425549	.00156662
185	34225	5760165	13.6231588	5.7553097	.00145109
186	34596	5806736	13.6614875	5.7680640	.00133516

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
187	34969	6539203	13.6747943	5.7184791	.005347594
188	35344	6644672	13.7113092	5.7266543	.005319149
189	35721	6751269	13.7477271	5.7387936	.005291005
190	36100	6859000	13.7840488	5.7488971	.005263158
191	36481	6967871	13.8202750	5.7589652	.005235602
192	36864	7077888	13.8564065	5.7689982	.005208333
193	37249	7189057	13.8924440	5.7789966	.005181347
194	37636	7301384	13.9283983	5.7889604	.005154639
195	38025	7414875	13.9642400	5.7988900	.005128205
196	38416	7529536	14.0000000	5.8087857	.005102041
197	38809	7645373	14.0356658	5.8186479	.005076142
198	39204	7762392	14.0712473	5.8284767	.005050505
199	39601	7880599	14.1067360	5.8382725	.005025126
200	40000	8000000	14.1421336	5.8480355	.005000000
201	40401	8120601	14.1774469	5.8577660	.004975124
202	40804	8242408	14.2126704	5.8674643	.004950495
203	41209	8365427	14.2478068	5.8771307	.004926108
204	41616	8489664	14.2828569	5.8867653	.004901961
205	42025	8615125	14.3178211	5.8963685	.004878049
206	42436	8741816	14.3527001	5.9059406	.004854369
207	42849	8869743	14.3874946	5.9154817	.004830918
208	43264	8998912	14.4222051	5.9249921	.004807692
209	43681	9129329	14.4568323	5.9344721	.004784689
210	44100	9261000	14.4913767	5.9439220	.004761905
211	44521	9393931	14.5258390	5.9533418	.004739336
212	44944	9528128	14.5602198	5.9627320	.004716981
213	45369	9663597	14.5945195	5.9720926	.004694836
214	45796	9800344	14.6287388	5.9814240	.004672897
215	46225	9938375	14.6628783	5.9907204	.004651163
216	46656	10077696	14.6969385	6.0000000	.004629630
217	47089	10218313	14.7309199	6.0092450	.004608295
218	47524	10360232	14.7648231	6.0184617	.004587156
219	47961	10503459	14.7986456	6.0276502	.004566210
220	48400	10648000	14.8323970	6.0368107	.004545455
221	48841	10793861	14.8660687	6.0459435	.004524887
222	49284	10941044	14.8996611	6.0550489	.004504505
223	49729	11089567	14.9331845	6.0641270	.004484305
224	50176	11239424	14.9666295	6.0731779	.004464246
225	50625	11390625	15.0000000	6.0822020	.004444444
226	51076	11543176	15.0332961	6.0911994	.004424779
227	51529	11697083	15.0665192	6.1001702	.004405286
228	51984	11852352	15.0996689	6.1091147	.004385965
229	52441	12008989	15.1327460	6.1180332	.004366812
230	52900	12167000	15.1657509	6.1269257	.004347826
231	53361	12326391	15.1986842	6.1357924	.004329004
232	53824	12487168	15.2315462	6.1446337	.004310345
233	54289	12649337	15.2643375	6.1534495	.004291845
234	54756	12812904	15.2970585	6.1622401	.004273504
235	55225	12977875	15.3297097	6.1710058	.004255319
236	55696	13144256	15.3622915	6.1797466	.004237288
237	56169	13312053	15.3948043	6.1884628	.004219409
238	56644	13481272	15.4272486	6.1971544	.004201681
239	57121	13651919	15.4596248	6.2058218	.004184100
240	57600	13824000	15.4919331	6.2144650	.004166667
241	58081	13997521	15.5241747	6.2230843	.004149378
242	58564	14172488	15.5563492	6.2316797	.004132231
243	59049	14348907	15.5884573	6.2402515	.004115226
244	59536	14526784	15.6204994	6.2487998	.004098361
245	60025	14706125	15.6524758	6.2573250	.004081633
246	60516	14886936	15.6843871	6.2658268	.004065041
247	61009	15069223	15.7162336	6.2743043	.004048583
248	61504	15252992	15.7480187	6.2827575	.004032258

142 TABLE XI. SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
219	62001	15138219	15.7797338	6.2911946	.004016064
270	62500	15625000	15.8113883	6.2996053	.004000000
251	63001	15813251	15.8429795	6.3079935	.003984064
252	63504	16 009008	15.8745079	6.3163596	.003968254
253	64009	16194277	15.9059737	6.3247035	.003952569
254	64516	16387064	15.9373775	6.3330256	.003937008
255	65025	16581375	15.9687194	6.3413257	.003921569
256	65536	16777216	16.0000000	6.3496042	.003906250
257	66049	16974593	16.0312195	6.3578611	.003891051
258	66564	17173512	16.0623794	6.3660963	.003875969
259	67081	17373979	16.0934769	6.3743111	.003861004
260	67600	17576000	16.1245155	6.3825043	.003846154
261	68121	17779581	16.1554914	6.3906765	.003831418
262	68644	17984723	16.1864141	6.3988279	.003816794
263	69169	18191447	16.2172747	6.4069595	.003802281
264	69696	18399744	16.2480763	6.4150697	.003787879
265	70225	18609625	16.2788206	6.4231593	.003773585
266	70756	18821096	16.3095064	6.4312276	.003759398
267	71289	19034163	16.3401316	6.4392767	.003745318
268	71824	19248832	16.3707055	6.4473057	.003731343
269	72361	19465109	16.4012195	6.4553143	.003717473
270	72900	19683000	16.4316767	6.4633011	.003703704
271	73441	19902511	16.4620776	6.4712736	.003690037
272	73984	20123648	16.4924225	6.4792236	.003676471
273	74529	20346417	16.5227116	6.4871511	.003663004
274	75076	20570824	16.5529451	6.4950653	.003649635
275	75625	20796875	16.5831240	6.5029572	.003636364
276	76176	21024576	16.6132477	6.5108300	.003623188
277	76729	21253933	16.6433170	6.5186839	.003610108
278	77284	21484952	16.6733320	6.5265189	.003597122
279	77841	21717639	16.7032931	6.5343351	.003584229
280	78400	21952000	16.7332005	6.5421326	.003571429
281	78961	22188011	16.7630546	6.5499116	.003558719
282	79524	22425668	16.7928556	6.5576722	.003546099
283	80089	22665187	16.8226133	6.5654144	.003533569
284	80656	22906304	16.8522995	6.5731345	.003521127
285	81225	23149125	16.8819430	6.5808413	.003508772
286	81796	23393656	16.9115315	6.5885323	.003496503
287	82369	23639903	16.9410743	6.5962023	.003484321
288	82944	23887872	16.9705627	6.6038545	.003472222
289	83521	24137569	17.0000000	6.6114890	.003460203
290	84100	24389000	17.0293861	6.6191060	.003448276
291	84681	24642171	17.0587221	6.6267054	.003436426
292	85264	24897088	17.0880075	6.6342874	.003424654
293	85849	25153757	17.1172428	6.6418522	.003412969
294	86436	25412184	17.1464282	6.6493998	.003401361
295	87025	25672375	17.1755640	6.6569302	.003389831
296	87616	25934336	17.2046505	6.6644437	.003378378
297	88209	26198073	17.2336879	6.6719403	.003367003
298	88804	26463592	17.2626765	6.6794200	.003355705
299	89401	26730999	17.2916165	6.6868831	.003344482
300	90000	27000000	17.3205081	6.6943295	.003333333
301	90601	27270901	17.3493516	6.7017593	.003322259
302	91204	27543604	17.3781472	6.7091720	.003311258
303	91809	27818127	17.4068952	6.7165700	.003300330
304	92416	28094464	17.4355958	6.7239509	.003289474
305	93025	28372625	17.4642492	6.7313155	.003278659
306	93636	28652616	17.4928557	6.7386641	.003267974
307	94249	28934443	17.5214155	6.7459967	.003257329
308	94864	29218112	17.5499298	6.7533134	.003246753
309	95481	29503629	17.5783968	6.7606143	.003236246
310	96100	29791000	17.6068169	6.7678995	.003225806

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
311	96721	30080231	17.6351921	6.7751690	.003215434
312	97344	30371328	17.6635217	6.7821229	.003225128
313	97969	30664297	17.6918060	6.7896613	.003194888
314	98596	30959144	17.7200451	6.7968444	.00318713
315	99225	31255875	17.7482393	6.8040921	.003174603
316	99856	31554196	17.7763888	6.8112847	.003161557
317	100489	31855013	17.8044938	6.8184620	.003154574
318	101124	32157432	17.8325545	6.8256242	.003144654
319	101761	32461759	17.8605711	6.8327714	.003134796
320	102400	32768000	17.8885438	6.8399037	.003125000
321	103041	33076161	17.9164729	6.8470213	.003115265
322	103684	33386248	17.9443584	6.8541240	.003105590
323	104329	33698267	17.9722008	6.8612120	.003095975
324	104976	34012224	18.0000000	6.8682855	.003086420
325	105625	34328125	18.0277564	6.8753443	.003076923
326	106276	34645976	18.0554701	6.8823888	.003067485
327	106929	34965783	18.0831413	6.8894188	.003058104
328	107584	35287552	18.1107703	6.8964345	.003048780
329	108241	35611289	18.1383571	6.9034359	.003039514
330	108900	35937000	18.1659021	6.9104232	.003030303
331	109561	36264691	18.1934054	6.9173964	.003021148
332	110224	36594368	18.2208672	6.9243556	.003012048
333	110889	36926037	18.2482876	6.9313008	.003003003
334	111556	37259704	18.2756669	6.9382321	.002994012
335	112225	37595375	18.3030052	6.9451496	.002985075
336	112896	37933056	18.3303028	6.9520533	.002976190
337	113569	38272753	18.3575598	6.9589434	.002967359
338	114244	38614472	18.3847763	6.9658198	.002958580
339	114921	38958219	18.4119526	6.9726826	.002949853
340	115600	39304000	18.4390889	6.9795321	.002941176
341	116281	39651821	18.4661853	6.9863681	.002932551
342	116964	40001684	18.4932420	6.9931906	.002923977
343	117649	40353607	18.5202592	7.0000000	.002915452
344	118336	40707584	18.5472370	7.0067962	.002906977
345	119025	41063625	18.5741756	7.0135791	.002898551
346	119716	41421736	18.6010752	7.0203490	.002890173
347	120409	41781923	18.6279360	7.0271058	.002881844
348	121104	42144192	18.6547581	7.0338497	.002873563
349	121801	42508549	18.6815117	7.0405806	.002865330
350	122500	42875000	18.7082869	7.0472987	.002857143
351	123201	43243551	18.7349940	7.0540041	.002849003
352	123904	43614208	18.7616630	7.0606967	.002840909
353	124609	43986977	18.7882942	7.0673767	.002832861
354	125316	44361864	18.8148877	7.0740440	.002824859
355	126025	44738875	18.8414437	7.0806988	.002816901
356	126736	45118016	18.8679623	7.0873411	.002809089
357	127449	45499293	18.8944436	7.0939709	.002801320
358	128164	45882712	18.9208879	7.1005885	.002793596
359	128881	46268279	18.9472953	7.1071937	.002785915
360	129600	46656000	18.9736660	7.1137866	.002778278
361	130321	47045881	19.0000000	7.1203674	.002770683
362	131044	47437928	19.0262976	7.1269360	.002763131
363	131769	47832147	19.0525589	7.1334925	.002755621
364	132496	48228511	19.0787840	7.1400370	.002748153
365	133225	48627025	19.1049732	7.1465695	.002740726
366	133956	49027696	19.1311265	7.1530901	.002733340
367	134689	49430523	19.1572441	7.1596988	.002726003
368	135424	49835512	19.1833261	7.1662957	.002718715
369	136161	50242669	19.2093727	7.1728809	.002711476
370	136900	50652000	19.2353841	7.1794544	.002704286
371	137641	51063521	19.2613601	7.1860163	.002697145
372	138384	51477232	19.2873107	7.1925666	.002690053

111 TABLE XI. SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocal.
373	138049	51856117	19.3132079	7.1984030	.00267985
374	138724	52078324	19.3207966	7.2048322	.00267337
375	139400	52303875	19.3284067	7.2112479	.00266685
376	140076	52531876	19.3360394	7.2176522	.00266034
377	140753	52762333	19.3436947	7.2240450	.00265383
378	141431	52995352	19.3513722	7.2304268	.00264733
379	142110	53230939	19.3590723	7.2367972	.00264083
380	142790	53469090	19.3667957	7.2431565	.00263433
381	143471	53709811	19.3745421	7.2495045	.00262783
382	144153	53953108	19.3823113	7.2558415	.00262133
383	144836	54198987	19.3901033	7.2621675	.00261483
384	145520	54447454	19.3979180	7.2684824	.00260833
385	146205	54698525	19.4057553	7.2747864	.00260183
386	146891	54952206	19.4136152	7.2810794	.00259533
387	147578	55208503	19.4214976	7.2873617	.00258883
388	148266	55467422	19.4294021	7.2936333	.00258233
389	148955	55728969	19.4373287	7.2998950	.00257583
390	149645	55993150	19.4452773	7.3061466	.00256933
391	150336	56259971	19.4532479	7.3123882	.00256283
392	151028	56529438	19.4612404	7.3186198	.00255633
393	151721	56801557	19.4692548	7.3248414	.00254983
394	152415	57075325	19.4772911	7.3310530	.00254333
395	153110	57351748	19.4853492	7.3372546	.00253683
396	153806	57630823	19.4934291	7.3434462	.00253033
397	154503	57912556	19.5015307	7.3496278	.00252383
398	155201	58196944	19.5096540	7.3557994	.00251733
399	155900	58483993	19.5177990	7.3619610	.00251083
400	156600	58773710	19.5259657	7.3681126	.00250433
401	157301	59066101	19.5341544	7.3742542	.00249783
402	158003	59361172	19.5423641	7.3803858	.00249133
403	158706	59658929	19.5505948	7.3865074	.00248483
404	159410	59959380	19.5588465	7.3926190	.00247833
405	160115	60262531	19.5671192	7.3987206	.00247183
406	160821	60568388	19.5754129	7.4048122	.00246533
407	161528	60876957	19.5837275	7.4108938	.00245883
408	162236	61188234	19.5920630	7.4169654	.00245233
409	162945	61502225	19.6004194	7.4230270	.00244583
410	163655	61818936	19.6087967	7.4290786	.00243933
411	164366	62138363	19.6171948	7.4351202	.00243283
412	165078	62460512	19.6256137	7.4411518	.00242633
413	165791	62785389	19.6340534	7.4471734	.00241983
414	166505	63112999	19.6425139	7.4531850	.00241333
415	167220	63443348	19.6509952	7.4591866	.00240683
416	167936	63776433	19.6594973	7.4651782	.00240033
417	168653	64113260	19.6680201	7.4711598	.00239383
418	169371	64452834	19.6765636	7.4771314	.00238733
419	170090	64795161	19.6851277	7.4830930	.00238083
420	170810	65139248	19.6937124	7.4890446	.00237433
421	171531	65486091	19.7023177	7.4949862	.00236783
422	172253	65835696	19.7109436	7.5009178	.00236133
423	172976	66188069	19.7195899	7.5068394	.00235483
424	173700	66543216	19.7282566	7.5127510	.00234833
425	174425	66901143	19.7369437	7.5186526	.00234183
426	175151	67261856	19.7456512	7.5245442	.00233533
427	175878	67625361	19.7543790	7.5304258	.00232883
428	176606	67991664	19.7631271	7.5362974	.00232233
429	177335	68360771	19.7718956	7.5421590	.00231583
430	178065	68732688	19.7806845	7.5480106	.00230933
431	178796	69107421	19.7894938	7.5538522	.00230283
432	179528	69484976	19.7983235	7.5596838	.00229633
433	180261	69865359	19.8071736	7.5655054	.00228983
434	181005	70248576	19.8160441	7.5713170	.00228333

Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
189225	82312875	20.8566536	7.5769849	.002298851
190036	82381856	20.8806130	7.5827865	.002293578
190969	83473153	20.9045450	7.5885793	.002288330
191844	84027672	20.9284495	7.5943633	.002283105
192721	84604519	20.9523268	7.6001385	.002277904
193600	85184000	20.9761770	7.6059049	.002272727
194481	85766121	21.0000000	7.6116626	.002267574
195364	86350888	21.0237960	7.6174116	.002262443
196249	86938307	21.0475652	7.6231519	.002257336
197136	87528384	21.0713075	7.6288837	.002252252
198025	88121125	21.0950231	7.6346067	.002247191
198916	88716536	21.1187121	7.6403213	.002242152
199809	89314623	21.1423745	7.6460272	.002237136
200704	89915392	21.1660105	7.6517247	.002232143
201601	90518849	21.1896201	7.6574138	.002227171
202500	91125000	21.2132034	7.6630943	.002222222
203401	91733851	21.2367606	7.6687665	.002217295
204304	92345408	21.2602916	7.6744303	.002212389
205209	92959677	21.2837967	7.6800857	.002207506
206116	93576664	21.3072758	7.6857328	.002202643
207025	94196375	21.3307290	7.6913717	.002197802
207936	94818816	21.3541565	7.6970023	.002192982
208849	95443993	21.3775583	7.7026246	.002188184
209764	96071912	21.4009346	7.7082388	.002183406
210681	96702579	21.4242853	7.7138448	.002178649
211600	97336000	21.4476106	7.7194426	.002173913
212521	97972181	21.4709106	7.7250325	.002169197
213444	98611128	21.4941853	7.7306141	.002164502
214369	99252847	21.5174348	7.7361877	.002159827
215296	99897344	21.5406592	7.7417532	.002155172
216225	10054625	21.5638587	7.7473109	.002150538
217156	10119496	21.5870331	7.7528606	.002145923
218089	101847563	21.6101823	7.7584023	.002141328
219024	102503232	21.6333077	7.7639361	.002136752
219961	103161709	21.6564078	7.7694620	.002132196
220900	103823000	21.6794834	7.7749901	.002127660
221841	104487111	21.7025344	7.7804904	.002123142
222784	105154018	21.7255610	7.7859928	.002118644
223729	105823817	21.7485632	7.7914875	.002114165
224676	106496424	21.7715411	7.7969745	.002109705
225625	107171875	21.7944947	7.8024538	.002105263
226576	107850176	21.8174242	7.8079254	.002100840
227529	108531333	21.8403297	7.8133892	.002096436
228484	109215352	21.8632111	7.8188456	.002092050
229441	109902239	21.8860686	7.8242942	.002087683
230400	110592000	21.9089023	7.8297353	.002083333
231361	111284611	21.9317122	7.8351688	.002079002
232324	111980168	21.9544984	7.8405949	.002074689
233289	112678587	21.9772610	7.8460134	.002070393
234256	113379904	22.0000000	7.8514244	.002066116
235225	114084125	22.0227155	7.8568281	.002061856
236196	114791256	22.0454077	7.8622242	.002057613
237169	115501303	22.0680765	7.8676130	.002053388
238144	116214272	22.0907220	7.8729944	.002049180
239121	116930169	22.1133444	7.8783684	.002044990
240100	117649000	22.1359436	7.8837352	.002040816
241081	118370771	22.1585198	7.8890946	.002036660
242064	119095488	22.1810730	7.8944463	.002032520
243049	119823167	22.2036033	7.8997917	.002028398
244036	120553784	22.2261108	7.9051294	.002024291
245025	121287375	22.2485955	7.9104599	.002020202
246016	122023936	22.2710575	7.9157832	.002016129

146 TABLE XI. SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocal.
497	247009	122763173	22.2931968	7.9210994	.002012073
498	248004	123505992	22.3159136	7.9264085	.002009032
499	249001	124251199	22.3383079	7.9317104	.002006008
500	250000	125000000	22.3606793	7.9370053	.002003000
501	251001	125751501	22.3830293	7.9422931	.001999908
502	252004	126506008	22.4053565	7.9475739	.001996802
503	253009	127263527	22.4276615	7.9528477	.001993702
504	254016	128024064	22.4499143	7.9581144	.001990617
505	255025	128787625	22.4722051	7.9633743	.001987549
506	256036	129554216	22.4944438	7.9686271	.001984496
507	257049	130323843	22.5166605	7.9738731	.001981457
508	258064	131096512	22.5388553	7.9791122	.001978434
509	259081	131872229	22.5610243	7.9843444	.001975426
510	260100	132651000	22.5831796	7.9895697	.001972434
511	261121	133432831	22.6053091	7.9947883	.001969457
512	262144	134217724	22.6274170	8.0000000	.001966496
513	263169	135005697	22.6495033	8.0052049	.001963549
514	264196	135796744	22.6715681	8.0104032	.001960617
515	265225	136590875	22.6936114	8.0155946	.001957699
516	266256	137388096	22.7156331	8.0207791	.001954796
517	267289	138188413	22.7376310	8.0259574	.001951908
518	268324	138991832	22.7596134	8.0311297	.001949034
519	269361	139798359	22.7815715	8.0362935	.001946174
520	270400	140608000	22.8035085	8.0414515	.001943328
521	271441	141420761	22.8255244	8.0466030	.001940496
522	272484	142235644	22.8475193	8.0517479	.001937678
523	273529	143053667	22.8694933	8.0568862	.001934874
524	274576	143874832	22.8914463	8.0620180	.001932084
525	275625	144700125	22.9133785	8.0671432	.001929308
526	276676	145528556	22.9352899	8.0722620	.001926546
527	277729	146360123	22.9571806	8.0773743	.001923798
528	278784	147194832	22.9790506	8.0824800	.001921064
529	279841	148032689	23.0009000	8.0875794	.001918344
530	280900	148873700	23.0227289	8.0926723	.001915638
531	281961	149717871	23.0445372	8.0977589	.001912946
532	283024	150565208	23.0663252	8.1028390	.001910268
533	284089	151415813	23.0880923	8.1079123	.001907604
534	285156	152269684	23.1098380	8.1129790	.001904954
535	286225	153126825	23.1315623	8.1180414	.001902318
536	287296	153987236	23.1532653	8.1230982	.001900000
537	288369	154850913	23.1749460	8.1281497	.001897696
538	289444	155717852	23.1966043	8.1331950	.001895406
539	290521	156588059	23.2182403	8.1382341	.001893130
540	291600	157461500	23.2398641	8.1432669	.001890868
541	292681	158338281	23.2614767	8.1482935	.001888620
542	293764	159218408	23.2830780	8.1533139	.001886386
543	294849	160101977	23.3046680	8.1583281	.001884166
544	295936	160988994	23.3262467	8.1633362	.001881959
545	297025	161879465	23.3478140	8.1683382	.001879774
546	298116	162773388	23.3693700	8.1733341	.001877601
547	299209	163670761	23.3909147	8.1783238	.001875440
548	300304	164571592	23.4124480	8.1833075	.001873291
549	301401	165475889	23.4339700	8.1882851	.001871154
550	302500	166383650	23.4554807	8.1932567	.001869030
551	303601	167294881	23.4769800	8.1982223	.001866918
552	304704	168209592	23.4984680	8.2031819	.001864828
553	305809	169127783	23.5199447	8.2081357	.001862750
554	306916	170049464	23.5414100	8.2130837	.001860684
555	308025	170974635	23.5628637	8.2180257	.001858630
556	309136	171903306	23.5843060	8.2229617	.001856588
557	310249	172835477	23.6057367	8.2278917	.001854558
558	311364	173771148	23.6271557	8.2328157	.001852539

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
550	312431	174676879	23.6131808	8.2376614	.001788909
550	313600	175616000	23.6643191	8.2425706	.001785714
551	314721	176558481	23.6854386	8.2474740	.001782531
552	315844	177504388	23.7065392	8.2523715	.001779359
553	316969	178453547	23.7276210	8.2572633	.001776199
554	318096	179406144	23.7486842	8.2621492	.001773050
555	319225	180362125	23.7697286	8.2670294	.001769912
556	320356	181321496	23.7907545	8.2719039	.001766784
557	321489	182284263	23.8117618	8.2767726	.001763663
558	322624	183250432	23.8327506	8.2816355	.001760563
559	323761	184220000	23.8537209	8.2864928	.001757469
570	321900	185193000	23.8746723	8.2913444	.001754356
571	323011	186169411	23.8956063	8.2961903	.001751313
572	324184	187149248	23.9165215	8.3010304	.001748252
573	325329	188132517	23.9374184	8.3058651	.001745201
574	326476	189119224	23.9582971	8.3106941	.001742161
575	327625	190109375	23.9791576	8.3155175	.001739130
576	328776	191102976	24.0000000	8.3203353	.001736111
577	329929	192100033	24.0208243	8.3251475	.001733102
578	331084	193100562	24.0416306	8.3299542	.001730104
579	332241	194104539	24.0624188	8.3347553	.001727116
580	333400	195112000	24.0831891	8.3395509	.001724139
581	334561	196122941	24.1039416	8.3443410	.001721170
582	335724	197137368	24.1246762	8.3491266	.001718213
583	336889	198155287	24.1453929	8.3539077	.001715266
584	338056	199176704	24.1660919	8.3586844	.001712329
585	339225	200201625	24.1867732	8.3634566	.001709402
586	340396	201230056	24.2074369	8.3682245	.001706485
587	341569	202262003	24.2280829	8.3729883	.001703578
588	342744	203297472	24.2487113	8.3777488	.001700680
589	343921	204336469	24.2693222	8.3825053	.001697793
590	345100	205379000	24.2899156	8.3872586	.001694915
591	346281	206425071	24.3104916	8.3919982	.001692047
592	347464	207474688	24.3310501	8.3967249	.001689189
593	348649	208527857	24.3515913	8.4014381	.001686341
594	349836	209584584	24.3721152	8.4061380	.001683502
595	351025	210644875	24.3926218	8.4108346	.001680672
596	352216	211708736	24.4131112	8.4155179	.001677852
597	353409	212776173	24.4335831	8.4202460	.001675042
598	354604	213847192	24.4540385	8.4249648	.001672241
599	355801	214921799	24.4744765	8.4296783	.001669449
600	360000	216000000	24.4948974	8.4343867	.001666667
601	361201	217081801	24.5153013	8.4390808	.001663894
602	362404	218167208	24.5356883	8.4437607	.001661130
603	363609	219256227	24.5560593	8.4484265	.001658375
604	364816	220348864	24.5764115	8.4530781	.001655629
605	366025	221445125	24.5967478	8.4577166	.001652892
606	367236	222545016	24.6170673	8.4623479	.001650165
607	368449	223648513	24.6373700	8.4669701	.001647446
608	369661	224755712	24.6576560	8.4715841	.001644737
609	370881	225866529	24.6779251	8.4761892	.001642036
610	372100	226981000	24.6981781	8.4807861	.001639344
611	373321	228099131	24.7184112	8.4853749	.001636661
612	374544	229220928	24.7386338	8.4900048	.001633987
613	375769	230346397	24.7588368	8.4945765	.001631321
614	376996	231475544	24.7790244	8.4991393	.001628664
615	378225	232608375	24.7991935	8.5036930	.001626016
616	379456	233744896	24.8193473	8.5082377	.001623377
617	380689	234885113	24.8394847	8.5127735	.001620746
618	381924	236029032	24.8596058	8.5173003	.001618123
619	383161	237176659	24.8797106	8.5218181	.001615509
620	384400	238328000	24.8997992	8.5263269	.001612903

148 TABLE XI. SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
621	335611	239433061	24.9198716	8.5316009	.001610306
622	336384	240641848	24.9399278	8.5361780	.001607717
623	338129	241801367	24.9599679	8.5407501	.001605136
624	339376	242970624	24.9799920	8.5453173	.001602564
625	393625	244140625	25.0000000	8.5498797	.001600000
626	391876	245314376	25.0199920	8.5544372	.001597444
627	393129	246491883	25.0399681	8.5589899	.001594896
628	394384	247673152	25.0599282	8.5635377	.001592357
629	395641	248858189	25.0798724	8.5680807	.001589825
630	396900	250047000	25.0998003	8.5726189	.001587302
631	398161	251239591	25.1197134	8.5771523	.001584786
632	399424	252435963	25.1396102	8.5816809	.001582278
633	400689	253636137	25.1594913	8.5862047	.001579779
634	401956	254841014	25.1793566	8.5907238	.001577287
635	403225	256047875	25.1992063	8.5952330	.001574803
636	404496	257259456	25.2190404	8.5997476	.001572327
637	405769	258474853	25.2388589	8.6042525	.001569859
638	407044	259694072	25.2586619	8.6087526	.001567393
639	408321	260917119	25.2784493	8.6132480	.001564945
640	409600	262144000	25.2982213	8.6177388	.001562500
641	410881	263374721	25.3179778	8.6222248	.001560062
642	412164	264609293	25.3377189	8.6267063	.001557632
643	413449	265847707	25.3574447	8.6311830	.001555210
644	414736	267089934	25.3771551	8.6356551	.001552795
645	416025	268336125	25.3968502	8.6401226	.001550388
646	417316	269586136	25.4165301	8.6445855	.001547988
647	418609	270840023	25.4361947	8.6490437	.001545595
648	419904	272097792	25.4558441	8.6534974	.001543210
649	421201	273359449	25.4754784	8.6579465	.001540832
650	422500	274625000	25.4950976	8.6623911	.001538462
651	423801	275894451	25.5147016	8.6668310	.001536098
652	425104	277167803	25.5342907	8.6712665	.001533742
653	426409	278445077	25.5538647	8.6756974	.001531394
654	427716	279726264	25.5734237	8.6801237	.001529052
655	429025	281011375	25.5929678	8.6845456	.001526718
656	430336	282300416	25.6124969	8.6889630	.001524390
657	431649	283593393	25.6320112	8.6933759	.001522070
658	432964	284890312	25.6515107	8.6977843	.001519757
659	434281	286191179	25.6709953	8.7021882	.001517451
660	435600	287496000	25.6904652	8.7065977	.001515152
661	436921	288804781	25.7099203	8.7109827	.001512859
662	438244	290117528	25.7293607	8.7153734	.001510574
663	439569	291434247	25.7487864	8.7197596	.001508296
664	440896	292754944	25.7681975	8.7241414	.001506024
665	442225	294079625	25.7875939	8.7285187	.001503759
666	443556	295408296	25.8069758	8.7328918	.001501502
667	444889	296740963	25.8263431	8.7372604	.001499250
668	446224	298077632	25.8456960	8.7416246	.001497006
669	447561	299418309	25.8650343	8.7459846	.001494768
670	448900	300763000	25.8843582	8.7503401	.001492537
671	450241	302111711	25.9036677	8.7546913	.001490313
672	451584	303464448	25.9229628	8.7590383	.001488095
673	452929	304821217	25.9422435	8.7633309	.001485884
674	454276	306182024	25.9615100	8.7676719	.001483680
675	455625	307546375	25.9807621	8.7720532	.001481481
676	456976	308914776	26.0000000	8.7763330	.001479290
677	458329	310287733	26.0192237	8.7807034	.001477105
678	459684	311665752	26.0384331	8.7850296	.001474926
679	461041	313046339	26.0576234	8.7893466	.001472754
680	462400	314432000	26.0768096	8.7936593	.001470588
681	463761	315821241	26.0959767	8.7979673	.001468429
682	465124	317214563	26.1151297	8.8022721	.001466276

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
633	466189	318611987	26.1342687	8.8065722	.001464129
634	467856	320013504	26.1533937	8.8108681	.001461988
635	469525	321419125	26.1725047	8.8151598	.001459854
636	471196	322828356	26.1916017	8.8194474	.001457726
637	472869	324242703	26.2106848	8.8237307	.001455604
638	474544	325660672	26.2297541	8.8280099	.001453488
639	476221	327082769	26.2488095	8.8322850	.001451379
640	477900	328509000	26.2678511	8.8365559	.001449275
641	479581	329939371	26.2868789	8.8408227	.001447178
642	481264	331373888	26.3058929	8.8450854	.001445087
643	482949	332812557	26.3248932	8.8493440	.001443001
644	484636	334255384	26.3438797	8.8535985	.001440922
645	486325	335702375	26.3628527	8.8578489	.001438849
646	488016	337153536	26.3818119	8.8620952	.001436782
647	489709	338608873	26.4007576	8.8663375	.001434720
648	491404	340068392	26.4196896	8.8705757	.001432665
649	493101	341532099	26.4386081	8.8748099	.001430615
700	490000	343000000	26.4575131	8.8790400	.001428571
701	491701	344472101	26.4764046	8.8832661	.001426534
702	493404	345948408	26.4952826	8.8874882	.001424501
703	495109	347428927	26.5141472	8.8917063	.001422475
704	496816	348913664	26.5329983	8.8959204	.001420455
705	498525	350402625	26.5518361	8.8901304	.001418440
706	499236	351895816	26.5706605	8.8943366	.001416431
707	500949	353393243	26.5894716	8.8985387	.001414427
708	501664	354895912	26.6082694	8.9027369	.001412429
709	502381	356403829	26.6270539	8.9069311	.001410437
710	504100	357911000	26.6458252	8.9211214	.001408451
711	505821	359428431	26.6645833	8.9253078	.001406470
712	506944	360946128	26.6833281	8.9294902	.001404494
713	508069	362464097	26.7020594	8.9336687	.001402525
714	509196	363982344	26.7207774	8.9378433	.001400560
715	510325	365500875	26.7394839	8.9420140	.001398601
716	511456	367019696	26.7581763	8.9461809	.001396648
717	512589	368538813	26.7768557	8.9503438	.001394700
718	513724	370058232	26.7955220	8.9545029	.001392758
719	514861	371577959	26.8141754	8.9586581	.001390821
720	516000	373097900	26.8328157	8.9628095	.001388889
721	517141	374618161	26.8514432	8.9669570	.001386963
722	518284	376138736	26.8700577	8.9711007	.001385042
723	519429	377659627	26.8886593	8.9752406	.001383126
724	520576	379180834	26.9072481	8.9793766	.001381215
725	521725	380702357	26.9258240	8.9835089	.001379310
726	522876	382224196	26.9443872	8.9876373	.001377410
727	524029	383746351	26.9629375	8.9917620	.001375516
728	525184	385268832	26.9814751	8.9958829	.001373626
729	526341	386791639	27.0000000	9.0000000	.001371742
730	532900	390017000	27.0185122	9.0041134	.001369863
731	534361	391543191	27.0370117	9.0082229	.001367989
732	535824	393069216	27.0554985	9.0123284	.001366120
733	537289	394595077	27.0739727	9.0164309	.001364256
734	538756	396120784	27.0924344	9.0205293	.001362398
735	540225	397646337	27.1108834	9.0246239	.001360544
736	541696	399171746	27.1293199	9.0287149	.001358696
737	543169	400697011	27.1477439	9.0328021	.001356852
738	544644	402222132	27.1661554	9.0368857	.001355014
739	546121	403747109	27.1845544	9.0409655	.001353180
740	547600	405272000	27.2029410	9.0450419	.001351351
741	549081	406796801	27.2213152	9.0491142	.001349528
742	550564	408321416	27.2396769	9.0531823	.001347709
743	552049	409845847	27.2580263	9.0572462	.001345895
744	553536	411370094	27.2763634	9.0613068	.001344086

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
745	555025	413493625	27.2946381	9.0653677	.001342232
746	556516	415160936	27.3130006	9.0694220	.001340483
747	558009	416832723	27.3313007	9.0734726	.001338658
748	559501	418508992	27.3495887	9.0775197	.001336893
749	561001	420189749	27.3678614	9.0815631	.001335113
750	562500	421875000	27.3861279	9.0856030	.001333333
751	564001	423564751	27.4043792	9.0896392	.001331553
752	565501	425259003	27.4226194	9.0936719	.001329737
753	567009	426957777	27.4408455	9.0977010	.001327921
754	568516	428661061	27.4590604	9.1017265	.001326260
755	570025	430368875	27.4772633	9.1057485	.001324503
756	571536	432081216	27.4954512	9.1097669	.001322751
757	573049	433798093	27.5136330	9.1137818	.001321004
758	574564	435519512	27.5317998	9.1177931	.001319261
759	576081	437245479	27.5499546	9.1218010	.001317523
760	577600	438976000	27.5680975	9.1258053	.001315789
761	579121	440711104	27.5862284	9.1298061	.001314060
762	580644	442450723	27.6043475	9.1338034	.001312336
763	582169	444194847	27.6224546	9.1377971	.001310616
764	583696	445943474	27.6405499	9.1417874	.001308901
765	585225	447696725	27.6586331	9.1457742	.001307190
766	586756	449454606	27.6767050	9.1497576	.001305483
767	588289	451217063	27.6947618	9.1537375	.001303781
768	589824	452984132	27.7128129	9.1577139	.001302083
769	591361	454755809	27.7308492	9.1616869	.001300390
770	592900	456532000	27.7488739	9.1656565	.001298701
771	594441	458312811	27.7668863	9.1696225	.001297017
772	595984	460098243	27.7848880	9.1735852	.001295337
773	597529	461888297	27.8028775	9.1775445	.001293661
774	599076	463682874	27.8208555	9.1815003	.001291990
775	600625	465482075	27.8388218	9.1854527	.001290323
776	602176	467285906	27.8567766	9.1894018	.001288660
777	603729	469094373	27.8747197	9.1933474	.001287001
778	605284	470907482	27.8926514	9.1972897	.001285347
779	606841	472725239	27.9105715	9.2012286	.001283697
780	608400	474547600	27.9284801	9.2051641	.001282051
781	609961	476374581	27.9463772	9.2090962	.001280410
782	611524	478206183	27.9642629	9.2130250	.001278773
783	613089	480042407	27.9821372	9.2169505	.001277139
784	614656	481883254	28.0000000	9.2208726	.001275510
785	616225	483728725	28.0178515	9.2247914	.001273885
786	617796	485579826	28.0357915	9.2287068	.001272265
787	619369	487435553	28.0537203	9.2326189	.001270649
788	620944	489295902	28.0716377	9.2365277	.001269036
789	622521	491161879	28.0895433	9.2404333	.001267427
790	624100	493033400	28.1074386	9.2443355	.001265823
791	625681	494909471	28.1253222	9.2482344	.001264223
792	627264	496790093	28.1431946	9.2521300	.001262626
793	628849	498675275	28.1610557	9.2560224	.001261031
794	630436	500565014	28.1789056	9.2599114	.001259446
795	632025	502459317	28.1967444	9.2637973	.001257863
796	633616	504358183	28.2145720	9.2676798	.001256281
797	635209	506261613	28.2323884	9.2715592	.001254705
798	636804	508169602	28.2501933	9.2754352	.001253133
799	638401	510082159	28.2680861	9.2793081	.001251564
800	640000	512000000	28.2859672	9.2831777	.001250000
801	641601	513922401	28.3038363	9.2870440	.001248439
802	643204	515849603	28.3216945	9.2909072	.001246883
803	644809	517781627	28.3395416	9.2947671	.001245330
804	646416	519718461	28.3573773	9.2986239	.001243781
805	648025	521660125	28.3752019	9.3024775	.001242236
806	649636	523606616	28.3930139	9.3063278	.001240696

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
807	651249	525557943	28.4077454	9.3101750	.001239157
808	652864	527514112	28.4253408	9.3140190	.001237624
809	654481	529475129	28.4429253	9.3178599	.001236094
810	656100	531441000	28.4601989	9.3216975	.001234563
811	657721	533411731	28.4780617	9.3255320	.001233046
812	659344	535387328	28.4956137	9.3293634	.001231527
813	660969	537367797	28.5131549	9.3331916	.001230012
814	662596	539353144	28.5306852	9.3370167	.001228501
815	664225	541343375	28.5482048	9.3408386	.001226994
816	665856	543338496	28.5657137	9.3446575	.001225490
817	667489	545338513	28.5832119	9.3484731	.001223990
818	669124	547343432	28.6006993	9.3522857	.001222494
819	670761	549353259	28.6181760	9.3560952	.001221001
820	672400	551369000	28.6356421	9.3599016	.001219512
821	674041	553387661	28.6530976	9.3637049	.001218027
822	675684	555412218	28.6705424	9.3675051	.001216545
823	677329	557441767	28.6879766	9.3713022	.001215067
824	678976	559476224	28.7054002	9.3750963	.001213592
825	680625	561515625	28.7228132	9.3788873	.001212121
826	682276	563559976	28.7402157	9.3826752	.001210654
827	683929	565609283	28.7576077	9.3864600	.001209190
828	685584	567663552	28.7749891	9.3902419	.001207729
829	687241	569722789	28.7923601	9.3940206	.001206273
830	688900	571787000	28.8097296	9.3977964	.001204819
831	690561	573856191	28.8270706	9.4015691	.001203369
832	692224	575930368	28.8444102	9.4053387	.001201923
833	693889	578009537	28.8617304	9.4091054	.001200480
834	695556	580093704	28.8790582	9.4128690	.001199041
835	697225	582182875	28.8963866	9.4166297	.001197605
836	698896	584277056	28.9136646	9.4203873	.001196172
837	700569	586376253	28.9309523	9.4241420	.001194743
838	702244	588480472	28.9482297	9.4278938	.001193317
839	703921	590589719	28.9654967	9.4316423	.001191895
840	705600	592704000	28.9827535	9.4353880	.001190476
841	707281	594823321	29.0000000	9.4391307	.001189061
842	708964	596947688	29.0172363	9.4428704	.001187648
843	710649	599077107	29.0344623	9.4466072	.001186240
844	712336	601211584	29.0516781	9.4503410	.001184834
845	714025	603351125	29.0688837	9.4540719	.001183432
846	715716	605495736	29.0860791	9.4577999	.001182033
847	717409	607645423	29.1032644	9.4615249	.001180638
848	719104	609800192	29.1204396	9.4652470	.001179245
849	720801	611960049	29.1376046	9.4689661	.001177856
850	722500	614125000	29.1547595	9.4726824	.001176471
851	724201	616295061	29.1719043	9.4763957	.001175088
852	725904	618470224	29.1890390	9.4801061	.001173709
853	727609	620650477	29.2061637	9.4838136	.001172333
854	729316	622835861	29.2232784	9.4875182	.001170960
855	731025	625026375	29.2403839	9.4912200	.001169591
856	732736	627222016	29.2574777	9.4949188	.001168224
857	734449	629422793	29.2745623	9.4986147	.001166861
858	736164	631628712	29.2916370	9.5023078	.001165501
859	737881	633839779	29.3087018	9.5059980	.001164144
860	739600	636056000	29.3257566	9.5096854	.001162791
861	741321	638277331	29.3428015	9.5133699	.001161440
862	743044	640503724	29.3598365	9.5170515	.001160093
863	744769	642735167	29.3768616	9.5207303	.001158749
864	746496	644972544	29.3938769	9.5244063	.001157407
865	748225	647214965	29.4108823	9.5280794	.001156068
866	749956	649462436	29.4278779	9.5317497	.001154734
867	751689	651714963	29.4448637	9.5354172	.001153403
868	753424	653972532	29.4618397	9.5390818	.001152074

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
869	755161	656231909	29.4788059	9.5427437	.001150748
870	756900	658503000	29.4957624	9.5464027	.001149425
871	758641	660776311	29.5127091	9.5500589	.001148106
872	760384	663054348	29.5296461	9.5537123	.001146789
873	762129	665338617	29.5465734	9.5573630	.001145475
874	763876	667627624	29.5634910	9.5610108	.001144165
875	765625	669921875	29.5803989	9.5646559	.001142857
876	767376	672221376	29.5972972	9.5682969	.001141553
877	769129	674526133	29.6141858	9.5719377	.001140251
878	770884	676836152	29.6310648	9.5755745	.001138952
879	772641	679151439	29.6479342	9.5792066	.001137656
880	774400	681472000	29.6647939	9.5828397	.001136364
881	776161	683797841	29.6816442	9.5864682	.001135074
882	777924	686128968	29.6984948	9.5900939	.001133787
883	779689	688465287	29.7153159	9.5937169	.001132503
884	781456	690807104	29.7321375	9.5973373	.001131222
885	783225	693154125	29.7489496	9.6009548	.001129944
886	784996	695506456	29.7657521	9.6045696	.001128668
887	786769	697864103	29.7825452	9.6081817	.001127396
888	788544	700227072	29.7993289	9.6117911	.001126126
889	790321	702595369	29.8161030	9.6153977	.001124859
890	792100	704969000	29.8328678	9.6190017	.001123596
891	793881	707347971	29.8496231	9.6226030	.001122334
892	795664	709732288	29.8663690	9.6262016	.001121076
893	797449	712121957	29.8831056	9.6297975	.001119821
894	799236	714516984	29.8998329	9.6333907	.001118568
895	801025	716917375	29.9165506	9.6369812	.001117318
896	802816	719323136	29.9332591	9.6405690	.001116071
897	804609	721734273	29.9499583	9.6441542	.001114827
898	806404	724150792	29.9666481	9.6477367	.001113586
899	808201	726572699	29.9833287	9.6513166	.001112347
900	810000	729000000	30.0000000	9.6548938	.001111111
901	811801	731432701	30.0166620	9.6584634	.001109878
902	813604	733870808	30.0333148	9.6620403	.001108647
903	815409	736314327	30.0499584	9.6656096	.001107420
904	817216	738763264	30.0665928	9.6691762	.001106195
905	819025	741217625	30.0832179	9.6727403	.001104972
906	820836	743677416	30.0998339	9.6763017	.001103751
907	822649	746142643	30.1164407	9.6798604	.001102536
908	824464	748613312	30.1330383	9.6834166	.001101322
909	826281	751089429	30.1496269	9.6869701	.001100110
910	828100	753571000	30.1662063	9.6905211	.001098901
911	829921	756058031	30.1827765	9.6940694	.001097695
912	831744	758550528	30.1993377	9.6976151	.001096491
913	833569	761048497	30.2158899	9.7011583	.001095290
914	835396	763551944	30.2324329	9.7046989	.001094092
915	837225	766060875	30.2489669	9.7082369	.001092896
916	839056	768575296	30.2654919	9.7117723	.001091703
917	840889	771095213	30.2820079	9.7153051	.001090513
918	842724	773620632	30.2985148	9.7188354	.001089325
919	844561	776151559	30.3150128	9.7223631	.001088139
920	846400	778688000	30.3315018	9.7258883	.001086957
921	848241	781229961	30.3479818	9.7294109	.001085776
922	850084	783777448	30.3644529	9.7329309	.001084599
923	851929	786330467	30.3809151	9.7364484	.001083423
924	853776	788889024	30.3973683	9.7399634	.001082251
925	855625	791453125	30.4138127	9.7434758	.001081081
926	857476	794022776	30.4302481	9.7469857	.001079914
927	859329	796597983	30.4466747	9.7504930	.001078749
928	861184	799178752	30.4630924	9.7539979	.001077586
929	863041	801765089	30.4795013	9.7575002	.001076426
930	864900	804357000	30.4959014	9.7610001	.001075269

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
931	866761	806954191	30.5122926	9.7644974	.001074114
932	868424	809557568	30.5286750	9.7679922	.001072961
933	870439	812166237	30.5450487	9.7714845	.001071811
934	872356	814780504	30.5614136	9.7749743	.001070664
935	874225	817400375	30.5777697	9.7784616	.001069519
936	876096	820025356	30.5941171	9.7825466	.001068376
937	877969	822656953	30.6104557	9.7864288	.001067236
938	879844	825293672	30.6267857	9.7899087	.001066098
939	881721	827936019	30.6431069	9.7923361	.001064963
940	883600	830594000	30.6594194	9.7953611	.001063830
941	885481	833237621	30.6757233	9.7993336	.001062699
942	887364	835896338	30.6920185	9.8023036	.001061571
943	889249	838561907	30.7083051	9.8062711	.001060445
944	891136	841232334	30.7245830	9.8097362	.001059322
945	893025	843904625	30.7408523	9.8131989	.001058201
946	894916	846590536	30.7571130	9.8166591	.001057082
947	896809	849278123	30.7733651	9.8201169	.001055966
948	898704	851971392	30.7896086	9.8235723	.001054852
949	900601	854670349	30.8058436	9.8270252	.001053741
950	902500	857375000	30.8220700	9.8304757	.001052632
951	904401	860085351	30.8382379	9.8339238	.001051525
952	906304	862801408	30.8544192	9.8373695	.001050420
953	908209	865523177	30.8706981	9.8408127	.001049318
954	910116	868250664	30.8868904	9.8442536	.001048218
955	912025	870983875	30.9030743	9.8476920	.001047120
956	913936	873722816	30.9192497	9.8511290	.001046025
957	915849	876467493	30.9354166	9.8545617	.001044932
958	917764	879217912	30.9515751	9.8579929	.001043841
959	919681	881974079	30.9677251	9.8614218	.001042753
960	921600	884736000	30.9838663	9.8648483	.001041667
961	923521	887503631	31.0000000	9.8682724	.001040583
962	925444	890277128	31.0161248	9.8716941	.001039501
963	927369	893056317	31.0322113	9.8751135	.001038422
964	929296	895841344	31.0483491	9.8785305	.001037341
965	931225	898632125	31.0644491	9.8819451	.001036269
966	933156	901428696	31.0805105	9.8853574	.001035197
967	935089	904231063	31.0966236	9.8887673	.001034126
968	937024	907039232	31.1126984	9.8921749	.001033058
969	938961	909853209	31.1287618	9.8955801	.001031992
970	940900	912673000	31.1448230	9.8989830	.001030928
971	942841	915498611	31.1608729	9.9023835	.001029866
972	944784	918330043	31.1769145	9.9057817	.001028807
973	946729	921167317	31.1929479	9.9091776	.001027749
974	948676	924010424	31.2089731	9.9125712	.001026694
975	950625	926859375	31.2249900	9.9159624	.001025641
976	952576	929714176	31.2410087	9.9193513	.001024590
977	954529	932574833	31.2569992	9.9227379	.001023541
978	956484	935441352	31.2729915	9.9261222	.001022495
979	958441	938313739	31.2889757	9.9295042	.001021450
980	960400	941192000	31.3049517	9.9328839	.001020408
981	962361	944076141	31.3209195	9.9362613	.001019368
982	964324	946966163	31.3368792	9.9396363	.001018330
983	966289	949862037	31.3528308	9.9430092	.001017291
984	968256	952763904	31.3687743	9.9463797	.001016260
985	970225	955671625	31.3847097	9.9497479	.001015228
986	972196	958585256	31.4006369	9.9531138	.001014199
987	974169	961504803	31.4165561	9.9564775	.001013171
988	976144	964430272	31.4324673	9.9598389	.001012146
989	978121	967361669	31.4483704	9.9631981	.001011122
990	980100	970299000	31.4642654	9.9665549	.001010101
991	982081	973242271	31.4801525	9.9699095	.001009082
992	984064	976191488	31.4960315	9.9732619	.001008065

Materials.	Specific Gravity.	Weight per Cubic Foot in lbs.	Tensile Strength per Square Inch in lbs.	Crushing Force per Square Inch in lbs.	Modulus of Rupture S in lbs.
<i>Metals.</i>					
Free cast, ...	8.399	524.94	17969 R.		
Upper, cast, ...	8.607	537.94	19072		
" rolled, ...	8.864 F.	554.00	32326 F.		
" wire-drawn, ...	8.878	554.87	61228		
Gold, ...	19.258 Ha.	1203.62			
	19.361 Ha.	1210.06			
Iron, cast, ...					
Cannon No. 2, cold blast,	7.066 H.	441.62	16633 H.	106375 H.	38556 H.
" " hot "	7.046 H.	440.37	13505 H.	108540 H.	37503 H.
Devon No. 3, cold "	7.295 H.	455.94			36288 H.
" " hot "	7.229 H.	451.81	21907 H.	145435 H.	43497 H.
Buffery No. 1, cold "	7.079 H.	442.44	17466 H.	93335 H.	37503 H.
" " hot "	6.998 H.	437.37	13434 H.	86397 H.	35316 H.
Iron, wrought, ...					
English bar, ...	7.700	481.25	57120 L.	56000? G.	54000 G.
Welsh "			61960 T.		
Swedish "			61960 T.		
" "	7.478 F.	467.37	58184 F.		
Lancaster No. 1, cold "	7.740 F.	483.75	58661 F.		
Tennessee "	7.805 F.	487.81	52099 F.		
Missouri "	7.722 F.	482.62	47909 F.		
Iron wire, ...					
English, No. 10, ...			80214 T.		
Phillips' No. 1, "	7.727 F.	482.94	84186 F.		
" " No. 19, "			73888 F.		
" " No. 17, "			89162 F.		
Lead, cast, ...	11.446 M.	715.37	1824 R.		
Lead wire, ...	11.317	707.31	2531 M.		
Mercury, ...	13.598 W.	849.87			
Platina, ...	22.669 Ha.	1218.75			
	22.669 Ha.	1218.75			
Silver, ...	10.474 Ha.	654.62	40902 M.		
Steel, soft, ...	7.780	486.25	120000		
razor-tempered, ...	7.840	490.30	160000		
Tin, cast, ...	7.291	456.62	6322 M.		
Zinc, fused, ...	7.050 W.	440.32			
" rolled, ...	7.540 W.	471.25			
<i>Woods.</i>					
Ash, English, ...	.760 B.	47.50	17000 B.	{ 8683 H. 9363 H. }	12156 B.
Birch, English, ...	.792 B.	49.50	15000	{ 3297 H. 6102 H. }	10920 B.
" American, ...	.648 B.	40.50		1663 H.	9621 B.
Box, ...	.960 B.	60.00	20000 B.	8771 H.	
Cedar, Canadian, ...	.909 C.	56.81	11400 B.	{ 6000 H. 5000 H. }	
Chestnut, ...	.657 Ro.	41.06	13300 Ro.		
Deal, Christiania mill, ...	.698 B.	43.62	12400		9364 B.
" Memel	.590 B.	36.87			10386 B.
" Norway Spruce, ...	.340	21.25	17600		
" English, ...	.470	29.37	7000		
Elm, seasoned, ...	.553 B.	34.56	13189 M.	10331 B.	8778 B.
Fir, New England, ...	.553 B.	34.56			8412 R.
" Riga, ...	.753 B.	47.06	12000 B.	{ 5740 F. 6586 H. }	8000 B.
Lignum-vitæ, ...	1.220	76.25	11800 M.		
Mahogany, Spanish, ...	.800	50.00	16500	{ 8198 F. 8193 H. }	

Materials.	Specific Gravity.	Weight per Cubic Foot in lbs.	Tensile Strength per Square Inch in lbs.	Crushing Force per Square Inch in lbs.	Modulus of Rupture S in lbs.
<i>Woods.</i>					
Oak, English, ...	.934 B.	58.37	10000 B.	{ 6184 H. 10058 H. }	10032 B.
" Canadian, ...	.872 B.	54.50	10253	{ 4231 H. 5982 H. }	10596 B.
Pine, pitch, ...	.660 B.	41.25	7818 M.	{ 6790 H. 6790 H. }	9792 B.
" red, ...	.657 B.	41.06		{ 5395 H. 7518 H. }	8046 B.
" American, white, .	.455 Br.	28.44			7829 Br.
" " Southern, .	.372 Br.	54.50			13937 Br.
Poplar, ...	.383 M.	23.94	7200 Be.	{ 3107 H. 5124 H. }	
Teak, ...	.745 B.	46.56	15000 B.	12101 H.	14772 B.
<i>Other Materials.</i>					
Brick, red, ...	2.168 R.	135.50	280	808 R.	340 W.
" pale red, ...	2.035 R.	130.31	300	562 R.	180 W.
Chalk, ...	{ 2.784 1.869 }	{ 174.00 116.81 }		501 R.	
Coal, Penn. anthracite, .	{ 1.327 Ta. 1.700 Ta. }	{ 82.94 106.25 }			
" " semi-bituminous,	1.453 Ta.	90.81			
" Md. " "	1.552 Ta.	97.00			
" Penn. bituminous, .	1.312 Ta.	82.00			
" Ohio " "	1.270 Ta.	79.37			
" English " "	1.259 Ta.	78.69			
Earth,					
loamy hard-stamped, fresh,	2.060 W.	123.75			
" " dry,	1.930 W.	120.62			
garden, fresh, ...	2.050 W.	123.12			
" dry, ...	1.630 W.	101.87			
dry, poor, ...	1.340 W.	83.75			
Glass, plate, ...	2.453	153.31	9420		
Gravel, ...	1.920	120.00			
Granite, Aberdeen, . . .	2.625 R.	164.06		10914 R.	
Ivory, ...	1.826	114.12	16626		
Limestone, ...	{ 2.400 W. 2.860 W. }	{ 150.00 178.75 }		1500 W. 6000 W.	700 W. 1700 W.
Marble, white Italian, . .	2.638 H.	164.87		9583 G.	1062
" black Galway, . . .	2.695 H.	163.44			2664
Masonry, quarry stone, dry,	2.400 W.	150.00			
" sandstone, " "	2.050 W.	123.12			
" brick, dry, ...	{ 1.470 W. 1.590 W. }	{ 91.87 99.37 }			
Ropes,					
hemp, under 1 inch diam.,			9280 W.		
" from 1 to 3 in. " "			7218 W.		
" over 3 inches " "			5156 W.		
Sand, river, ...	1.886	117.87			
Sandstone, ...	{ 1.900 W. 2.700 W. }	{ 118.75 163.75 }		1400 W. 13000 W.	600 W. 800 W.
" Dundee, ...	2.530 R.	158.12		6630 R.	
" Derby, red and friable,	2.316 R.	144.75		3142 R.	
Slate, Welsh, ...	2.883	180.50	12300		
" Scotch, ...			9600		

No.	0	1	2	3	4	5	6	7	8	9
100	000000	000131	000261	000391	000521	000651	000781	000911	001041	001171
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174
2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415
3	012837	013259	013680	014100	014521	014940	015359	015777	016194	016610
4	017026	017441	017856	018270	018684	019097	019509	019920	020331	020741
5	021150	021559	021967	022374	022781	023187	023592	024000	024405	024810
6	025214	025618	026021	026424	026826	027228	027629	028029	028428	028826
7	029224	029621	030017	030412	030806	031199	031591	031982	032372	032761
8	033150	033537	033923	034308	034691	035073	035454	035834	036213	036590
9	036966	037341	037715	038088	038460	038831	039199	039566	039931	040295
110	011303	011787	012182	012576	012969	013362	013755	014148	014540	014932
1	5423	5714	6105	6495	6885	7275	7664	8053	8442	8830
2	9218	9606	9993	030380	050766	061153	071538	081924	092309	102694
3	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524
4	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083
6	064458	064832	065205	065578	065951	066324	066697	067069	067441	067815
7	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514
8	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182
9	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819
120	079181	079543	079904	080265	080626	080987	081347	081707	082067	082426
1	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004
2	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552
3	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071
4	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562
5	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462
7	103804	104146	104487	104828	105169	105509	105849	106188	106527	106865
8	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253
9	110590	110926	111261	111599	111934	112270	112605	112940	113275	113609
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940
1	117271	117603	117934	118265	118595	118925	119255	119585	119915	120245
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525
3	123852	124178	124504	124829	125155	125481	125806	126131	126456	126781
4	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219
6	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403
7	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564
8	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702
9	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911
1	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982
2	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032
3	155336	155640	155943	156246	156549	156852	157155	157457	157759	158061
4	158363	158665	158967	159268	159569	159870	160171	160471	160772	161073
5	161373	161673	161973	162273	162573	162873	163173	163473	163773	164073
6	164373	164673	164973	165273	165573	165873	166173	166473	166773	167073
7	167373	167673	167973	168273	168573	168873	169173	169473	169773	170073
8	170373	170673	170973	171273	171573	171873	172173	172473	172773	173073
9	173373	173673	173973	174273	174573	174873	175173	175473	175773	176073
150	176373	176673	176973	177273	177573	177873	178173	178473	178773	179073
1	179373	179673	179973	180273	180573	180873	181173	181473	181773	182073
2	182373	182673	182973	183273	183573	183873	184173	184473	184773	185073
3	185373	185673	185973	186273	186573	186873	187173	187473	187773	188073
4	188373	188673	188973	189273	189573	189873	190173	190473	190773	191073
5	191373	191673	191973	192273	192573	192873	193173	193473	193773	194073
6	194373	194673	194973	195273	195573	195873	196173	196473	196773	197073
7	197373	197673	197973	198273	198573	198873	199173	199473	199773	200073
8	200373	200673	200973	201273	201573	201873	202173	202473	202773	203073
9	203373	203673	203973	204273	204573	204873	205173	205473	205773	206073
No.	0	1	2	3	4	5	6	7	8	9

TABLE XII.

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000

No.	0	1	2	3	4	5	6	7	8	9	DM
10	000000	000000	000000	000000	0001734	0002166	0002598	0003020	0003461	0003891	00
1	4321	1751	5181	5000	6038	6466	6894	7321	7748	8174	01
2	8600	9936	9151	9876	010300	010734	011147	011570	011993	012415	02
3	012415	012838	013261	013684	014107	014530	014953	015376	015799	016222	03
4	7032	7455	7878	8301	8724	9147	9570	9993	020361	020775	04
5	021100	021523	021946	022369	022792	023215	023638	024061	024484	024907	05
6	5306	5729	6152	6575	6998	7421	7844	8267	8690	9113	06
7	9331	9754	0000	0000	0000	0000	0000	0000	0000	0000	07
8	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	08
9	7120	7543	7966	8389	8812	9235	9658	0000	0000	0000	09
10	011000	011423	011846	012269	012692	013115	013538	013961	014384	014807	10
1	5723	6146	6569	6992	7415	7838	8261	8684	9107	9530	11
2	9218	9641	0000	0000	0000	0000	0000	0000	0000	0000	12
3	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	13
4	6245	6668	7091	7514	7937	8360	8783	9206	9629	0000	14
5	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	15
6	4158	4581	5004	5427	5850	6273	6696	7119	7542	7965	16
7	8181	8604	9027	9450	9873	0000	0000	0000	0000	0000	17
8	0718	0718	0718	0718	0718	0718	0718	0718	0718	0718	18
9	5517	5940	6363	6786	7209	7632	8055	8478	8901	9324	19
10	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	20
1	1025	0811	0597	0383	0169	4219	4576	4934	5291	5647	21
2	6299	6716	7071	7426	7781	8136	8490	8845	9198	9552	22
3	9905	0000	0000	0000	0000	0000	0000	0000	0000	0000	23
4	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	24
5	6010	7257	7604	7951	8298	8644	8990	9335	9681	10026	25
6	10031	10073	10115	10157	10199	10241	10283	10325	10367	10409	26
7	8011	4146	4187	4228	5169	5510	5851	6191	6531	6871	27
8	7210	7519	7828	8137	8446	8755	9064	9373	9682	10000	28
9	11050	11092	11134	11176	11218	11260	11302	11344	11386	11428	29
10	11090	11132	11174	11216	11258	11300	11342	11384	11426	11468	30
1	7211	7634	7931	8228	8525	8822	9119	9416	9713	10010	31
2	12051	12093	12135	12177	12219	12261	12303	12345	12387	12429	32
3	3052	4178	4591	4904	5217	5530	5843	6156	6469	6782	33
4	7105	7429	7753	8076	8399	8722	9045	9368	9691	10014	34
5	13051	13093	13135	13177	13219	13261	13303	13345	13387	13429	35
6	3339	3588	4177	4496	4814	5133	5451	5769	6086	6403	36
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	37
8	9079	10191	10508	10822	11136	11450	11763	12076	12389	12702	38
9	13015	3327	3639	3951	4263	4574	4885	5196	5507	5818	39
10	116125	116438	116748	117058	117367	117676	117985	118294	118603	118911	40
1	9219	9527	9835	10142	10449	10756	11063	11370	11676	11982	41
2	152288	152591	152900	153205	3510	3815	4120	4424	4728	5032	42
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	43
4	8362	8661	8965	9266	9567	9868	10168	10469	10769	11068	44
5	161368	161667	161967	162266	162564	162863	3181	3480	3778	4075	45
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	46
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	47
8	170962	170555	170448	171141	171434	171726	172019	172311	172603	172895	48
9	3106	3178	3269	3360	4351	4641	4932	5222	5512	5802	49
10	176001	176381	176670	176959	177248	177536	177825	178113	178401	178689	50
1	9977	9264	9552	9839	180126	180413	180699	180986	181272	181558	51
2	181411	182129	182415	182700	2955	3270	3585	3899	4213	4407	52
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	53
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	10051	54
5	190332	190612	190892	191171	191451	191730	192009	192289	192567	2846	55
6	3125	3103	3681	3959	4237	4514	4792	5069	5346	5623	56
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	57
8	6537	5932	6206	9181	9755	20029	200303	200577	200850	201124	58
9	210600	210600	210600	210600	202488	2761	3033	3305	3577	3848	59
No.	0	1	2	3	4	5	6	7	8	9	DM

No.	0	1	2	3	4	5	6	7	8	9	Diff.
160	201120	201331	201533	201731	201924	202115	202302	202486	202666	202842	271
1	6326	7035	7365	7631	7901	8173	8441	8710	8979	9247	269
2	9515	9743	210151	210319	210486	210653	211121	211388	211654	211921	267
3	212188	212451	2725	2986	3252	3518	3785	4049	4311	4579	266
4	4511	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	2716	2976	3235	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5827	6084	6342	6600	6858	7115	7372	7630	258
9	7357	8111	8369	8627	8883	9139	9396	9652	9908	230193	256
170	230149	230701	230959	231215	231470	231724	231979	232231	232484	232742	255
1	2396	3250	3501	3757	4011	4261	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7541	7795	252
3	8146	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
5	3138	3286	3531	3782	4030	4277	4525	4772	5019	5266	248
6	5313	5559	6005	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8461	8709	8951	9198	9443	9687	9932	250176	245
8	250129	250661	250908	251151	251395	251638	251881	252125	252368	2610	243
9	2553	3096	3335	3589	3822	4061	4306	4541	4790	5031	242
180	255273	255511	255755	255996	256237	256477	256718	256958	257198	257439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260171	260310	260518	260787	261025	261263	261501	261739	261976	262214	238
3	2451	2698	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4518	5051	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9981	270123	270146	270679	270912	271144	271377	271609	233
7	271842	272071	272406	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	272751	273982	279211	279439	279667	279895	280123	280351	280578	280806	228
1	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
2	3391	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7351	7578	225
4	7802	8026	8249	8473	8696	8919	9143	9366	9589	9812	223
5	200135	200257	200380	200502	200625	200747	200869	200991	201113	201234	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6881	7101	7321	7542	7761	7980	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	30061	30037	300595	300813	218
200	301139	301217	301461	301681	301898	302114	302331	302547	302761	302980	217
1	31196	3112	3328	3541	3753	3965	4176	4387	4598	4808	216
2	5351	5566	5781	5995	6211	6425	6639	6853	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311751	311966	2177	2389	2600	2812	3023	3231	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6600	6810	7018	7227	7436	7646	7851	209
8	8067	8272	8481	8689	8898	9106	9311	9522	9730	9938	208
9	320164	320351	320532	320709	320877	321041	321201	321359	321515	321672	207
210	322219	322426	322631	322830	323016	323202	323385	323565	323741	323917	206
1	32422	4188	4601	4850	5105	5359	5616	5872	6126	6381	205
2	6535	6781	6945	7150	7355	7560	7767	7972	8177	8381	204
3	8584	8788	8991	9194	9398	9601	9805	330008	330211	330414	203
4	330411	330617	330819	331022	331225	331427	331629	331832	2831	2934	202
5	2138	2610	2842	3011	3216	3417	3619	3850	4051	4253	201
6	4451	4655	4859	5057	5257	5458	5658	5859	6059	6259	200
7	6469	6669	6869	7069	7269	7469	7669	7868	8068	8267	199
8	8466	8666	8865	9065	9265	9465	9665	9865	340015	340216	198
9	340411	340612	340811	341012	341213	341415	341616	341817	2828	2929	197
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	DIF.
220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
1	4392	4599	475	491	5178	5374	5570	5766	5962	6157	198
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	199
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
4	350218	351112	350006	350899	351023	351216	351410	351603	351796	351989	195
5	2184	2375	2568	2761	2954	3147	3339	3532	3724	3916	196
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	197
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	198
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	199
9	9535	360025	360215	360404	360593	360783	360972	361161	361350	361539	194
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	195
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	196
2	5188	5375	5562	5749	5936	6123	6310	6496	6683	6869	197
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	198
4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	199
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	194
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	195
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	196
8	6577	6759	6941	7124	7306	7488	7670	7852	8034	8216	197
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	198
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	199
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	194
2	3845	3995	4174	4353	4533	4712	4891	5070	5249	5428	195
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	196
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	197
5	9166	9343	9520	9697	9875	390051	390228	390405	390582	390759	198
6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	199
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	194
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	195
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	196
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	197
1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	198
2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	199
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	194
4	4811	5005	5176	5346	5517	5688	5858	6029	6199	6370	195
5	6549	6719	6889	7059	7229	7399	7569	7739	7909	8079	196
6	8249	8419	8589	8749	8918	9087	9257	9426	9595	9764	197
7	9833	110192	110271	410440	410609	410777	410946	411114	411283	411451	198
8	111620	1788	1956	2124	2293	2461	2629	2796	2964	3132	199
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	194
260	411973	412140	412307	412474	412641	412808	412974	413141	413308	413474	195
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8136	196
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	197
3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	198
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	199
5	3216	3410	3574	3737	3901	4065	4228	4392	4555	4718	194
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	195
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	196
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	197
9	9752	5014	430975	430236	430398	430559	430720	430881	431042	431203	198
270	431361	431525	431688	431851	432014	432177	432338	432498	432659	432809	199
1	2269	3130	3290	3450	3610	3770	3930	4090	4249	4409	194
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	195
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	196
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	197
5	9353	9491	9648	9806	9964	10122	440279	440437	440594	440752	198
6	440909	110065	110221	441381	441538	10165	1865	1865	2166	2323	199
7	2189	2637	2793	2950	3106	3263	3419	3576	3732	3889	194
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	195
9	5601	5760	5915	6071	6226	6382	6537	6692	6848	7003	196
No.	0	1	2	3	4	5	6	7	8	9	DIF.

TABLE XII. LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
230	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
2	450219	450403	450587	450771	450955	451139	451322	451506	451689	1633	153
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	160146	160296	160447	160597	160748	151
9	460398	461048	461198	461348	461499	1649	1799	1948	2098	2248	150
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6368	7016	7161	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2871	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5541	5680	5819	5958	6097	6236	6375	6514	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	500099	500236	500373	500511	500648	500785	500922	137
7	501059	501196	501333	1470	1607	1744	1880	2017	2154	2291	137
8	2127	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	510009	510143	510277	510411	134
4	510515	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
5	1833	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7856	7987	8119	8251	8382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
1	9823	9939	520090	520221	520353	520484	520615	520745	520876	521007	131
2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	1351	128
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
160	662753	662852	662947	663041	663135	663230	663324	663418	663512	663607	91
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	91
2	4612	4736	4830	4924	5018	5112	5206	5299	5393	5487	91
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	91
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	91
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	9988	1080	93
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
170	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6691	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
180	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
190	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
200	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
1	9839	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5861	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
210	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0379	85
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	81
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716003	716037	716170	716254	716337	716421	716504	716588	716671	716754	83
1	6333	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0.003	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724686	724767	724849	724931	725013	82
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730732	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5839	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7937	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748183	748266	748348	748421	748498	748576	748653	748731	748808	748885	77
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
2	9736	9811	9891	9968	750045	750123	750200	750277	750354	750431	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2018	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4343	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755375	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
1	6836	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	76
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2528	2603	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	ME.
10	662753	662852	662917	663041	663135	663230	663321	663418	663512	663607	94
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
2	4612	4795	4889	4984	5078	5172	5266	5359	5453	5547	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
5	7153	7246	7339	7433	7526	7619	7713	7806	7899	7992	94
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	94
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	94
8	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	94
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	94
470	672098	672190	672283	672375	672467	672559	672652	672744	672836	672929	94
1	3021	3113	3205	3297	3389	3482	3574	3666	3758	3850	94
2	3912	4004	4096	4188	4280	4372	4464	4556	4647	4739	94
3	4831	4923	5015	5107	5199	5291	5383	5475	5567	5659	94
4	5778	5870	5962	6054	6146	6238	6330	6422	6514	6606	94
5	6691	6783	6875	6967	7059	7151	7243	7335	7427	7519	94
6	7607	7699	7791	7883	7975	8067	8159	8251	8343	8435	94
7	8518	8610	8702	8794	8886	8978	9070	9162	9254	9346	94
8	9128	9220	9312	9404	9496	9588	9680	9772	9864	9956	94
9	680336	680428	680520	680612	680704	680796	680888	680980	0970	1060	94
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	94
1	2145	2235	2325	2415	2506	2596	2686	2777	2867	2957	94
2	3017	3107	3197	3287	3377	3467	3557	3647	3737	3827	94
3	3917	4007	4097	4187	4277	4367	4457	4547	4637	4727	94
4	4815	4905	4995	5085	5175	5265	5355	5445	5535	5625	94
5	5722	5812	5902	5992	6082	6172	6262	6352	6442	6532	94
6	6638	6728	6818	6908	6998	7088	7178	7268	7358	7448	94
7	7529	7619	7709	7799	7889	7979	8069	8159	8249	8339	94
8	8420	8509	8599	8689	8779	8869	8959	9049	9139	9229	94
9	9309	9398	9488	9578	9668	9758	9848	9938	690019	690107	94
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	94
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	94
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	94
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	94
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	94
5	4605	4693	4781	4869	4956	5044	5131	5219	5307	5394	94
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	94
7	6356	6443	6531	6618	6706	6793	6880	6968	7055	7142	94
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	94
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	94
500	690970	691057	691144	691231	691317	691404	691491	691578	691664	691751	94
1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	94
2	700701	700790	0877	0963	1050	1136	1222	1309	1395	1482	94
3	1565	1651	1741	1827	1913	1999	2086	2172	2258	2344	94
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	94
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	94
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	94
7	5005	5091	5177	5263	5349	5435	5521	5607	5693	5778	94
8	5861	5949	6035	6121	6207	6293	6379	6465	6551	6637	94
9	6718	6803	6889	6974	7059	7144	7229	7315	7400	7485	94
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	94
1	8421	8506	8591	8676	8761	8846	8931	9016	9100	9185	94
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	94
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	94
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	94
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	94
6	2650	2734	2818	2902	2986	3070	3154	3238	3322	3406	94
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	94
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	94
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	94
No.	0	1	2	3	4	5	6	7	8	9	ME.

TABLE XII. LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716083	716187	716170	716254	716337	716421	716504	716588	716671	716754	83
1	683	6921	7001	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7921	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0.003	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3621	3702	3784	3866	3948	4030	4112	4194	82
530	721276	721358	721440	721522	721604	721685	721767	721849	721931	722013	82
1	5085	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7511	7593	7674	7755	7836	7918	8000	8081	8162	8243	81
5	8354	8435	8516	8597	8678	8759	8840	8921	9002	9083	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9894	81
7	9974	730155	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730782	0853	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732391	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	741063	741142	741221	741300	741378	741457	741536	741615	741694	741773	79
1	1152	1231	1309	1388	1467	1546	1624	1703	1782	1861	79
2	1939	2018	2096	2175	2254	2332	2411	2490	2569	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7490	7567	7645	7722	7800	7878	7955	8033	8110	78
560	741888	741966	742044	742121	742198	742276	742353	742431	742508	742585	77
1	8933	9010	9088	9165	9242	9320	9397	9474	9552	9629	77
2	9706	9783	9861	9938	750015	750092	750169	750246	750323	750400	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5493	5570	5646	5722	5799	76
570	755575	755651	755727	755803	755879	755955	756031	756107	756183	756259	76
1	6946	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9365	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760015	760121	760227	760332	760437	75
6	761422	761498	761573	761649	761724	0790	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2528	2603	75
9	2679	2754	2829	2904	2978	3053	3128	3202	3277	3352	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
700	84508	84560	84622	84684	84746	84808	84870	84932	84994	85056	62
1	5718	5740	5762	5784	5806	5828	5850	5872	5894	5916	62
2	6337	6359	6381	6403	6425	6447	6469	6491	6513	6535	62
3	6955	7017	7079	7141	7203	7265	7327	7389	7451	7513	62
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
9	0616	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851808	61
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3638	61
4	3699	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4974	5034	5095	5156	5216	5277	5338	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6669	61
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
5	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863798	863858	59
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5281	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5873	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
8	8056	8115	8174	8233	8292	8351	8410	8469	8528	8586	59
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	58
2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3843	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
1	5610	5668	5726	5784	5841	5899	5957	6015	6072	6130	58
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9669	9726	9784	9841	9898	9956	880070	880127	880185	880242	57
9	880299	880356	880413	880471	880528	0585	0642	0699	0756	0813	57
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE XII. LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
760	890814	890871	890923	890985	891042	891099	891156	891213	891271	891328	57
1	1355	1412	1499	1566	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	896191	896547	896601	896660	896716	896773	896829	896885	896942	896998	56
1	7051	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9133	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	990030	990086	990141	990197	990253	990309	990365	56
7	890421	890477	890533	0539	0645	0700	0756	0812	0868	0924	56
8	0930	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	56
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	56
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	990039	990094	990149	990203	990258	990312	55
5	990367	990422	990476	990531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
810	903990	903141	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3903	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908155	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	990037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	2234	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	990019	990071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9300	9301	9302	9303	9304	9305	9306	9307	9308	9309	51
2	9310	9311	9312	9313	9314	9315	9316	9317	9318	9319	51
3	9320	9321	9322	9323	9324	9325	9326	9327	9328	9329	51
4	9330	9331	9332	9333	9334	9335	9336	9337	9338	9339	51
5	9340	9341	9342	9343	9344	9345	9346	9347	9348	9349	51
6	9350	9351	9352	9353	9354	9355	9356	9357	9358	9359	51
7	9360	9361	9362	9363	9364	9365	9366	9367	9368	9369	51
8	9370	9371	9372	9373	9374	9375	9376	9377	9378	9379	51
9	9380	9381	9382	9383	9384	9385	9386	9387	9388	9389	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE XII. LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
890	944183	944532	944581	944631	944640	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5169	5518	5567	5616	5665	5715	5761	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
990	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
1	9478	9526	9575	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0755	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
990	954213	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6841	6889	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7560	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8469	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959011	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9581	9586	9591	9596	9601	9606	9611	9616	9621	9626	48
2	9631	9636	9641	9646	9651	9656	9661	9666	9671	9676	48
3	9681	9686	9691	9696	9701	9706	9711	9716	9721	9726	48
4	9731	9736	9741	9746	9751	9756	9761	9766	9771	9776	48
5	9781	9786	9791	9796	9801	9806	9811	9816	9821	9826	48
6	9831	9836	9841	9846	9851	9856	9861	9866	9871	9876	48
7	9881	9886	9891	9896	9901	9906	9911	9916	9921	9926	48
8	9931	9936	9941	9946	9951	9956	9961	9966	9971	9976	48
9	9981	9986	9991	9996	10001	10006	10011	10016	10021	10026	48
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4269	4307	4354	4401	4448	4495	4542	4589	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	964483	964530	964576	964623	964670	964716	964763	964810	964856	964903	47
1	8959	8996	9033	9070	9106	9143	9179	9216	9253	9290	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	47
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	16
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	16
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	16
8	2203	2249	2295	2342	2388	2434	2481	2527	2574	2620	16
9	2666	2712	2758	2804	2851	2897	2944	2990	3037	3082	16
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
94	97312	97317	97322	97326	97331	97335	97340	97345	97349	97353	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7221	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	97772	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	90003	90009	90014	90019	90024	90029	90034	90039	90044	90049	46
6	0158	0193	0239	0284	0330	0375	0421	0466	0512	0557	46
7	0612	0657	0703	0748	0794	0839	0884	0929	0975	1020	46
8	1066	1111	1156	1201	1247	1292	1337	1383	1428	1473	46
9	1519	1564	1609	1654	1700	1745	1790	1836	1881	1926	46
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982679	46
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	46
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	46
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	46
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	46
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	46
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	46
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5831	46
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	46
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	46
970	9	6772	986817	986861	986906	986951	986996	987040	987085	987130	46
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	46
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	46
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	46
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	46
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	46
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	46
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	46
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	46
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	46
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	46
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	46
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	46
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	46
4	2995	3039	3083	3127	3171	3216	3260	3304	3348	3392	46
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	46
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	46
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	46
8	4757	4801	4845	4889	4933	4977	5021	5065	5109	5152	46
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	46
990	995635	995679	995723	995767	995811	995855	995898	995942	995986	996030	46
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	46
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6905	46
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	46
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	46
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	46
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8651	46
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	46
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	46
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	46
	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE XIII.

LOGARITHMIC SINES, COSINES, TANGENTS.

AND

COTANGENTS.

M.	Sine.	D. 1'	Cosine.	D. 1'	Tang.	D. 1'	Cotang.	M.
0	Inf. neg.		0.000000	.00	Inf. neg.		Infinite.	60
1	6.463726	5017.17	.000000	.00	6.463726	5017.17	3.536274	59
2	.764756	2934.85	.000000	.00	.764756	2934.85	.235244	58
3	.940847	2082.31	.000000	.00	.940847	2082.31	.059153	57
4	7.065786	1615.17	.000000	.00	7.065786	1615.17	2.934214	56
5	.162696	1319.69	.000000	.00	.162696	1319.69	.837304	55
6	.241877	1115.78	.999999	.00	.241877	1115.78	.763122	54
7	.308824	966.53	.999999	.00	.308825	966.54	.691175	53
8	.366416	852.54	.999999	.01	.366817	852.55	.633183	52
9	.417968	762.62	.999999	.01	.417970	762.63	.582030	51
10	7.163726	639.88	.999998	.01	7.463727	639.88	2.536273	50
11	.505118	629.81	.999998	.01	.505120	629.81	.494880	49
12	.542906	579.37	.999997	.01	.542909	579.37	.457091	48
13	.577663	536.41	.999997	.01	.577672	536.42	.422328	47
14	.609353	499.38	.999996	.01	.609357	499.39	.390143	46
15	.639316	467.14	.999996	.01	.639320	467.15	.360180	45
16	.667845	438.81	.999995	.01	.667849	438.82	.332151	44
17	.694173	413.72	.999995	.01	.694179	413.73	.305821	43
18	.718997	391.35	.999991	.01	.719003	391.36	.280997	42
19	.742478	371.27	.999993	.01	.742484	371.28	.257516	41
20	7.764754	353.15	.999993	.01	7.764761	353.16	2.235239	40
21	.785943	336.72	.999992	.01	.785951	336.73	.214049	39
22	.806146	321.75	.999991	.01	.806155	321.76	.193845	38
23	.825451	303.05	.999990	.01	.825460	303.07	.174540	37
24	.843934	295.47	.999989	.02	.843944	295.49	.156056	36
25	.861662	283.88	.999989	.02	.861674	283.90	.138326	35
26	.878695	273.17	.999988	.02	.878708	273.18	.121292	34
27	.895035	263.23	.999987	.02	.895049	263.25	.104901	33
28	.910879	253.99	.999986	.02	.910894	254.01	.089108	32
29	.926119	245.38	.999985	.02	.926134	245.40	.073866	31
30	7.940842	237.33	.999983	.02	7.940858	237.35	2.059142	30
31	.955082	229.80	.999982	.02	.955100	229.82	.044900	29
32	.963870	222.73	.999981	.02	.963889	222.75	.031111	28
33	.972233	216.08	.999980	.02	.972253	216.10	.017747	27
34	.981198	209.81	.999979	.02	.981219	209.83	.004781	26
35	8.007787	203.90	.999977	.02	8.007809	203.92	1.992191	25
36	.020021	193.31	.999976	.02	.020044	198.33	.979966	24
37	.031919	193.02	.999975	.02	.031945	193.05	.969055	23
38	.043501	188.01	.999973	.02	.043527	188.03	.956473	22
39	.054781	183.25	.999972	.02	.054809	183.27	.945191	21
40	8.065776	178.72	.999971	.02	8.065806	178.75	1.934194	20
41	.076500	174.42	.999969	.03	.076533	174.44	.923469	19
42	.086965	170.31	.999968	.03	.086997	170.34	.913003	18
43	.097183	166.39	.999966	.03	.097217	166.42	.902783	17
44	.107167	162.65	.999964	.03	.107203	162.68	.892797	16
45	.116926	159.08	.999963	.03	.116963	159.11	.883037	15
46	.126471	155.66	.999961	.03	.126510	155.69	.873490	14
47	.135810	152.38	.999959	.03	.135851	152.41	.864149	13
48	.144953	149.24	.999958	.03	.144996	149.27	.855004	12
49	.153907	146.22	.999956	.03	.153952	146.25	.846048	11
50	8.162681	143.33	.999954	.03	8.162727	143.36	1.837273	10
51	.171230	140.54	.999952	.03	.171325	140.57	.825672	9
52	.179713	137.86	.999950	.03	.179763	137.90	.820237	8
53	.187935	135.29	.999948	.03	.188036	135.32	.811964	7
54	.196102	132.80	.999946	.03	.196156	132.84	.803944	6
55	.204070	130.41	.999944	.03	.204125	130.44	.795874	5
56	.211895	128.10	.999942	.03	.211953	128.14	.788047	4
57	.219581	125.87	.999940	.04	.219641	125.91	.780359	3
58	.227134	123.72	.999938	.04	.227195	123.76	.772905	2
59	.234557	121.61	.999936	.04	.234621	121.63	.765379	1
60	.241855		.999934	.04	.241921		.758079	0
M.	Cosine.	D. 1'	Sine.	D. 1'	Cotang.	D. 1'	Tang.	M.

M.	Sine	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	8.241855	119.63	9.999934	.04	8.241921	119.67	1.758079	60
1	.249033	117.69	.999932	.04	.249102	117.72	.758088	59
2	.256094	115.80	.999929	.04	.256165	115.84	.743835	58
3	.263042	113.93	.999927	.04	.263115	114.02	.736885	57
4	.269881	112.21	.999925	.04	.269956	112.25	.730044	56
5	.276614	110.50	.999922	.04	.276691	110.54	.723309	55
6	.283243	108.83	.999920	.04	.283323	108.87	.716677	54
7	.289773	107.22	.999918	.04	.289856	107.26	.710144	53
8	.296207	105.66	.999915	.04	.296292	105.70	.703708	52
9	.302546	104.13	.999913	.04	.302634	104.18	.697366	51
10	8.308794	102.66	9.999910	.04	8.308884	102.70	1.691116	50
11	.314954	101.22	.999907	.04	.315046	101.26	.684954	49
12	.321027	99.82	.999905	.04	.321122	99.87	.678878	48
13	.327016	98.47	.999902	.05	.327114	98.51	.672886	47
14	.332924	97.14	.999899	.05	.333025	97.19	.666975	46
15	.338753	95.86	.999897	.05	.338861	95.90	.661114	45
16	.344504	94.60	.999894	.05	.344610	94.65	.655390	44
17	.350181	93.38	.999891	.05	.350289	93.43	.649711	43
18	.355783	92.19	.999888	.05	.355895	92.24	.644105	42
19	.361315	91.03	.999885	.05	.361430	91.08	.638570	41
20	8.366777	89.90	9.999882	.05	8.366895	89.95	1.633105	40
21	.372171	88.80	.999879	.05	.372292	88.85	.632708	39
22	.377499	87.72	.999876	.05	.377622	87.77	.622378	38
23	.382762	86.67	.999873	.05	.382889	86.72	.617111	37
24	.387962	85.64	.999870	.05	.388092	85.70	.611908	36
25	.393101	84.64	.999867	.05	.393234	84.69	.606766	35
26	.398179	83.66	.999864	.05	.398315	83.71	.601685	34
27	.403199	82.71	.999861	.05	.403338	82.76	.596662	33
28	.408161	81.77	.999858	.05	.408304	81.82	.591696	32
29	.413063	80.86	.999854	.05	.413213	80.91	.586787	31
30	8.417919	79.96	9.999851	.06	8.418068	80.02	1.581932	30
31	.422717	79.09	.999848	.06	.422869	79.14	.577131	29
32	.427462	78.23	.999844	.06	.427618	78.29	.572332	28
33	.432156	77.40	.999841	.06	.432315	77.45	.567536	27
34	.436800	76.58	.999838	.06	.436962	76.63	.563038	26
35	.441394	75.77	.999834	.06	.441560	75.83	.558440	25
36	.445941	74.99	.999831	.06	.446110	75.05	.553890	24
37	.450440	74.22	.999827	.06	.450613	74.23	.549357	23
38	.454893	73.47	.999824	.06	.455070	73.53	.544930	22
39	.459301	72.73	.999820	.06	.459481	72.79	.540519	21
40	8.463665	72.00	9.999816	.06	8.463849	72.06	1.536151	20
41	.467935	71.29	.999813	.06	.468172	71.35	.531828	19
42	.472263	70.60	.999809	.06	.472454	70.66	.527516	18
43	.476493	69.91	.999805	.06	.476693	69.98	.523307	17
44	.480693	69.24	.999801	.06	.480892	69.31	.519108	16
45	.484848	68.59	.999797	.06	.485050	68.65	.514950	15
46	.488963	67.94	.999794	.07	.489170	68.01	.510830	14
47	.493040	67.31	.999790	.07	.493250	67.38	.506750	13
48	.497078	66.69	.999786	.07	.497293	66.76	.502707	12
49	.501080	66.08	.999782	.07	.501293	66.15	.498702	11
50	8.505015	65.48	9.999778	.07	8.505267	65.55	1.494733	10
51	.508974	64.89	.999774	.07	.509200	64.96	.490800	9
52	.512867	64.32	.999769	.07	.513099	64.39	.486902	8
53	.516726	63.75	.999765	.07	.516961	63.82	.483039	7
54	.520551	63.19	.999761	.07	.520790	63.26	.479210	6
55	.524343	62.65	.999757	.07	.524586	62.72	.475414	5
56	.528102	62.11	.999753	.07	.528349	62.18	.471651	4
57	.531823	61.58	.999748	.07	.532090	61.65	.467920	3
58	.535523	61.06	.999744	.07	.535779	61.13	.464221	2
59	.539186	60.55	.999740	.07	.539447	60.62	.460553	1
60	.542819		.999735		.543084		.456916	0
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.019235	20.00	9.997614	22	9.021620	20.23	0.978380	60
1	.021435	19.95	.997601	22	.022834	20.17	.977166	59
2	.021632	19.89	.997588	22	.024044	20.12	.975956	58
3	.022825	19.84	.997574	22	.025251	20.06	.974749	57
4	.024016	19.78	.997561	22	.026455	20.01	.973545	56
5	.025203	19.73	.997547	22	.027655	19.95	.972345	55
6	.026386	19.67	.997534	23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	23	.030046	19.85	.969954	53
8	.028741	19.57	.997507	23	.031237	19.79	.968763	52
9	.029918	19.51	.997493	23	.032425	19.74	.967575	51
10	9.031089	19.46	9.997480	23	9.033609	19.69	0.966391	50
11	.032257	19.41	.997466	23	.034791	19.64	.965209	49
12	.033421	19.36	.997452	23	.035969	19.63	.964031	48
13	.034582	19.30	.997439	23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	23	.040651	19.38	.959349	44
17	.039197	19.10	.997383	23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	23	.044130	19.23	.955867	41
20	9.042625	18.95	9.997341	23	9.045281	19.18	0.954716	40
21	.043762	18.90	.997327	23	.046434	19.13	.953566	39
22	.044895	18.85	.997313	24	.047582	19.08	.952413	38
23	.046026	18.80	.997299	24	.048727	19.03	.951273	37
24	.047151	18.75	.997285	24	.049869	18.98	.950131	36
25	.048279	18.70	.997271	24	.051008	18.93	.948992	35
26	.049400	18.65	.997257	24	.052144	18.89	.947856	34
27	.050519	18.60	.997242	24	.053277	18.84	.946723	33
28	.051635	18.55	.997228	24	.054407	18.79	.945593	32
29	.052749	18.50	.997214	24	.055535	18.74	.944465	31
30	9.053859	18.46	9.997199	24	9.056659	18.70	0.943341	30
31	.054965	18.41	.997185	24	.057781	18.65	.942219	29
32	.056071	18.36	.997170	24	.058900	18.60	.941100	28
33	.057172	18.31	.997156	24	.060016	18.56	.939984	27
34	.058271	18.27	.997141	24	.061130	18.51	.938870	26
35	.059367	18.22	.997127	24	.062240	18.46	.937760	25
36	.060460	18.17	.997112	24	.063348	18.42	.936652	24
37	.061551	18.13	.997098	24	.064453	18.37	.935547	23
38	.062639	18.08	.997083	25	.065556	18.33	.934444	22
39	.063724	18.04	.997068	25	.066655	18.28	.933345	21
40	9.064816	17.99	9.997053	25	9.067752	18.24	0.932248	20
41	.065885	17.95	.997039	25	.068846	18.19	.931154	19
42	.066962	17.90	.997024	25	.069938	18.15	.930062	18
43	.068036	17.86	.997009	25	.071027	18.10	.928973	17
44	.069107	17.81	.996994	25	.072113	18.06	.927887	16
45	.070176	17.77	.996979	25	.073197	18.02	.926803	15
46	.071242	17.72	.996964	25	.074278	17.97	.925722	14
47	.072306	17.68	.996949	25	.075356	17.93	.924644	13
48	.073366	17.64	.996934	25	.076432	17.89	.923568	12
49	.074424	17.59	.996919	25	.077505	17.84	.922495	11
50	9.075480	17.55	9.996904	25	9.078576	17.80	0.921424	10
51	.076533	17.51	.996889	25	.079644	17.76	.920356	9
52	.077583	17.46	.996874	25	.080710	17.72	.919290	8
53	.078631	17.42	.996858	25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	26	.082833	17.63	.917167	6
55	.080719	17.34	.996828	26	.083891	17.59	.916109	5
56	.081759	17.29	.996812	26	.084947	17.55	.915053	4
57	.082797	17.25	.996797	26	.086000	17.51	.914000	3
58	.083832	17.21	.996782	26	.087050	17.47	.912950	2
59	.084861	17.17	.996766	26	.088098	17.43	.911902	1
60	.085891		.996751	26	.089144		.910856	
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.085891	17.13	9.996751	.26	9.089144	17.39	0.910856	60
1	.086922	17.09	.996735	.26	.090187	17.35	.909813	59
2	.087917	17.05	.996720	.26	.091228	17.31	.908772	58
3	.088970	17.00	.996704	.26	.092266	17.27	.907734	57
4	.089990	16.96	.996688	.26	.093302	17.23	.906698	56
5	.091008	16.92	.996673	.26	.094336	17.19	.905661	55
6	.092024	16.88	.996657	.26	.095367	17.15	.904633	54
7	.093037	16.84	.996641	.26	.096395	17.11	.903605	53
8	.094047	16.80	.996625	.26	.097422	17.07	.902578	52
9	.095056	16.76	.996610	.26	.098446	17.03	.901554	51
10	9.096062	16.73	9.996594	.27	9.099468	16.99	0.900532	50
11	.097065	16.69	.996578	.27	.100487	16.95	.899513	49
12	.098066	16.65	.996562	.27	.101504	16.91	.898496	48
13	.099065	16.61	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.84	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.49	.996498	.27	.105550	16.76	.894450	44
17	.103037	16.46	.996482	.27	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.69	.892441	42
19	.105010	16.38	.996449	.27	.108560	16.65	.891440	41
20	9.105992	16.34	9.996433	.27	9.109559	16.61	0.890441	40
21	.106973	16.30	.996417	.27	.110556	16.58	.889444	39
22	.107951	16.27	.996400	.27	.111551	16.54	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.19	.996368	.27	.113533	16.47	.886467	36
25	.110873	16.16	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.39	.884493	34
27	.112809	16.08	.996318	.28	.116491	16.36	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.32	.882528	32
29	.114737	16.01	.996285	.28	.118452	16.29	.881548	31
30	9.115698	15.98	9.996269	.28	9.119429	16.25	0.880571	30
31	.116656	15.94	.996252	.28	.120404	16.22	.879596	29
32	.117613	15.90	.996235	.28	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.11	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.76	.996168	.28	.125249	16.04	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.01	.873789	23
38	.123306	15.69	.996134	.28	.127172	15.98	.872828	22
39	.124248	15.66	.996117	.28	.128130	15.94	.871870	21
40	9.125187	15.62	9.996100	.28	9.129087	15.91	0.870913	20
41	.126125	15.59	.996083	.28	.130041	15.87	.869959	19
42	.127060	15.56	.996066	.28	.130994	15.84	.869006	18
43	.127993	15.52	.996049	.29	.131944	15.81	.868056	17
44	.128925	15.49	.996032	.29	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.29	.133839	15.74	.866161	15
46	.130781	15.42	.995998	.29	.134784	15.71	.865216	14
47	.131706	15.39	.995980	.29	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.29	.136667	15.64	.863333	12
49	.133551	15.32	.995946	.29	.137605	15.61	.862395	11
50	9.134470	15.29	9.995928	.29	9.138512	15.58	0.861458	10
51	.135387	15.26	.995911	.29	.139476	15.55	.860524	9
52	.136303	15.22	.995894	.29	.140409	15.51	.859591	8
53	.137216	15.19	.995876	.29	.141310	15.48	.858660	7
54	.138128	15.16	.995859	.29	.142269	15.45	.857731	6
55	.139037	15.13	.995841	.29	.143196	15.42	.856804	5
56	.139944	15.09	.995823	.29	.144111	15.39	.855879	4
57	.140850	15.06	.995806	.29	.145011	15.36	.854956	3
58	.141754	15.03	.995788	.29	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.29	.853115	1
60	.143555	15.00	.995753	.30	.147803		.852197	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine	D. 1".	Cosine.	D. 1'	Tang.	D. 1".	Cotang.	M.
0	9.143555	14.97	9.995753	.30	9.147803	15.26	0.852197	60
1	.144453	14.93	.995735	.30	.147718	15.23	.8512-2	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850363	58
3	.146243	14.87	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.84	.995681	.30	.151454	15.14	.848546	56
5	.148026	14.81	.995664	.30	.152363	15.11	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.75	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.30	.155077	15.02	.844923	52
9	.151569	14.69	.995591	.30	.155978	14.99	.844022	51
10	9.152451	14.66	9.995573	.30	9.156877	14.96	0.843123	50
11	.153330	14.63	.995555	.30	.157775	14.93	.842225	49
12	.154208	14.60	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.54	.995501	.30	.160457	14.84	.839543	46
15	.156830	14.51	.995482	.31	.161347	14.81	.838653	45
16	.157700	14.48	.995464	.31	.162236	14.78	.837764	44
17	.158569	14.45	.995446	.31	.163123	14.75	.836877	43
18	.159435	14.42	.995427	.31	.164008	14.73	.835992	42
19	.160301	14.39	.995409	.31	.164892	14.70	.835108	41
20	9.161164	14.36	9.995390	.31	9.165774	14.67	0.834226	40
21	.162025	14.33	.995372	.31	.166654	14.64	.833346	39
22	.162885	14.30	.995353	.31	.167532	14.61	.832468	38
23	.163743	14.27	.995334	.31	.168409	14.58	.831591	37
24	.164600	14.24	.995316	.31	.169284	14.56	.830716	36
25	.165454	14.22	.995297	.31	.170157	14.53	.829843	35
26	.166307	14.19	.995278	.31	.171029	14.50	.828971	34
27	.167159	14.16	.995260	.31	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.31	.172767	14.44	.827233	32
29	.168856	14.10	.995222	.31	.173634	14.42	.826366	31
30	9.169702	14.07	9.995203	.31	9.174499	14.39	0.825501	30
31	.170547	14.05	.995184	.32	.175362	14.36	.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	13.99	.995146	.32	.177084	14.31	.822916	27
34	.173070	13.96	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.94	.995108	.32	.178799	14.25	.821201	25
36	.174744	13.91	.995089	.32	.179655	14.23	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.17	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.15	.817789	21
40	9.178072	13.80	9.995013	.32	9.183059	14.12	0.816941	20
41	.178900	13.77	.994993	.32	.183907	14.09	.816093	19
42	.179726	13.75	.994974	.32	.184752	14.07	.815248	18
43	.180551	13.72	.994955	.32	.185597	14.04	.814403	17
44	.181374	13.69	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.32	.187280	13.99	.812720	15
46	.183016	13.64	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.61	.994877	.33	.188959	13.94	.811042	13
48	.184651	13.59	.994857	.33	.189794	13.91	.810206	12
49	.185466	13.56	.994838	.33	.190629	13.89	.809371	11
50	9.186280	13.54	9.994818	.33	9.191462	13.86	0.808538	10
51	.187092	13.51	.994798	.33	.192294	13.84	.807706	9
52	.187903	13.48	.994779	.33	.193124	13.81	.806876	8
53	.188712	13.46	.994759	.33	.193953	13.79	.806047	7
54	.189519	13.43	.994739	.33	.194780	13.76	.805220	6
55	.190325	13.41	.994720	.33	.195606	13.74	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.71	.803570	4
57	.191933	13.36	.994680	.33	.197253	13.69	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.66	.801926	2
59	.193534	13.31	.994640	.33	.198894	13.64	.801106	1
60	.194332		.994620	.33	.199713		.800287	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M	Sine.	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	9.194332	13.28	9.994620	.33	9.199713	13.62	0.800237	60
1	.195129	13.26	.994600	.33	.200529	13.59	.799471	59
2	.195925	13.23	.994580	.34	.201345	13.57	.799655	58
3	.196719	13.21	.994560	.34	.202159	13.54	.797841	57
4	.197511	13.18	.994540	.34	.202971	13.52	.797029	56
5	.198302	13.16	.994519	.34	.203782	13.49	.796218	55
6	.199091	13.13	.994499	.34	.204592	13.47	.795408	54
7	.199879	13.11	.994479	.34	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.34	.206207	13.42	.793793	52
9	.201451	13.06	.994438	.34	.207013	13.40	.792987	51
10	9.202234	13.04	9.994418	.34	9.207817	13.38	0.792183	50
11	.203017	13.01	.994398	.34	.208619	13.35	.791381	49
12	.203797	12.99	.994377	.34	.209420	13.33	.790580	48
13	.204577	12.96	.994357	.34	.210220	13.31	.789780	47
14	.205354	12.94	.994336	.34	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.34	.211815	13.26	.788185	45
16	.206906	12.89	.994295	.34	.212611	13.24	.787389	44
17	.207679	12.87	.994274	.34	.213405	13.21	.786595	43
18	.208452	12.85	.994254	.35	.214199	13.19	.785802	42
19	.209222	12.82	.994233	.35	.214993	13.17	.785011	41
20	9.209992	12.80	9.994212	.35	9.215780	13.15	0.784220	40
21	.210769	12.78	.994191	.35	.216588	13.12	.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.08	.781858	37
24	.213056	12.71	.994129	.35	.218926	13.06	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.66	.994087	.35	.220492	13.01	.779508	34
27	.215338	12.64	.994066	.35	.221272	12.99	.778723	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777943	32
29	.216854	12.59	.994024	.35	.222830	12.95	.777170	31
30	9.217609	12.57	9.994003	.35	9.223607	12.92	0.776393	30
31	.218363	12.55	.993982	.35	.224382	12.90	.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774841	28
33	.219868	12.50	.993939	.35	.225929	12.86	.774071	27
34	.220618	12.48	.993918	.36	.226700	12.84	.773300	26
35	.221367	12.46	.993897	.36	.227471	12.82	.772529	25
36	.222115	12.44	.993875	.36	.228239	12.79	.771761	24
37	.222861	12.42	.993854	.36	.229007	12.77	.770993	23
38	.223606	12.39	.993832	.36	.229773	12.75	.770227	22
39	.224349	12.37	.993811	.36	.230539	12.73	.769461	21
40	9.225092	12.35	9.993789	.36	9.231302	12.71	0.768698	20
41	.225833	12.33	.993768	.36	.232065	12.69	.767935	19
42	.226573	12.31	.993746	.36	.232826	12.67	.767174	18
43	.227311	12.29	.993725	.36	.233586	12.65	.766414	17
44	.228048	12.26	.993703	.36	.234345	12.63	.765655	16
45	.228784	12.24	.993681	.36	.235103	12.61	.764897	15
46	.229518	12.22	.993660	.36	.235859	12.59	.764141	14
47	.230252	12.20	.993638	.36	.236614	12.56	.763386	13
48	.230984	12.18	.993616	.36	.237368	12.54	.762632	12
49	.231715	12.16	.993594	.36	.238120	12.52	.761880	11
50	9.232444	12.14	9.993572	.37	9.238872	12.50	0.761128	10
51	.233172	12.12	.993550	.37	.239622	12.48	.760378	9
52	.233909	12.10	.993528	.37	.240371	12.46	.759629	8
53	.234645	12.07	.993506	.37	.241118	12.44	.758882	7
54	.235379	12.05	.993484	.37	.241865	12.42	.758135	6
55	.236113	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236845	12.01	.993440	.37	.243354	12.38	.756646	4
57	.237576	11.99	.993418	.37	.244097	12.36	.755903	3
58	.238305	11.97	.993396	.37	.244839	12.34	.755161	2
59	.239033	11.95	.993374	.37	.245579	12.32	.754421	1
60	.239761		.993351	.37	.246319		.753681	0
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.239670	11.93	9.993351	.37	9.246319	12.30	0.753681	60
1	.240386	11.91	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.89	.993307	.37	.247794	12.26	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.24	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.22	.750736	56
5	.243237	11.83	.993240	.37	.249998	12.20	.750002	55
6	.243947	11.81	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.79	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.75	.993149	.38	.252920	12.13	.747080	51
10	9.246775	11.73	9.993127	.38	9.253649	12.11	0.746352	50
11	.247478	11.71	.993104	.38	.254374	12.09	.745626	49
12	.248181	11.69	.993081	.38	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.01	.742731	45
16	.250980	11.61	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.59	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.58	.992944	.38	.259429	11.96	.740571	42
19	.253067	11.56	.992921	.38	.260146	11.94	.739854	41
20	9.253761	11.54	9.992898	.38	9.260863	11.92	0.739137	40
21	.254453	11.52	.992875	.38	.261578	11.90	.738422	39
22	.255144	11.50	.992852	.39	.262292	11.89	.737708	38
23	.255834	11.48	.992829	.39	.263005	11.87	.736995	37
24	.256523	11.46	.992806	.39	.263717	11.85	.736283	36
25	.257211	11.44	.992783	.39	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.39	.265138	11.81	.734862	34
27	.258583	11.41	.992736	.39	.265847	11.79	.734153	33
28	.259268	11.39	.992713	.39	.266555	11.78	.733445	32
29	.259951	11.37	.992690	.39	.267261	11.76	.732739	31
30	9.260633	11.35	9.992666	.39	9.267967	11.74	0.732033	30
31	.261314	11.33	.992643	.39	.268671	11.72	.731329	29
32	.261994	11.31	.992619	.39	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.39	.270077	11.69	.729923	27
34	.263351	11.28	.992572	.39	.270779	11.67	.729221	26
35	.264027	11.26	.992549	.39	.271479	11.65	.728521	25
36	.264703	11.24	.992525	.39	.272178	11.64	.727822	24
37	.265377	11.22	.992501	.39	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.19	.992454	.40	.274269	11.58	.725731	21
40	9.267395	11.17	9.992430	.40	9.274964	11.57	0.725036	20
41	.268065	11.15	.992406	.40	.275658	11.55	.724342	19
42	.268734	11.13	.992382	.40	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.51	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.06	.992287	.40	.279113	11.46	.720887	14
47	.272064	11.05	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.40	.280488	11.43	.719512	12
49	.273388	11.01	.992214	.40	.281174	11.41	.718826	11
50	9.274049	10.99	9.992190	.40	9.281858	11.40	0.718142	10
51	.274708	10.98	.992166	.40	.282542	11.38	.717458	9
52	.275367	10.96	.992142	.40	.283225	11.36	.716775	8
53	.276025	10.94	.992118	.41	.283907	11.35	.716093	7
54	.276681	10.92	.992093	.41	.284588	11.33	.715412	6
55	.277337	10.91	.992069	.41	.285268	11.31	.714732	5
56	.277991	10.89	.992044	.41	.285947	11.30	.714053	4
57	.278645	10.87	.992020	.41	.286624	11.28	.713376	3
58	.279297	10.86	.991996	.41	.287301	11.26	.712699	2
59	.279948	10.84	.991971	.41	.287977	11.25	.712023	1
60	.280599		.991947		.288652		.711348	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	9.230599	10.82	9.991917	.41	9.230652	11.23	0.711348	60
1	.231243	10.81	.991922	.41	.239326	11.22	.710674	59
2	.231897	10.79	.991897	.41	.239999	11.20	.710001	58
3	.232544	10.77	.991873	.41	.290671	11.18	.709329	57
4	.233190	10.76	.991848	.41	.291342	11.17	.708658	56
5	.233836	10.74	.991823	.41	.292013	11.17	.707987	55
6	.234480	10.72	.991799	.41	.292682	11.15	.707318	54
7	.235124	10.71	.991774	.41	.293350	11.14	.706650	53
8	.235766	10.69	.991749	.41	.294017	11.12	.705983	52
9	.236408	10.67	.991724	.42	.294684	11.11	.705316	51
10	9.237043	10.66	9.991699	.42	9.295319	11.09	0.704651	50
11	.237633	10.64	.991674	.42	.295983	11.07	.703987	49
12	.238326	10.63	.991649	.42	.296677	11.06	.703323	48
13	.238961	10.61	.991624	.42	.297339	11.04	.702661	47
14	.239600	10.59	.991599	.42	.298001	11.03	.701999	46
15	.240236	10.58	.991574	.42	.298662	11.01	.701338	45
16	.240370	10.56	.991549	.42	.299322	11.00	.700678	44
17	.241504	10.55	.991524	.42	.299980	10.98	.700020	43
18	.24137	10.53	.991498	.42	.300638	10.97	.699362	42
19	.242763	10.51	.991473	.42	.301295	10.95	.698705	41
20	9.293399	10.50	9.991448	.42	9.301951	10.93	0.698049	40
21	.291029	10.48	.991422	.42	.302607	10.92	.697393	39
22	.291658	10.47	.991397	.42	.303261	10.90	.696739	38
23	.292296	10.45	.991372	.42	.303914	10.89	.696086	37
24	.292913	10.43	.991346	.42	.304567	10.87	.695433	36
25	.293539	10.42	.991321	.42	.305218	10.86	.694782	35
26	.294161	10.40	.991295	.43	.305869	10.84	.694131	34
27	.294788	10.39	.991270	.43	.306519	10.83	.693481	33
28	.295412	10.37	.991244	.43	.307168	10.81	.692832	32
29	.296034	10.36	.991218	.43	.307816	10.80	.692184	31
30	9.299655	10.34	9.991193	.43	9.309463	10.78	0.691537	30
31	.300276	10.33	.991167	.43	.309109	10.77	.690891	29
32	.300895	10.31	.991141	.43	.309754	10.76	.690246	28
33	.301514	10.30	.991115	.43	.310399	10.74	.689601	27
34	.302132	10.28	.991090	.43	.311042	10.73	.688955	26
35	.302748	10.26	.991064	.43	.311685	10.71	.688315	25
36	.303361	10.25	.991038	.43	.312327	10.70	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.68	.687032	23
38	.304593	10.22	.990986	.43	.313609	10.67	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.65	.685753	21
40	9.315819	10.19	9.990931	.41	9.314885	10.64	0.685115	20
41	.306439	10.17	.990908	.41	.315523	10.62	.684477	19
42	.307041	10.16	.990882	.41	.316159	10.61	.683841	18
43	.307650	10.14	.990855	.41	.316795	10.60	.683205	17
44	.308255	10.13	.990829	.41	.317430	10.58	.682570	16
45	.308867	10.12	.990803	.41	.318064	10.57	.681936	15
46	.309474	10.10	.990777	.41	.318697	10.55	.681303	14
47	.310090	10.09	.990750	.41	.319330	10.54	.680670	13
48	.310685	10.07	.990724	.41	.319961	10.53	.680039	12
49	.311289	10.06	.990697	.41	.320592	10.51	.679408	11
50	9.311893	10.04	9.990671	.41	9.321222	10.50	0.678778	10
51	.312495	10.03	.990645	.41	.321851	10.49	.678149	9
52	.313097	10.01	.990618	.41	.322479	10.47	.677521	8
53	.313698	10.00	.990591	.41	.323106	10.46	.676894	7
54	.314297	9.98	.990565	.41	.323733	10.44	.676267	6
55	.314897	9.97	.990538	.41	.324358	10.43	.675642	5
56	.315495	9.96	.990511	.41	.324983	10.41	.675017	4
57	.316092	9.94	.990485	.41	.325607	10.40	.674393	3
58	.316689	9.93	.990458	.41	.326231	10.39	.673769	2
59	.317284	9.91	.990431	.41	.326853	10.37	.673147	1
60	.317879	9.91	.990404	.41	.327475	10.36	.672525	0
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	Tang.	M.

M.	Sine	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.317879	9.90	9.990404	.45	9.327475	10.35	0.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.86	.990324	.45	.329334	10.31	.670666	57
4	.320249	9.84	.990297	.45	.329953	10.29	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.81	.990243	.45	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.79	.990188	.45	.332418	10.24	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.23	.666967	51
10	9.323780	9.76	9.990134	.45	9.333646	10.21	0.666354	50
11	.324366	9.75	.990107	.45	.334259	10.20	.665741	49
12	.324950	9.73	.990079	.46	.334871	10.19	.665129	48
13	.325534	9.72	.990052	.46	.335482	10.17	.664518	47
14	.326117	9.70	.990025	.46	.336093	10.16	.663907	46
15	.326700	9.69	.989997	.46	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.46	.337311	10.14	.662689	44
17	.327862	9.66	.989942	.46	.337919	10.12	.662081	43
18	.328442	9.65	.989915	.46	.338527	10.11	.661473	42
19	.329021	9.64	.989887	.46	.339133	10.10	.660867	41
20	9.329599	9.62	9.989860	.46	9.339739	10.08	0.660261	40
21	.330176	9.61	.989832	.46	.340344	10.07	.659656	39
22	.330753	9.60	.989804	.46	.340948	10.06	.659052	38
23	.331329	9.58	.989777	.46	.341552	10.05	.658448	37
24	.331903	9.57	.989749	.46	.342155	10.03	.657845	36
25	.332478	9.57	.989721	.46	.342757	10.02	.657243	35
26	.333051	9.56	.989693	.46	.343358	10.01	.656642	34
27	.333624	9.53	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.52	.989637	.47	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	9.335337	9.49	9.989582	.47	9.345755	9.96	0.654245	30
31	.335906	9.48	.989553	.47	.346353	9.95	.653647	29
32	.336475	9.46	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.92	.652455	27
34	.337610	9.44	.989469	.47	.348141	9.91	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.41	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.47	.349922	9.87	.650078	23
38	.339871	9.39	.989356	.47	.350514	9.86	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	9.340996	9.36	9.989300	.47	9.351697	9.84	0.648303	20
41	.341558	9.35	.989271	.47	.352287	9.82	.647713	19
42	.342119	9.34	.989243	.47	.352876	9.81	.647124	18
43	.342679	9.32	.989214	.48	.353465	9.80	.646535	17
44	.343239	9.31	.989186	.48	.354053	9.79	.645947	16
45	.343797	9.30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9.29	.989128	.48	.355227	9.76	.644773	14
47	.344912	9.27	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.26	.989071	.48	.356398	9.74	.643602	12
49	.346024	9.25	.989042	.48	.356982	9.73	.643018	11
50	9.346579	9.24	9.989014	.48	9.357566	9.72	0.642434	10
51	.347134	9.22	.988985	.48	.358149	9.70	.641851	9
52	.347687	9.21	.988956	.48	.358731	9.69	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.68	.640687	7
54	.348792	9.19	.988898	.48	.359893	9.67	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.66	.639526	5
56	.349893	9.16	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.14	.988782	.49	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.49	.362787	9.61	.637213	1
60	.352088	9.13	.988724	.49	.363364		.636636	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.352038	9.11	9.936724	.49	9.363364	9.60	0.636636	60
1	.352636	9.10	.936996	.49	.363940	9.59	.636060	59
2	.353181	9.09	.936666	.49	.364516	9.58	.635486	58
3	.353726	9.08	.936336	.49	.365090	9.57	.634910	57
4	.354271	9.07	.936007	.49	.365664	9.57	.634336	56
5	.354816	9.06	.935678	.49	.366237	9.56	.633763	55
6	.355368	9.05	.935348	.49	.366810	9.54	.633190	54
7	.355901	9.04	.935019	.49	.367382	9.53	.632618	53
8	.356443	9.03	.934689	.49	.367953	9.52	.632047	52
9	.356984	9.01	.934360	.49	.368524	9.51	.631476	51
10	9.357524	8.99	9.934030	.49	9.369094	9.49	0.630906	50
11	.358064	8.98	.933701	.49	.369663	9.48	.630337	49
12	.358603	8.97	.933371	.49	.370232	9.47	.629768	48
13	.359141	8.96	.933042	.50	.370799	9.46	.629199	47
14	.359678	8.96	.932712	.50	.371367	9.45	.628633	46
15	.360216	8.94	.932382	.50	.371933	9.44	.628067	45
16	.360752	8.92	.932052	.50	.372499	9.43	.627501	44
17	.361287	8.91	.931723	.50	.373064	9.42	.626936	43
18	.361822	8.90	.931393	.50	.373629	9.41	.626371	42
19	.362356	8.89	.931063	.50	.374193	9.40	.625807	41
20	9.362899	8.88	9.930733	.50	9.374756	9.38	0.625244	40
21	.363422	8.87	.930403	.50	.375319	9.37	.624681	39
22	.363964	8.86	.930073	.50	.375881	9.36	.624119	38
23	.364486	8.84	.929743	.50	.376442	9.35	.623558	37
24	.365016	8.83	.929413	.50	.377003	9.35	.622997	36
25	.365546	8.82	.929083	.50	.377563	9.33	.622437	35
26	.366076	8.81	.928753	.50	.378122	9.32	.621878	34
27	.366604	8.80	.928423	.50	.378681	9.31	.621319	33
28	.367131	8.79	.928092	.50	.379239	9.30	.620761	32
29	.367659	8.78	.927762	.51	.379797	9.29	.620203	31
30	9.368185	8.76	9.927432	.51	9.380354	9.27	0.619646	30
31	.368711	8.75	.927101	.51	.380910	9.26	.619090	29
32	.369236	8.74	.926771	.51	.381466	9.25	.618534	28
33	.369761	8.73	.926440	.51	.382021	9.24	.617979	27
34	.370285	8.72	.926110	.51	.382575	9.24	.617425	26
35	.370808	8.71	.925779	.51	.383129	9.23	.616871	25
36	.371330	8.70	.925449	.51	.383682	9.22	.616318	24
37	.371852	8.69	.925118	.51	.384234	9.21	.615766	23
38	.372373	8.68	.924788	.51	.384786	9.20	.615214	22
39	.372894	8.66	.924457	.51	.385337	9.19	.614663	21
40	9.373414	8.65	9.924126	.51	9.385898	9.17	0.614112	20
41	.373933	8.64	.923796	.51	.386458	9.16	.613562	19
42	.374452	8.63	.923465	.51	.386997	9.15	.613013	18
43	.374970	8.62	.923134	.51	.387536	9.14	.612464	17
44	.375487	8.61	.922803	.51	.388084	9.12	.611916	16
45	.376003	8.60	.922472	.52	.388631	9.11	.611369	15
46	.376519	8.59	.922141	.52	.389178	9.10	.610822	14
47	.377035	8.58	.921810	.52	.389724	9.09	.610276	13
48	.377549	8.57	.921479	.52	.390270	9.08	.609730	12
49	.378063	8.56	.921148	.52	.390815	9.07	.609185	11
50	9.378577	8.55	9.920817	.52	9.391360	9.06	0.608640	10
51	.379099	8.53	.920486	.52	.391903	9.06	.608097	9
52	.379601	8.52	.920155	.52	.392447	9.04	.607553	8
53	.380113	8.51	.919824	.52	.392989	9.03	.607011	7
54	.380624	8.50	.919492	.52	.393531	9.02	.606469	6
55	.381134	8.49	.919161	.52	.394073	9.01	.605927	5
56	.381643	8.48	.918830	.52	.394614	9.00	.605386	4
57	.382152	8.47	.918499	.52	.395154	8.99	.604846	3
58	.382661	8.46	.918168	.52	.395694	8.98	.604306	2
59	.383169	8.45	.917837	.52	.396233	8.97	.603767	1
60	.383676		.917506	.52	.396771		.603229	0
C.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.38675	8.44	9.96004	.53	9.396771	8.96	0.603229	60
1	384132	8.43	986873	.53	397309	8.96	.602691	59
2	381657	8.42	986841	.53	397846	8.96	.602154	58
3	385192	8.41	986809	.53	398383	8.95	.601617	57
4	385697	8.40	986778	.53	398919	8.93	.601081	56
5	386201	8.39	986746	.53	399455	8.92	.600545	55
6	386704	8.38	986714	.53	399990	8.92	.600010	54
7	387207	8.37	986683	.53	400524	8.91	.599476	53
8	387709	8.36	986651	.53	401058	8.90	.598942	52
9	388210	8.35	986619	.53	401591	8.88	.598409	51
10	9.393711	8.34	9.986587	.53	9.402124	8.97	0.597876	50
11	389211	8.33	986555	.53	402656	8.86	.597344	49
12	389711	8.32	986523	.53	403187	8.85	.596813	48
13	389210	8.31	986491	.53	403718	8.84	.596282	47
14	389708	8.30	986459	.53	404249	8.83	.595751	46
15	391206	8.29	986427	.54	404778	8.82	.595222	45
16	391703	8.28	986395	.54	405308	8.81	.594692	44
17	392199	8.27	986363	.54	405836	8.80	.594164	43
18	392695	8.26	986331	.54	406364	8.79	.593636	42
19	393191	8.25	986299	.54	406892	8.78	.593108	41
20	9.393635	8.24	9.986266	.54	9.407419	8.77	0.592581	40
21	394179	8.23	986231	.54	407945	8.76	.592055	39
22	394673	8.22	986202	.54	408471	8.75	.591529	38
23	395166	8.21	986169	.54	408996	8.75	.591004	37
24	395658	8.20	986137	.54	409521	8.74	.590479	36
25	396150	8.19	986104	.54	410045	8.73	.589955	35
26	396641	8.18	986072	.54	410569	8.72	.589431	34
27	397132	8.17	986039	.54	411092	8.71	.588908	33
28	397621	8.16	986007	.54	411615	8.70	.588385	32
29	398111	8.15	985974	.54	412137	8.69	.587863	31
30	9.393600	8.14	9.985942	.54	9.412658	8.68	0.587342	30
31	399088	8.13	985909	.55	413179	8.67	.586821	29
32	399575	8.12	985876	.55	413699	8.67	.586301	28
33	400062	8.11	985843	.55	414219	8.66	.585781	27
34	400549	8.10	985811	.55	414738	8.65	.585262	26
35	401035	8.09	985778	.55	415257	8.64	.584743	25
36	401520	8.08	985745	.55	415775	8.64	.584225	24
37	402005	8.07	985712	.55	416293	8.63	.583707	23
38	402489	8.06	985679	.55	416810	8.62	.583190	22
39	402972	8.05	985646	.55	417326	8.60	.582674	21
40	9.403455	8.04	9.985613	.55	9.417842	8.59	0.582158	20
41	403938	8.03	985580	.55	418358	8.58	.581642	19
42	404420	8.02	985547	.55	418873	8.57	.581127	18
43	404901	8.01	985514	.55	419387	8.56	.580613	17
44	405382	8.00	985480	.55	419901	8.56	.580099	16
45	405862	7.99	985447	.55	420415	8.55	.579585	15
46	406341	7.98	985414	.55	420927	8.55	.579073	14
47	406820	7.97	985381	.56	421440	8.54	.578560	13
48	407299	7.96	985347	.56	421952	8.53	.578048	12
49	407777	7.96	985314	.56	422463	8.51	.577537	11
50	9.408251	7.95	9.985280	.56	9.422974	8.50	0.577026	10
51	408731	7.94	985247	.56	423184	8.49	.576516	9
52	409207	7.93	985213	.56	423693	8.49	.576007	8
53	409682	7.92	985180	.56	424203	8.48	.575497	7
54	410157	7.91	985146	.56	424711	8.47	.574989	6
55	410632	7.90	985113	.56	425219	8.47	.574481	5
56	411106	7.89	985079	.56	425727	8.46	.573973	4
57	411579	7.88	985045	.56	426235	8.45	.573466	3
58	412052	7.87	985011	.56	426741	8.44	.572959	2
59	412524	7.86	984978	.56	427247	8.43	.572453	1
60	412996	7.86	984944	.56	427752	8.43	.571948	0
M.	Cosine.	D. 1"	Sine.	D. 1"	Cotang.	D. 1"	Tang.	

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.412996		9.984944		9.428052		0.571948	60
1	.413467	7.85	.984910	.56	.428558	8.42	.571442	59
2	.413938	7.84	.984876	.57	.429062	8.41	.570938	58
3	.414408	7.84	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.83	.984808	.57	.430070	8.39	.569930	56
5	.415347	7.82	.984774	.57	.430573	8.38	.569427	55
6	.415815	7.81	.984740	.57	.431075	8.38	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.79	.984672	.57	.432079	8.36	.567921	52
9	.417217	7.78	.984638	.57	.432580	8.35	.567420	51
		7.77		.57		8.34		
10	9.417694		9.984603		9.433080		0.566920	50
11	.418150	7.76	.984569	.57	.433580	8.33	.566420	49
12	.418615	7.75	.984535	.57	.434080	8.33	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.74	.984466	.57	.435078	8.31	.564922	46
15	.420007	7.73	.984432	.57	.435576	8.30	.564424	45
16	.420470	7.72	.984397	.57	.436073	8.29	.563927	44
17	.420933	7.71	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.58	.437067	8.28	.562933	42
19	.421857	7.69	.984294	.58	.437563	8.27	.562437	41
		7.68		.58		8.26		
20	9.422318		9.984259		9.438059		0.561911	40
21	.422778	7.67	.984224	.58	.438554	8.25	.561416	39
22	.423238	7.67	.984190	.58	.439048	8.24	.560922	38
23	.423697	7.66	.984155	.58	.439543	8.24	.560427	37
24	.424156	7.65	.984120	.58	.440036	8.23	.559934	36
25	.424615	7.64	.984085	.58	.440529	8.22	.559441	35
26	.425073	7.63	.984050	.58	.441022	8.21	.558948	34
27	.425530	7.62	.984015	.58	.441514	8.20	.558456	33
28	.425987	7.61	.983981	.58	.442006	8.20	.557964	32
29	.426443	7.61	.983946	.58	.442497	8.19	.557473	31
		7.60		.58		8.18		
30	9.426899		9.983911		9.442988		0.557012	30
31	.427354	7.59	.983875	.58	.443479	8.17	.556521	29
32	.427809	7.58	.983840	.58	.443968	8.16	.556032	28
33	.428263	7.57	.983805	.59	.444458	8.16	.555542	27
34	.428717	7.56	.983770	.59	.444947	8.15	.555053	26
35	.429170	7.55	.983735	.59	.445435	8.14	.554565	25
36	.429623	7.55	.983700	.59	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.59	.446411	8.13	.553589	23
38	.430527	7.52	.983629	.59	.446898	8.12	.553102	22
39	.430978	7.52	.983594	.59	.447384	8.11	.552616	21
		7.51		.59		8.10		
40	9.431429		9.983558		9.447870		0.552130	20
41	.431879	7.50	.983523	.59	.448356	8.09	.551644	19
42	.432329	7.49	.983487	.59	.448841	8.09	.551159	18
43	.432778	7.48	.983452	.59	.449326	8.08	.550674	17
44	.433226	7.47	.983416	.59	.449810	8.07	.550190	16
45	.433675	7.47	.983381	.59	.450294	8.06	.549706	15
46	.434122	7.46	.983345	.59	.450777	8.06	.549223	14
47	.434569	7.45	.983309	.59	.451260	8.05	.548740	13
48	.435016	7.44	.983273	.60	.451743	8.04	.548257	12
49	.435462	7.44	.983238	.60	.452225	8.03	.547775	11
		7.43		.60		8.03		
50	9.435908		9.983202		9.452706		0.547294	10
51	.436353	7.42	.983166	.60	.453187	8.02	.546813	9
52	.436798	7.41	.983130	.60	.453668	8.01	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.40	.983058	.60	.454628	8.00	.545372	6
55	.438129	7.39	.983022	.60	.455107	7.99	.544893	5
56	.438572	7.38	.982986	.60	.455586	7.98	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543935	3
58	.439456	7.36	.982914	.60	.456542	7.97	.543458	2
59	.439897	7.36	.982878	.60	.457019	7.96	.542981	1
60	.440338	7.35	.982842	.60	.457496	7.95	.542504	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.440338	7.34	9.982842	.60	9.457196	7.94	0.542504	60
1	.440778	7.33	.982805	.60	.457973	7.94	.542027	59
2	.441218	7.32	.982769	.61	.458419	7.93	.541551	58
3	.441658	7.31	.982733	.61	.458865	7.92	.541075	57
4	.442096	7.31	.982696	.61	.459300	7.91	.540600	56
5	.442535	7.30	.982660	.61	.459745	7.91	.540125	55
6	.442973	7.29	.982624	.61	.460189	7.90	.539651	54
7	.443410	7.28	.982587	.61	.460633	7.89	.539177	53
8	.443847	7.27	.982551	.61	.461077	7.88	.538703	52
9	.444284	7.27	.982514	.61	.461521	7.88	.538229	51
10	9.444720	7.26	9.982477	.61	9.462242	7.87	0.537758	50
11	.445156	7.25	.982441	.61	.462715	7.86	.537285	49
12	.445590	7.24	.982404	.61	.463186	7.86	.536814	48
13	.446025	7.24	.982367	.61	.463658	7.85	.536342	47
14	.446459	7.23	.982331	.61	.464128	7.84	.535872	46
15	.446893	7.22	.982294	.61	.464599	7.83	.535401	45
16	.447326	7.21	.982257	.61	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.81	.533992	42
19	.448623	7.19	.982146	.62	.466477	7.81	.533523	41
20	9.449054	7.18	9.982109	.62	9.466945	7.80	0.533055	40
21	.449485	7.17	.982072	.62	.467413	7.79	.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.16	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.14	.981924	.62	.469280	7.76	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.76	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.62	.470676	7.74	.529324	32
29	.452915	7.11	.981774	.62	.471141	7.74	.528859	31
30	9.453342	7.10	9.981737	.62	9.471605	7.73	0.528395	30
31	.453768	7.10	.981700	.62	.472069	7.72	.527931	29
32	.454191	7.09	.981662	.63	.472532	7.71	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.71	.527006	27
34	.455041	7.07	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.63	.473919	7.69	.526081	25
36	.455893	7.06	.981512	.63	.474381	7.69	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.04	.981436	.63	.475303	7.67	.524697	22
39	.457162	7.04	.981399	.63	.475763	7.67	.524237	21
40	9.457584	7.03	9.981361	.63	9.476223	7.66	0.523777	20
41	.458006	7.02	.981323	.63	.476683	7.65	.523317	19
42	.458427	7.01	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.01	.981247	.63	.477601	7.64	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	6.99	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.64	.479432	7.61	.520568	13
48	.460946	6.97	.981057	.64	.479889	7.61	.520111	12
49	.461364	6.96	.981019	.64	.480345	7.60	.519655	11
50	9.461782	6.96	9.980981	.64	9.480801	7.59	0.519199	10
51	.462199	6.95	.980942	.64	.481257	7.59	.518743	9
52	.462616	6.94	.980904	.64	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.64	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.64	.482621	7.57	.517379	6
55	.463861	6.92	.980789	.64	.483075	7.56	.516925	5
56	.464279	6.91	.980750	.64	.483529	7.55	.516471	4
57	.464691	6.90	.980712	.64	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.64	.484435	7.54	.515565	2
59	.465522	6.89	.980635	.64	.484887	7.53	.515113	1
60	.465935	6.89	.980596	.64	.485339	7.53	.514661	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

COSINES, TANGENTS, AND COTANGENTS.

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M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.465935	6.88	9.930506	.61	9.483339	7.53	0.514061	60
1	.466313	6.88	.930558	.61	.483791	7.52	.514209	59
2	.466761	6.87	.930519	.65	.484242	7.51	.513753	58
3	.467173	6.86	.930480	.65	.484693	7.51	.513307	57
4	.467585	6.85	.930442	.65	.485143	7.50	.512857	56
5	.467996	6.85	.930403	.65	.485593	7.50	.512407	55
6	.468407	6.84	.930364	.65	.486043	7.50	.511957	54
7	.468817	6.84	.930325	.65	.486492	7.49	.511503	53
8	.469227	6.83	.930286	.65	.486941	7.48	.511059	52
9	.469637	6.82	.930247	.65	.487390	7.47	.510610	51
10	9.470046	6.81	9.930208	.65	9.487839	7.46	0.510162	50
11	.470455	6.81	.930169	.65	.488286	7.46	.509714	49
12	.470863	6.80	.930130	.65	.488733	7.45	.509267	48
13	.471271	6.79	.930091	.65	.489180	7.44	.508820	47
14	.471679	6.78	.930052	.65	.489627	7.44	.508373	46
15	.472086	6.78	.930012	.65	.490073	7.43	.507927	45
16	.472492	6.77	.929973	.65	.490519	7.43	.507481	44
17	.472898	6.76	.929934	.66	.490965	7.42	.507035	43
18	.473304	6.76	.929895	.66	.491410	7.41	.506590	42
19	.473710	6.75	.929855	.66	.491854	7.41	.506146	41
20	9.474115	6.74	9.929816	.66	9.492299	7.40	0.505701	40
21	.474519	6.74	.929776	.66	.492743	7.39	.505257	39
22	.474923	6.73	.929737	.66	.493186	7.39	.504814	38
23	.475327	6.72	.929697	.66	.493630	7.39	.504370	37
24	.475730	6.72	.929658	.66	.494073	7.39	.503927	36
25	.476133	6.71	.929618	.66	.494515	7.39	.503485	35
26	.476536	6.71	.929579	.66	.494957	7.37	.503043	34
27	.476939	6.70	.929539	.66	.495399	7.36	.502601	33
28	.477340	6.69	.929499	.66	.495841	7.35	.502159	32
29	.477741	6.68	.929459	.66	.496282	7.34	.501718	31
30	9.478142	6.67	9.929420	.66	9.496722	7.34	0.501278	30
31	.478542	6.67	.929380	.66	.497163	7.33	.500837	29
32	.478942	6.66	.929340	.67	.497603	7.33	.500397	28
33	.479342	6.65	.929300	.67	.498042	7.32	.499955	27
34	.479741	6.65	.929260	.67	.498481	7.32	.499519	26
35	.480140	6.65	.929220	.67	.498920	7.31	.499080	25
36	.480539	6.64	.929180	.67	.499359	7.31	.498641	24
37	.480937	6.63	.929140	.67	.499797	7.30	.498203	23
38	.481334	6.63	.929100	.67	.500235	7.30	.497765	22
39	.481731	6.61	.929059	.67	.500672	7.29	.497328	21
40	9.482128	6.61	9.929019	.67	9.501109	7.28	0.496891	20
41	.482525	6.60	.928979	.67	.501546	7.27	.496454	19
42	.482921	6.59	.928939	.67	.501982	7.27	.496018	18
43	.483316	6.59	.928898	.67	.502419	7.26	.495582	17
44	.483712	6.58	.928858	.67	.502854	7.25	.495146	16
45	.484107	6.57	.928817	.67	.503289	7.25	.494711	15
46	.484501	6.57	.928777	.67	.503724	7.24	.494276	14
47	.484895	6.56	.928737	.67	.504159	7.24	.493841	13
48	.485289	6.55	.928696	.68	.504593	7.24	.493407	12
49	.485682	6.55	.928655	.68	.505027	7.23	.492973	11
50	9.486075	6.54	9.928615	.68	9.505460	7.22	0.492540	10
51	.486467	6.54	.928574	.68	.505893	7.21	.492107	9
52	.486860	6.53	.928533	.68	.506326	7.21	.491674	8
53	.487251	6.52	.928493	.68	.506759	7.20	.491241	7
54	.487643	6.52	.928452	.68	.507191	7.20	.490808	6
55	.488034	6.51	.928411	.68	.507622	7.19	.490375	5
56	.488424	6.51	.928370	.68	.508054	7.19	.489946	4
57	.488814	6.50	.928329	.68	.508485	7.18	.489515	3
58	.489204	6.50	.928288	.68	.508916	7.18	.489084	2
59	.489593	6.49	.928247	.68	.509346	7.17	.488654	1
60	.489982	6.48	.928206	.68	.509776	7.17	.488224	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.48982	6.48	9.978206	.63	9.511776	7.16	0.488224	60
1	.490371	6.47	.978165	.69	.512206	7.16	.487794	59
2	.490759	6.46	.978124	.69	.512635	7.16	.487365	58
3	.491147	6.46	.978083	.69	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.69	.513493	7.14	.486507	56
5	.491922	6.45	.978001	.69	.513921	7.14	.486079	55
6	.492308	6.45	.977959	.69	.514349	7.13	.485651	54
7	.492695	6.44	.977918	.69	.514777	7.13	.485223	53
8	.493081	6.43	.977877	.69	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.69	.515631	7.11	.484369	51
10	9.493851	6.41	9.977794	.69	9.516057	7.10	0.483943	50
11	.494236	6.41	.977752	.69	.516484	7.10	.483516	49
12	.494621	6.40	.977711	.69	.516910	7.09	.483090	48
13	.495005	6.39	.977669	.69	.517335	7.09	.482665	47
14	.495389	6.39	.977628	.69	.517761	7.09	.482239	46
15	.495772	6.38	.977586	.69	.518186	7.08	.481814	45
16	.496154	6.38	.977544	.70	.518610	7.08	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.36	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.36	.977419	.70	.519882	7.06	.480118	41
20	9.497682	6.35	9.977377	.70	9.520305	7.05	0.479695	40
21	.498064	6.31	.977335	.70	.520728	7.05	.479272	39
22	.498444	6.31	.977293	.70	.521151	7.04	.478849	38
23	.498825	6.33	.977251	.70	.521573	7.04	.478427	37
24	.499201	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499581	6.32	.977167	.70	.522417	7.03	.477583	35
26	.499963	6.31	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.31	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.01	.476320	32
29	.501099	6.30	.976999	.70	.524100	7.00	.475900	31
30	9.501476	6.29	9.976957	.70	9.524520	6.99	0.475480	30
31	.501854	6.28	.976914	.71	.524940	6.99	.475060	29
32	.502231	6.28	.976872	.71	.525359	6.98	.474641	28
33	.502607	6.27	.976830	.71	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.71	.526197	6.97	.473803	26
35	.503360	6.26	.976745	.71	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.71	.527033	6.96	.472967	24
37	.504110	6.25	.976660	.71	.527451	6.96	.472549	23
38	.504485	6.24	.976617	.71	.527869	6.95	.472132	22
39	.504860	6.24	.976574	.71	.528285	6.95	.471716	21
40	9.505234	6.23	9.976532	.71	9.528702	6.94	0.471298	20
41	.505608	6.22	.976489	.71	.529119	6.94	.470881	19
42	.505981	6.22	.976446	.71	.529535	6.93	.470465	18
43	.506354	6.21	.976404	.71	.529951	6.93	.470049	17
44	.506727	6.21	.976361	.71	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.91	.469219	15
46	.507471	6.19	.976275	.72	.531196	6.91	.468804	14
47	.507843	6.19	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.89	.467561	11
50	9.508956	6.17	9.976103	.72	9.532853	6.89	0.467147	10
51	.509326	6.16	.976060	.72	.533266	6.88	.466731	9
52	.509696	6.16	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.72	.534092	6.87	.465908	7
54	.510431	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.14	.975887	.72	.534916	6.86	.465084	5
56	.511172	6.14	.975841	.72	.535328	6.86	.464672	4
57	.511540	6.13	.975790	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.72	.536561	6.84	.463439	1
60	.512642	6.12	.975670	.72	.536972	6.84	.463028	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.512612	6.11	9.975670	.73	9.536972	6.84	0.463023	60
1	.513009	6.11	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462203	58
3	.513711	6.09	.975539	.73	.538202	6.82	.461792	57
4	.514107	6.09	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.09	.975452	.73	.539020	6.81	.460980	55
6	.514837	6.08	.975408	.73	.539429	6.81	.460571	54
7	.515202	6.08	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.06	.975277	.73	.540653	6.79	.459347	51
10	9.516294	6.05	9.975233	.73	9.541061	6.79	0.458939	50
11	.516657	6.05	.975189	.73	.541465	6.78	.458532	49
12	.517020	6.04	.975145	.73	.541875	6.78	.458125	48
13	.517382	6.04	.975101	.73	.542281	6.77	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.03	.975013	.74	.543094	6.76	.456906	45
16	.518468	6.02	.974969	.74	.543501	6.76	.456501	44
17	.518829	6.02	.974925	.74	.543905	6.75	.456095	43
18	.519190	6.01	.974880	.74	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.74	.544715	6.74	.455285	41
20	9.519911	6.00	9.974792	.74	9.545119	6.74	0.454881	40
21	.520271	5.99	.974748	.74	.545524	6.73	.454476	39
22	.520631	5.99	.974703	.74	.545928	6.73	.454072	38
23	.520990	5.98	.974659	.74	.546331	6.72	.453669	37
24	.521349	5.98	.974614	.74	.546735	6.72	.453265	36
25	.521707	5.97	.974570	.74	.547138	6.71	.452862	35
26	.522066	5.97	.974525	.74	.547540	6.71	.452460	34
27	.522421	5.96	.974481	.74	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.74	.548345	6.70	.451655	32
29	.523133	5.95	.974391	.75	.548747	6.69	.451253	31
30	9.523495	5.94	9.974347	.75	9.549149	6.69	0.450851	30
31	.523852	5.94	.974302	.75	.549550	6.68	.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524561	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.67	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.67	.448847	25
36	.525630	5.91	.974077	.75	.551552	6.66	.448448	24
37	.525984	5.90	.974032	.75	.551952	6.66	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.89	.973942	.75	.552750	6.65	.447250	21
40	9.527016	5.89	9.973897	.75	9.553149	6.64	0.446851	20
41	.527400	5.88	.973852	.75	.553548	6.64	.446452	19
42	.527753	5.88	.973807	.75	.553946	6.63	.446051	18
43	.528105	5.87	.973761	.75	.554344	6.63	.445656	17
44	.528458	5.87	.973716	.76	.554741	6.62	.445259	16
45	.528810	5.86	.973671	.76	.555139	6.62	.444861	15
46	.529161	5.86	.973625	.76	.555536	6.61	.444461	14
47	.529513	5.85	.973580	.76	.555933	6.61	.444067	13
48	.529861	5.85	.973535	.76	.556329	6.60	.443671	12
49	.530215	5.84	.973489	.76	.556725	6.60	.443275	11
50	9.530565	5.83	9.973444	.76	9.557121	6.59	0.442879	10
51	.530915	5.83	.973398	.76	.557517	6.59	.442483	9
52	.531265	5.82	.973352	.76	.557913	6.59	.442087	8
53	.531614	5.82	.973307	.76	.558308	6.58	.441692	7
54	.531963	5.81	.973261	.76	.558703	6.58	.441297	6
55	.532312	5.81	.973215	.76	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.76	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.76	.559885	6.56	.440115	3
58	.533357	5.79	.973078	.77	.560279	6.56	.439721	2
59	.533704	5.79	.973032	.77	.560673	6.55	.439327	1
60	.534052		.972986		.561066		.438934	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	Tang.	M.	

M.	Sine.	D. 1 ^u .	Cosine.	D. 1 ^u .	Tang.	D. 1 ^u .	Cotang.	M.
0	9.534052		9.972986		9.561066		0.438934	60
1	.534399	5.78	.972940	.77	.561459	6.55	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.54	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.54	.437756	57
4	.535438	5.76	.972802	.77	.562636	6.53	.437364	56
5	.535783	5.76	.972755	.77	.563023	6.53	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.52	.436581	54
7	.536474	5.75	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.74	.972617	.77	.564202	6.51	.435798	52
9	.537163	5.74	.972570	.77	.564593	6.51	.435407	51
10	9.537507		9.972524		9.564983		0.435017	50
11	.537851	5.73	.972478	.77	.565373	6.50	.434627	49
12	.538194	5.72	.972431	.78	.565763	6.50	.434237	48
13	.538538	5.71	.972385	.78	.566153	6.49	.433847	47
14	.538880	5.71	.972338	.78	.566542	6.49	.433456	46
15	.539223	5.70	.972291	.78	.566932	6.48	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.69	.972198	.78	.567709	6.47	.432291	43
18	.540249	5.69	.972151	.78	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.46	.431514	41
20	9.540931		9.972058		9.568873		0.431127	40
21	.541272	5.67	.972011	.78	.569261	6.46	.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.66	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.66	.971870	.78	.570422	6.44	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.44	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.64	.971729	.79	.571581	6.43	.428419	33
28	.543649	5.64	.971682	.79	.571967	6.43	.428033	32
29	.543987	5.63	.971635	.79	.572352	6.42	.427648	31
30	9.544325		9.971588		9.572738		0.427262	30
31	.544663	5.62	.971540	.79	.573123	6.42	.426877	29
32	.545000	5.62	.971493	.79	.573507	6.41	.426493	28
33	.545338	5.61	.971446	.79	.573892	6.40	.426108	27
34	.545674	5.61	.971398	.79	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.79	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.79	.575044	6.39	.424956	24
37	.546683	5.59	.971256	.79	.575427	6.39	.424573	23
38	.547019	5.59	.971208	.79	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.79	.576193	6.38	.423807	21
40	9.547689		9.971113		9.576576		0.423424	20
41	.548024	5.57	.971066	.80	.576959	6.37	.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.56	.970970	.80	.577723	6.36	.422277	17
44	.549027	5.56	.970922	.80	.578104	6.36	.421896	16
45	.549360	5.55	.970874	.80	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.34	.420752	13
48	.550359	5.54	.970731	.80	.579629	6.34	.420371	12
49	.550692	5.54	.970683	.80	.580009	6.34	.419991	11
50	9.551024		9.970635		9.580389		0.419611	10
51	.551356	5.53	.970586	.80	.580769	6.33	.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.51	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.51	.970394	.81	.582286	6.31	.417714	5
56	.553010	5.50	.970345	.81	.582665	6.31	.417335	4
57	.553341	5.50	.970297	.81	.583044	6.30	.416956	3
58	.553670	5.49	.970249	.81	.583422	6.30	.416578	2
59	.554000	5.49	.970200	.81	.583800	6.30	.416200	1
60	.554329	5.49	.970152		.584177		.415823	0
M.	Cosine.	D. 1 ^u .	Sine.	D. 1 ^u .	Cotang.	D. 1 ^u .	Tang.	M.

M.	Sine.	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	9.554329	5.48	9.970152	.81	9.584177	6.29	0.415823	60
1	.554658	5.48	.970103	.81	.584555	6.29	.415445	59
2	.554987	5.47	.970055	.81	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.81	.585309	6.28	.414691	57
4	.555643	5.46	.969957	.81	.585686	6.28	.414314	56
5	.555971	5.46	.969909	.81	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.81	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.81	.586815	6.27	.413185	53
8	.556953	5.44	.969762	.81	.587190	6.26	.412810	52
9	.557280	5.44	.969714	.81	.587566	6.26	.412434	51
10	9.557606	5.44	9.969665	.82	9.587941	6.25	0.412059	50
11	.557932	5.43	.969616	.82	.588316	6.25	.411684	49
12	.558258	5.43	.969567	.82	.588691	6.24	.411309	48
13	.558583	5.42	.969518	.82	.589066	6.24	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.24	.410560	46
15	.559234	5.41	.969420	.82	.589814	6.24	.410186	45
16	.559558	5.41	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.23	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.39	.969223	.82	.591308	6.22	.408692	41
20	9.560855	5.39	9.969173	.82	9.591631	6.21	0.408319	40
21	.561178	5.38	.969124	.82	.592054	6.21	.407946	39
22	.561501	5.38	.969075	.82	.592426	6.20	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.20	.406829	36
25	.562468	5.37	.968926	.83	.593542	6.20	.406458	35
26	.562790	5.36	.968877	.83	.593914	6.19	.406086	34
27	.563112	5.36	.968827	.83	.594285	6.19	.405715	33
28	.563433	5.35	.968777	.83	.594656	6.18	.405344	32
29	.563755	5.35	.968728	.83	.595027	6.18	.404973	31
30	9.564075	5.34	9.968678	.83	9.595398	6.17	0.404602	30
31	.564396	5.34	.968628	.83	.595768	6.17	.404232	29
32	.564716	5.33	.968578	.83	.596138	6.16	.403862	28
33	.565036	5.33	.968528	.83	.596509	6.16	.403492	27
34	.565356	5.32	.968479	.83	.596878	6.16	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.31	.968329	.83	.597985	6.15	.402015	23
38	.566632	5.31	.968278	.84	.598354	6.14	.401646	22
39	.566951	5.30	.968228	.84	.598722	6.14	.401277	21
40	9.567269	5.30	9.968178	.84	9.599091	6.13	0.400909	20
41	.567587	5.29	.968128	.84	.599459	6.13	.400541	19
42	.567904	5.29	.968078	.84	.599827	6.13	.400173	18
43	.568222	5.28	.968027	.84	.600194	6.12	.399806	17
44	.568539	5.28	.967977	.84	.600562	6.12	.399438	16
45	.568856	5.28	.967927	.84	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.84	.601296	6.11	.398704	14
47	.569488	5.27	.967826	.84	.601663	6.11	.398337	13
48	.569804	5.26	.967775	.84	.602029	6.10	.397971	12
49	.570120	5.26	.967725	.84	.602395	6.10	.397605	11
50	9.570435	5.25	9.967674	.84	9.602761	6.10	0.397239	10
51	.570751	5.25	.967624	.84	.603127	6.09	.396873	9
52	.571066	5.24	.967573	.85	.603493	6.09	.396507	8
53	.571380	5.24	.967522	.85	.603858	6.09	.396142	7
54	.571695	5.24	.967471	.85	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.23	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.22	.967319	.85	.605317	6.07	.394682	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.21	.967217	.85	.606046	6.06	.393954	1
60	.573575	5.21	.967166	.85	.606410		.393590	
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	Tang.	

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.573575	5.21	9.967166	.85	9.606110	6.06	0.393590
1	.573588	5.20	.967115	.85	.606773	6.06	.393227
2	.574200	5.20	.967064	.85	.607137	6.05	.392863
3	.574512	5.20	.967013	.85	.607500	6.05	.392500
4	.574824	5.19	.966961	.85	.607863	6.05	.392137
5	.575136	5.19	.966910	.85	.608225	6.04	.391775
6	.575447	5.18	.966859	.86	.608588	6.04	.391412
7	.575758	5.18	.966808	.86	.608950	6.03	.391050
8	.576069	5.17	.966756	.86	.609312	6.03	.390688
9	.576379	5.17	.966705	.86	.609674	6.03	.390326
10	9.576689	5.17	9.966653	.86	9.610036	6.02	0.389964
11	.576999	5.16	.966602	.86	.610397	6.02	.389603
12	.577309	5.16	.966550	.86	.610759	6.02	.389241
13	.577618	5.15	.966499	.86	.611120	6.01	.388880
14	.577927	5.15	.966447	.86	.611480	6.01	.388520
15	.578236	5.14	.966395	.86	.611841	6.01	.388159
16	.578545	5.14	.966344	.86	.612201	6.00	.387799
17	.578853	5.11	.966292	.86	.612561	6.00	.387439
18	.579162	5.13	.966240	.86	.612921	6.00	.387079
19	.579470	5.13	.966188	.86	.613281	5.99	.386719
20	9.579777	5.12	9.966136	.87	9.613641	5.99	0.386359
21	.580085	5.12	.966085	.87	.614000	5.98	.386000
22	.580393	5.11	.966033	.87	.614359	5.98	.385641
23	.580699	5.11	.965981	.87	.614718	5.98	.385282
24	.581005	5.11	.965929	.87	.615077	5.97	.384923
25	.581312	5.10	.965876	.87	.615435	5.97	.384565
26	.581618	5.10	.965824	.87	.615793	5.97	.384207
27	.581924	5.09	.965772	.87	.616151	5.96	.383849
28	.582229	5.09	.965720	.87	.616509	5.96	.383491
29	.582535	5.09	.965668	.87	.616867	5.96	.383133
30	9.582840	5.08	9.965615	.87	9.617224	5.95	0.382776
31	.583145	5.08	.965563	.87	.617582	5.95	.382418
32	.583449	5.07	.965511	.87	.617939	5.95	.382061
33	.583754	5.07	.965458	.87	.618295	5.94	.381705
34	.584058	5.06	.965406	.88	.618652	5.94	.381348
35	.584361	5.06	.965353	.88	.619008	5.94	.380992
36	.584665	5.06	.965301	.88	.619364	5.93	.380636
37	.584968	5.05	.965248	.88	.619720	5.93	.380280
38	.585272	5.05	.965195	.88	.620076	5.93	.379924
39	.585574	5.04	.965143	.88	.620432	5.92	.379568
40	9.585877	5.04	9.965090	.88	9.620787	5.92	0.379213
41	.586179	5.04	.965037	.88	.621142	5.92	.378858
42	.586482	5.03	.964984	.88	.621497	5.91	.378503
43	.586783	5.03	.964931	.88	.621852	5.91	.378148
44	.587085	5.02	.964879	.88	.622207	5.91	.377793
45	.587386	5.02	.964826	.88	.622561	5.90	.377439
46	.587688	5.01	.964773	.88	.622915	5.90	.377085
47	.587989	5.01	.964720	.88	.623269	5.90	.376731
48	.588289	5.01	.964666	.89	.623623	5.89	.376377
49	.588590	5.00	.964613	.89	.623976	5.89	.376024
50	9.588890	5.00	9.964560	.89	9.624330	5.89	0.375670
51	.589190	4.99	.964507	.89	.624683	5.88	.375317
52	.589489	4.99	.964454	.89	.625036	5.88	.374964
53	.589788	4.99	.964400	.89	.625388	5.88	.374612
54	.590088	4.98	.964347	.89	.625741	5.87	.374259
55	.590387	4.98	.964294	.89	.626093	5.87	.373907
56	.590686	4.97	.964240	.89	.626445	5.87	.373555
57	.590984	4.97	.964187	.89	.626797	5.86	.373203
58	.591282	4.97	.964133	.89	.627149	5.86	.372851
59	.591580	4.96	.964080	.89	.627501	5.86	.372499
60	.591878	4.96	.964026	.89	.627852	5.86	.372148
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.

M.	Sine.	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	9.591878	4.96	9.964026	.89	9.627852	5.85	0.372148	60
1	.592176	4.95	.963972	.89	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.84	.371095	57
4	.593067	4.94	.963811	.90	.629255	5.84	.370745	56
5	.593363	4.94	.963757	.90	.629606	5.84	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.83	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.82	.368995	51
10	9.594842	4.92	9.963488	.90	9.631355	5.82	0.368645	50
11	.595137	4.91	.963434	.90	.631704	5.82	.368296	49
12	.595432	4.91	.963379	.90	.632053	5.81	.367947	48
13	.595727	4.91	.963325	.90	.632402	5.81	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.81	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.81	.366901	45
16	.596609	4.89	.963163	.91	.633447	5.80	.366553	44
17	.596903	4.89	.963108	.91	.633795	5.80	.366205	43
18	.597196	4.89	.963054	.91	.634143	5.80	.365857	42
19	.597490	4.88	.962999	.91	.634490	5.79	.365510	41
20	9.597783	4.88	9.962945	.91	9.634838	5.79	0.365162	40
21	.598075	4.88	.962890	.91	.635185	5.78	.364815	39
22	.598368	4.87	.962836	.91	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.91	.635879	5.78	.364121	37
24	.598952	4.86	.962727	.91	.636225	5.78	.363774	36
25	.599244	4.86	.962672	.91	.636572	5.77	.363428	35
26	.599536	4.86	.962617	.91	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.91	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.91	.637611	5.76	.362389	32
29	.600409	4.84	.962453	.92	.637956	5.76	.362044	31
30	9.600700	4.84	9.962398	.92	9.638302	5.76	0.361698	30
31	.600990	4.84	.962343	.92	.638647	5.75	.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.74	.360318	26
35	.602150	4.82	.962123	.92	.640027	5.74	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.74	.359629	24
37	.602728	4.81	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.81	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.81	.961902	.92	.641404	5.73	.358596	21
40	9.603594	4.80	9.961846	.92	9.641747	5.73	0.358253	20
41	.603882	4.80	.961791	.92	.642091	5.72	.357909	19
42	.604170	4.79	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.79	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.79	.961624	.93	.643120	5.71	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.71	.356537	15
46	.605319	4.78	.961513	.93	.643806	5.71	.356194	14
47	.605606	4.78	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.77	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	9.606465	4.76	9.961290	.93	9.645174	5.69	0.354826	10
51	.606751	4.76	.961235	.93	.645516	5.69	.354484	9
52	.607036	4.76	.961179	.93	.645857	5.69	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.69	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.74	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.74	.960955	.93	.647222	5.68	.352778	4
57	.608461	4.74	.960899	.94	.647562	5.67	.352438	3
58	.608745	4.73	.960843	.94	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.94	.648243	5.67	.351757	1
60	.609313	4.73	.960730	.94	.648583	5.67	.351417	0
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.609313	4.73	9.960730	.94	9.648583	5.67	0.351417	60
1	.609397	4.72	.960674	.94	.648923	5.66	.351077	59
2	.609480	4.72	.960618	.94	.649263	5.66	.350737	58
3	.610161	4.72	.960561	.94	.649602	5.66	.350398	57
4	.610147	4.71	.960505	.94	.649942	5.65	.350058	56
5	.610729	4.71	.960448	.94	.650281	5.65	.349719	55
6	.611012	4.71	.960392	.94	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.94	.650959	5.64	.349041	53
8	.611576	4.70	.960279	.94	.651297	5.64	.348703	52
9	.611858	4.69	.960222	.94	.651636	5.64	.348364	51
10	9.612140	4.69	9.960165	.95	9.651974	5.64	0.348026	50
11	.612121	4.69	.960109	.95	.652312	5.63	.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612933	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.95	.653326	5.62	.346674	46
15	.613345	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613325	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614335	4.66	.959711	.95	.654674	5.61	.345326	42
19	.614665	4.66	.959654	.95	.655011	5.61	.344989	41
20	9.614944	4.65	9.959596	.95	9.655348	5.61	0.344652	40
21	.615223	4.65	.959539	.95	.655684	5.61	.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.64	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.64	.959368	.96	.656692	5.60	.343308	36
25	.616348	4.64	.959310	.96	.657028	5.59	.342972	35
26	.616626	4.63	.959253	.96	.657364	5.59	.342636	34
27	.616904	4.63	.959195	.96	.657699	5.59	.342301	33
28	.617172	4.63	.959138	.96	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.96	.658369	5.58	.341631	31
30	9.617727	4.62	9.959023	.96	9.658704	5.58	0.341296	30
31	.618001	4.61	.958965	.96	.659039	5.58	.340961	29
32	.618281	4.61	.958908	.96	.659373	5.57	.340627	28
33	.618558	4.61	.958850	.96	.659708	5.57	.340292	27
34	.618831	4.60	.958792	.96	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.96	.660376	5.56	.339624	25
36	.619386	4.60	.958677	.96	.660710	5.56	.339290	24
37	.619662	4.59	.958619	.97	.661043	5.56	.338957	23
38	.619938	4.59	.958561	.97	.661377	5.56	.338623	22
39	.620213	4.59	.958503	.97	.661710	5.55	.338290	21
40	9.620488	4.58	9.958445	.97	9.662043	5.55	0.337957	20
41	.620763	4.58	.958387	.97	.662376	5.55	.337624	19
42	.621038	4.58	.958329	.97	.662709	5.54	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.54	.336958	17
44	.621587	4.57	.958213	.97	.663375	5.54	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.54	.336293	15
46	.622135	4.56	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.56	.958038	.97	.664371	5.53	.335629	13
48	.622682	4.56	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.53	.334965	11
50	9.623229	4.55	9.957863	.97	9.665366	5.52	0.334634	10
51	.623502	4.54	.957804	.98	.665698	5.52	.334302	9
52	.623774	4.54	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.54	.957687	.98	.666360	5.51	.333640	7
54	.624319	4.53	.957628	.98	.666691	5.51	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.51	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.51	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.50	.332318	3
58	.625406	4.52	.957393	.98	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	.625948	4.52	.957276	.98	.668673	5.50	.331327	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	9.625948	4.51	9.957276	.98	9.665673	5.50	0.331327	60
1	.626219	4.51	.957217	.98	.669002	5.49	.330998	59
2	.626490	4.51	.957158	.98	.669332	5.49	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.49	.330339	57
4	.627030	4.50	.957040	.99	.669991	5.49	.330009	56
5	.627300	4.50	.956981	.99	.670320	5.49	.329680	55
6	.627570	4.49	.956921	.99	.670649	5.48	.329351	54
7	.627840	4.49	.956862	.99	.670977	5.48	.329023	53
8	.628109	4.49	.956803	.99	.671306	5.47	.328694	52
9	.628378	4.48	.956744	.99	.671635	5.47	.328365	51
10	9.628647	4.48	9.956684	.99	9.671963	5.47	0.328037	50
11	.628917	4.48	.956625	.99	.672291	5.47	.327709	49
12	.629185	4.47	.956566	.99	.672619	5.46	.327381	48
13	.629453	4.47	.956506	.99	.672947	5.46	.327053	47
14	.629721	4.47	.956447	.99	.673274	5.46	.326726	46
15	.629989	4.46	.956387	.99	.673602	5.46	.326398	45
16	.630257	4.46	.956327	.99	.673929	5.45	.326071	44
17	.630524	4.46	.956268	.99	.674257	5.45	.325743	43
18	.630792	4.45	.956208	.99	.674584	5.45	.325416	42
19	.631059	4.45	.956148	1.00	.674911	5.45	.325089	41
20	9.631326	4.45	9.956089	1.00	9.675237	5.44	0.324763	40
21	.631593	4.44	.956029	1.00	.675564	5.44	.324436	39
22	.631859	4.44	.955969	1.00	.675890	5.44	.324110	38
23	.632125	4.44	.955909	1.00	.676217	5.44	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.43	.955789	1.00	.676869	5.43	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.42	.322480	33
28	.633454	4.42	.955609	1.00	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	9.633984	4.41	9.955488	1.00	9.678496	5.42	0.321504	30
31	.634249	4.41	.955428	1.01	.678821	5.41	.321179	29
32	.634514	4.41	.955368	1.01	.679146	5.41	.320854	28
33	.634778	4.40	.955307	1.01	.679471	5.41	.320529	27
34	.635042	4.40	.955247	1.01	.679795	5.41	.320205	26
35	.635306	4.40	.955186	1.01	.680120	5.41	.319880	25
36	.635570	4.39	.955126	1.01	.680444	5.40	.319556	24
37	.635834	4.39	.955065	1.01	.680768	5.40	.319232	23
38	.636097	4.39	.955005	1.01	.681092	5.40	.318909	22
39	.636360	4.38	.954944	1.01	.681416	5.39	.318584	21
40	9.636623	4.38	9.954883	1.01	9.681740	5.39	0.318260	20
41	.636886	4.38	.954823	1.01	.682063	5.39	.317937	19
42	.637148	4.37	.954762	1.01	.682387	5.39	.317613	18
43	.637411	4.37	.954701	1.01	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.36	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.36	.954518	1.02	.683679	5.38	.316321	14
47	.638458	4.36	.954457	1.02	.684001	5.37	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	9.639242	4.35	9.954274	1.02	9.684969	5.37	0.315032	10
51	.639503	4.34	.954213	1.02	.685290	5.36	.314710	9
52	.639764	4.34	.954152	1.02	.685612	5.36	.314388	8
53	.640024	4.34	.954090	1.02	.685934	5.36	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.36	.313745	6
55	.640544	4.33	.953968	1.02	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.32	.953845	1.02	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.03	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60	.641842	4.32	.953660	1.03	.688182	5.35	.311818	0
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	Tang.	M.

TABLE XIII. LOGARITHMIC SINES,

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.641812	4.32	9.951660	1.03	9.688182	5.34	0.31181
1	.642101	4.31	.953599	1.03	.688502	5.34	.31149
2	.642360	4.31	.953537	1.03	.688823	5.34	.31117
3	.642618	4.31	.953475	1.03	.689143	5.34	.31085
4	.642877	4.30	.953413	1.03	.689463	5.33	.31053
5	.643135	4.30	.953352	1.03	.689783	5.33	.31021
6	.643393	4.30	.953290	1.03	.690103	5.33	.30989
7	.643650	4.29	.953228	1.03	.690423	5.33	.30957
8	.643908	4.29	.953166	1.03	.690742	5.32	.30925
9	.644165	4.29	.953104	1.03	.691062	5.32	.30893
10	9.644123	4.28	9.953042	1.03	9.691381	5.32	0.30861
11	.644680	4.28	.952980	1.04	.691700	5.32	.30830
12	.644936	4.28	.952918	1.04	.692019	5.31	.30798
13	.645193	4.27	.952855	1.04	.692338	5.31	.30766
14	.645450	4.27	.952793	1.04	.692656	5.31	.30734
15	.645706	4.27	.952731	1.04	.692975	5.31	.30702
16	.645962	4.26	.952669	1.04	.693293	5.30	.30670
17	.646218	4.26	.952606	1.04	.693612	5.30	.30638
18	.646474	4.26	.952544	1.04	.693930	5.30	.30607
19	.646729	4.26	.952481	1.04	.694248	5.30	.30575
20	9.646944	4.25	9.952119	1.04	9.694566	5.29	0.30543
21	.647210	4.25	.952356	1.04	.694883	5.29	.30511
22	.647494	4.25	.952294	1.04	.695201	5.29	.30479
23	.647749	4.24	.952231	1.04	.695518	5.29	.30448
24	.648004	4.24	.952169	1.05	.695836	5.29	.30416
25	.648253	4.24	.952106	1.05	.696153	5.29	.30384
26	.648512	4.23	.952043	1.05	.696470	5.28	.30353
27	.648766	4.23	.951980	1.05	.696787	5.28	.30321
28	.649020	4.23	.951917	1.05	.697103	5.28	.30289
29	.649274	4.22	.951854	1.05	.697420	5.27	.30258
30	9.649527	4.22	9.951791	1.05	9.697736	5.27	0.30226
31	.649781	4.22	.951728	1.05	.698053	5.27	.30194
32	.650034	4.22	.951665	1.05	.698369	5.27	.30163
33	.650287	4.21	.951602	1.05	.698685	5.26	.30131
34	.650539	4.21	.951539	1.05	.699001	5.26	.30099
35	.650792	4.21	.951476	1.05	.699316	5.26	.30068
36	.651044	4.20	.951412	1.05	.699632	5.26	.30036
37	.651297	4.20	.951349	1.06	.699947	5.26	.30005
38	.651549	4.20	.951286	1.06	.700263	5.25	.29973
39	.651800	4.19	.951222	1.06	.700578	5.25	.29942
40	9.652052	4.19	9.951159	1.06	9.700893	5.25	0.29910
41	.652304	4.19	.951096	1.06	.701208	5.25	.29879
42	.652555	4.18	.951032	1.06	.701523	5.24	.29847
43	.652806	4.18	.950968	1.06	.701837	5.24	.29816
44	.653057	4.18	.950905	1.06	.702152	5.24	.29784
45	.653308	4.18	.950841	1.06	.702466	5.24	.29753
46	.653558	4.17	.950778	1.06	.702781	5.24	.29721
47	.653808	4.17	.950714	1.06	.703095	5.23	.29690
48	.654059	4.17	.950650	1.06	.703409	5.23	.29659
49	.654309	4.16	.950586	1.06	.703722	5.23	.29627
50	9.654558	4.16	9.950522	1.07	9.704036	5.23	0.29596
51	.654808	4.16	.950458	1.07	.704350	5.22	.29565
52	.655058	4.15	.950394	1.07	.704663	5.22	.29533
53	.655307	4.15	.950330	1.07	.704976	5.22	.29502
54	.655556	4.15	.950266	1.07	.705290	5.22	.29471
55	.655805	4.15	.950202	1.07	.705603	5.22	.29439
56	.656054	4.14	.950138	1.07	.705916	5.21	.29408
57	.656302	4.14	.950074	1.07	.706228	5.21	.29377
58	.656551	4.14	.950010	1.07	.706541	5.21	.29345
59	.656799	4.13	.949945	1.07	.706854	5.21	.29314
60	.657047		.949881	1.07	.707166		.29283
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.

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M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.637017	4.13	9.949831	1.07	9.707166	5.20	0.292334	60
1	.657295	4.13	.919316	1.07	.707478	5.20	.292522	59
2	.657512	4.12	.919752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.919638	1.08	.708102	5.20	.291598	57
4	.658037	4.12	.919623	1.08	.708414	5.20	.291586	56
5	.658254	4.12	.919558	1.08	.708726	5.19	.291274	55
6	.658531	4.11	.919491	1.08	.709037	5.19	.290963	54
7	.658778	4.11	.919429	1.08	.709319	5.19	.290651	53
8	.659025	4.11	.919364	1.08	.709660	5.19	.290340	52
9	.659271	4.10	.919300	1.08	.709971	5.18	.290029	51
10	9.659517	4.10	9.919235	1.08	9.710282	5.18	0.289718	50
11	.659763	4.10	.919170	1.08	.710593	5.18	.289407	49
12	.660009	4.10	.919105	1.08	.710901	5.18	.289096	48
13	.660255	4.09	.919040	1.08	.711215	5.18	.288785	47
14	.660501	4.09	.918975	1.08	.711525	5.17	.288475	46
15	.660746	4.09	.918910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.918845	1.09	.712146	5.17	.287854	44
17	.661236	4.08	.918780	1.09	.712456	5.17	.287544	43
18	.661481	4.08	.918715	1.09	.712766	5.17	.287231	42
19	.661726	4.08	.918650	1.09	.713076	5.16	.286924	41
20	9.661970	4.07	9.918584	1.09	9.713386	5.16	0.286614	40
21	.662214	4.07	.918519	1.09	.713696	5.16	.286304	39
22	.662459	4.07	.918454	1.09	.714003	5.16	.285995	38
23	.662703	4.06	.918388	1.09	.714314	5.15	.285686	37
24	.662946	4.06	.918323	1.09	.714624	5.15	.285376	36
25	.663190	4.06	.918257	1.09	.714933	5.15	.285067	35
26	.663433	4.05	.918192	1.09	.715242	5.15	.284758	34
27	.663677	4.05	.918126	1.09	.715551	5.15	.284449	33
28	.663920	4.05	.918060	1.09	.715860	5.14	.284140	32
29	.664163	4.05	.917995	1.10	.716168	5.14	.283832	31
30	9.664406	4.04	9.917929	1.10	9.716477	5.14	0.283523	30
31	.664643	4.04	.917863	1.10	.716785	5.14	.283215	29
32	.664891	4.04	.917797	1.10	.717093	5.14	.282907	28
33	.665133	4.03	.917731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.917665	1.10	.717709	5.13	.282291	26
35	.665617	4.03	.917600	1.10	.718017	5.13	.281983	25
36	.665859	4.03	.917533	1.10	.718325	5.13	.281675	24
37	.666100	4.02	.917467	1.10	.718633	5.13	.281367	23
38	.666342	4.02	.917401	1.10	.718940	5.12	.281060	22
39	.666583	4.02	.917335	1.10	.719248	5.12	.280752	21
40	9.666824	4.01	9.917269	1.10	9.719555	5.12	0.280445	20
41	.667065	4.01	.917203	1.11	.719862	5.12	.280138	19
42	.667305	4.01	.917136	1.11	.720169	5.11	.279831	18
43	.667546	4.01	.917070	1.11	.720476	5.11	.279524	17
44	.667786	4.00	.917004	1.11	.720783	5.11	.279217	16
45	.668027	4.00	.916937	1.11	.721089	5.11	.278911	15
46	.668267	4.00	.916871	1.11	.721396	5.11	.278604	14
47	.668506	3.99	.916804	1.11	.721702	5.10	.278298	13
48	.668746	3.99	.916738	1.11	.722009	5.10	.277991	12
49	.668986	3.99	.916671	1.11	.722315	5.10	.277685	11
50	9.669225	3.99	9.916604	1.11	9.722621	5.10	0.277379	10
51	.669464	3.98	.916538	1.11	.722927	5.10	.277073	9
52	.669703	3.98	.916471	1.11	.723232	5.09	.276768	8
53	.669942	3.98	.916404	1.11	.723538	5.09	.276462	7
54	.670181	3.98	.916337	1.12	.723844	5.09	.276156	6
55	.670419	3.97	.916270	1.12	.724149	5.09	.275851	5
56	.670658	3.97	.916203	1.12	.724454	5.09	.275546	4
57	.670896	3.97	.916136	1.12	.724760	5.08	.275240	3
58	.671134	3.96	.916069	1.12	.725065	5.08	.274935	2
59	.671372	3.96	.916002	1.12	.725370	5.08	.274630	1
60	.671609	3.96	.915935	1.12	.725674	5.08	.274325	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	0.27432
1	.671847	3.96	.945863	1.12	.725979	5.08	.27406
2	.672084	3.95	.945800	1.12	.726284	5.07	.27371
3	.672321	3.95	.945733	1.12	.726598	5.07	.27341
4	.672553	3.95	.945666	1.12	.726892	5.07	.27310
5	.672795	3.94	.945598	1.12	.727197	5.07	.27280
6	.673032	3.94	.945531	1.12	.727501	5.07	.27249
7	.673263	3.94	.945461	1.12	.727805	5.06	.27218
8	.673505	3.94	.945396	1.13	.728109	5.06	.27188
9	.673741	3.93	.945328	1.13	.728412	5.06	.27158
10	9.673977	3.93	9.945261	1.13	9.728716	5.06	0.27128
11	.674213	3.93	.945193	1.13	.729020	5.06	.27098
12	.674448	3.93	.945125	1.13	.729323	5.05	.27067
13	.674684	3.92	.945058	1.13	.729626	5.05	.27037
14	.674919	3.92	.944990	1.13	.729929	5.05	.27007
15	.675155	3.92	.944922	1.13	.730233	5.05	.26976
16	.675390	3.91	.944854	1.13	.730535	5.05	.26946
17	.675624	3.91	.944786	1.13	.730838	5.05	.26916
18	.675859	3.91	.944718	1.13	.731141	5.04	.26885
19	.676094	3.91	.944650	1.13	.731444	5.04	.26855
20	9.676328	3.90	9.944582	1.14	9.731746	5.04	0.26825
21	.676562	3.90	.944514	1.14	.732048	5.04	.26795
22	.676796	3.90	.944446	1.14	.732351	5.04	.26764
23	.677030	3.90	.944377	1.14	.732653	5.03	.26734
24	.677264	3.90	.944309	1.14	.732955	5.03	.26704
25	.677498	3.90	.944241	1.14	.733257	5.03	.26674
26	.677731	3.90	.944172	1.14	.733558	5.03	.26644
27	.677964	3.90	.944104	1.14	.733860	5.03	.26614
28	.678197	3.90	.944036	1.14	.734162	5.02	.26583
29	.678430	3.90	.943967	1.14	.734463	5.02	.26553
30	9.678663	3.90	9.943899	1.14	9.734764	5.02	0.26523
31	.678895	3.90	.943830	1.14	.735066	5.02	.26493
32	.679128	3.90	.943761	1.14	.735367	5.02	.26463
33	.679360	3.90	.943693	1.15	.735668	5.01	.26433
34	.679592	3.90	.943624	1.15	.735969	5.01	.26403
35	.679824	3.90	.943555	1.15	.736269	5.01	.26373
36	.680056	3.90	.943486	1.15	.736570	5.01	.26343
37	.680288	3.90	.943417	1.15	.736870	5.01	.26313
38	.680519	3.90	.943348	1.15	.737171	5.01	.26283
39	.680750	3.90	.943279	1.15	.737471	5.00	.26253
40	9.680982	3.90	9.943210	1.15	9.737771	5.00	0.26223
41	.681213	3.90	.943141	1.15	.738071	5.00	.26193
42	.681443	3.90	.943072	1.15	.738371	5.00	.26163
43	.681674	3.90	.943003	1.15	.738671	5.00	.26133
44	.681905	3.90	.942934	1.15	.738971	4.99	.26103
45	.682135	3.90	.942864	1.16	.739271	4.99	.26073
46	.682365	3.90	.942795	1.16	.739570	4.99	.26043
47	.682595	3.90	.942726	1.16	.739870	4.99	.26013
48	.682825	3.90	.942656	1.16	.740169	4.99	.25983
49	.683055	3.90	.942587	1.16	.740468	4.98	.25953
50	9.683284	3.90	9.942517	1.16	9.740768	4.98	0.25923
51	.683514	3.90	.942447	1.16	.741066	4.98	.25893
52	.683743	3.90	.942377	1.16	.741365	4.98	.25863
53	.683973	3.90	.942307	1.16	.741664	4.98	.25833
54	.684202	3.90	.942237	1.16	.741962	4.98	.25803
55	.684431	3.90	.942167	1.16	.742261	4.98	.25773
56	.684660	3.90	.942097	1.16	.742559	4.98	.25743
57	.684889	3.90	.942027	1.16	.742858	4.98	.25713
58	.685118	3.90	.941957	1.17	.743156	4.98	.25683
59	.685347	3.90	.941887	1.17	.743454	4.98	.25653
60	.685576	3.90	.941817	1.17	.743752	4.98	.25623
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.

M	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.635571	3.90	9.941819	1.17	9.743752	4.96	0.256248	60
1	.635799	3.79	.941749	1.17	.741050	4.96	.255950	59
2	.636027	3.79	.941679	1.17	.741348	4.96	.255652	58
3	.636254	3.79	.941609	1.17	.741645	4.96	.255355	57
4	.636482	3.79	.941539	1.17	.741943	4.96	.255057	56
5	.636709	3.78	.941469	1.17	.742240	4.96	.254760	55
6	.636936	3.78	.941398	1.17	.742538	4.95	.254462	54
7	.637163	3.78	.941328	1.17	.742835	4.95	.254165	53
8	.637390	3.78	.941258	1.17	.743132	4.95	.253868	52
9	.637616	3.77	.941187	1.17	.743429	4.95	.253571	51
10	9.637843	3.77	9.941117	1.18	9.743726	4.95	0.253274	50
11	.638069	3.77	.941046	1.18	.747023	4.95	.252977	49
12	.638295	3.77	.940975	1.18	.747319	4.94	.252681	48
13	.638521	3.76	.940905	1.18	.747616	4.94	.252384	47
14	.638747	3.76	.940834	1.18	.747913	4.94	.252087	46
15	.638972	3.76	.940763	1.18	.748209	4.94	.251791	45
16	.639198	3.76	.940693	1.18	.748505	4.94	.251495	44
17	.639423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.639648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.639873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	9.639098	3.75	9.940409	1.18	9.749689	4.93	0.250311	40
21	.639323	3.74	.940338	1.18	.749985	4.93	.250015	39
22	.639548	3.74	.940267	1.19	.750281	4.93	.249719	38
23	.639772	3.74	.940196	1.19	.750576	4.92	.249424	37
24	.639996	3.74	.940125	1.19	.750872	4.92	.249128	36
25	.640220	3.73	.940054	1.19	.751167	4.92	.248833	35
26	.640444	3.73	.939982	1.19	.751462	4.92	.248538	34
27	.640668	3.73	.939911	1.19	.751757	4.92	.248243	33
28	.640892	3.73	.939840	1.19	.752052	4.92	.247948	32
29	.641115	3.72	.939768	1.19	.752347	4.91	.247653	31
30	9.641339	3.72	9.939697	1.19	9.752642	4.91	0.247358	30
31	.641562	3.72	.939625	1.19	.752937	4.91	.247063	29
32	.641785	3.72	.939554	1.19	.753231	4.91	.246769	28
33	.642008	3.71	.939482	1.19	.753526	4.91	.246474	27
34	.642231	3.71	.939410	1.19	.753820	4.91	.246180	26
35	.642453	3.71	.939339	1.20	.754115	4.90	.245885	25
36	.642676	3.71	.939267	1.20	.754409	4.90	.245591	24
37	.642898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.643120	3.70	.939123	1.20	.755000	4.90	.245003	22
39	.643342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	9.643564	3.70	9.938980	1.20	9.755585	4.89	0.244415	20
41	.643786	3.69	.938908	1.20	.755878	4.89	.244122	19
42	.644007	3.69	.938836	1.20	.756172	4.89	.243828	18
43	.644228	3.69	.938763	1.20	.756465	4.89	.243535	17
44	.644448	3.69	.938691	1.20	.756759	4.89	.243241	16
45	.644668	3.68	.938619	1.20	.757052	4.89	.242948	15
46	.644888	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.645107	3.68	.938475	1.21	.757638	4.88	.242362	13
48	.645326	3.68	.938402	1.21	.757931	4.88	.242069	12
49	.645544	3.67	.938330	1.21	.758224	4.88	.241776	11
50	9.645765	3.67	9.938258	1.21	9.758517	4.88	0.241483	10
51	.645985	3.67	.938185	1.21	.758810	4.88	.241190	9
52	.646203	3.67	.938113	1.21	.759102	4.87	.240898	8
53	.646421	3.66	.938040	1.21	.759395	4.87	.240605	7
54	.646638	3.66	.937967	1.21	.759687	4.87	.240313	6
55	.646855	3.66	.937895	1.21	.759979	4.87	.240020	5
56	.647071	3.66	.937822	1.21	.760272	4.87	.239728	4
57	.647287	3.65	.937749	1.21	.760564	4.87	.239436	3
58	.647502	3.65	.937676	1.21	.760856	4.86	.239144	2
59	.647717	3.65	.937604	1.22	.761148	4.86	.238852	1
60	.647931	3.65	.937531		.761439		.238561	

M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.
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TABLE XIII. LOGARITHMIC SINES,

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotan
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	0.2042
1	.724412	3.37	.928442	1.32	.796070	4.68	.2039
2	.724614	3.36	.928263	1.32	.796351	4.68	.2036
3	.724816	3.36	.928183	1.32	.796632	4.68	.2033
4	.725017	3.36	.928104	1.32	.796913	4.68	.2030
5	.725219	3.36	.928025	1.32	.797194	4.68	.2028
6	.725420	3.36	.927946	1.32	.797474	4.68	.2025
7	.725622	3.35	.927867	1.32	.797755	4.68	.2022
8	.725823	3.35	.927787	1.32	.798036	4.67	.2019
9	.726024	3.35	.927708	1.32	.798316	4.67	.2016
10	9.726225	3.35	9.927629	1.32	9.798596	4.67	0.2014
11	.726426	3.34	.927549	1.33	.798877	4.67	.2011
12	.726626	3.34	.927470	1.33	.799157	4.67	.2008
13	.726827	3.34	.927390	1.33	.799437	4.67	.2005
14	.727027	3.34	.927310	1.33	.799717	4.67	.2002
15	.727228	3.34	.927231	1.33	.799997	4.66	.2000
16	.727428	3.33	.927151	1.33	.800277	4.66	.1997
17	.727628	3.33	.927071	1.33	.800557	4.66	.1994
18	.727828	3.33	.926991	1.33	.800836	4.66	.1991
19	.728027	3.33	.926911	1.33	.801116	4.66	.1988
20	9.728227	3.33	9.926831	1.33	9.801396	4.66	0.1986
21	.728427	3.32	.926751	1.33	.801675	4.66	.1983
22	.728626	3.32	.926671	1.33	.801955	4.66	.1980
23	.728825	3.32	.926591	1.34	.802234	4.65	.1977
24	.729024	3.32	.926511	1.34	.802513	4.65	.1974
25	.729223	3.31	.926431	1.34	.802792	4.65	.1972
26	.729422	3.31	.926351	1.34	.803072	4.65	.1969
27	.729621	3.31	.926270	1.34	.803351	4.65	.1966
28	.729820	3.31	.926190	1.34	.803630	4.65	.1963
29	.730018	3.31	.926110	1.34	.803909	4.65	.1960
30	9.730217	3.30	9.926029	1.34	9.804187	4.65	0.1958
31	.730415	3.30	.925949	1.34	.804466	4.64	.1955
32	.730613	3.30	.925868	1.34	.804745	4.64	.1952
33	.730811	3.30	.925788	1.34	.805023	4.64	.1949
34	.731009	3.30	.925707	1.35	.805302	4.64	.1946
35	.731206	3.29	.925626	1.35	.805580	4.64	.1943
36	.731404	3.29	.925545	1.35	.805859	4.64	.1940
37	.731602	3.29	.925465	1.35	.806137	4.64	.1937
38	.731799	3.29	.925384	1.35	.806415	4.64	.1934
39	.731996	3.28	.925303	1.35	.806693	4.63	.1931
40	9.732193	3.28	9.925222	1.35	9.806971	4.63	0.1930
41	.732390	3.28	.925141	1.35	.807249	4.63	.1927
42	.732587	3.28	.925060	1.35	.807527	4.63	.1924
43	.732784	3.28	.924979	1.35	.807805	4.63	.1921
44	.732980	3.27	.924897	1.35	.808083	4.63	.1918
45	.733177	3.27	.924816	1.35	.808361	4.63	.1915
46	.733373	3.27	.924735	1.35	.808638	4.63	.1912
47	.733569	3.27	.924654	1.36	.808916	4.62	.1909
48	.733765	3.27	.924572	1.36	.809193	4.62	.1906
49	.733961	3.26	.924491	1.36	.809471	4.62	.1903
50	9.734157	3.26	9.924409	1.36	9.809748	4.62	0.1901
51	.734353	3.26	.924328	1.36	.810025	4.62	.1898
52	.734549	3.26	.924246	1.36	.810302	4.62	.1895
53	.734744	3.26	.924164	1.36	.810580	4.62	.1892
54	.734939	3.25	.924083	1.36	.810857	4.62	.1889
55	.735135	3.25	.924001	1.36	.811134	4.61	.1886
56	.735330	3.25	.923919	1.36	.811410	4.61	.1883
57	.735525	3.25	.923837	1.37	.811687	4.61	.1880
58	.735719	3.25	.923755	1.37	.811964	4.61	.1877
59	.735914	3.24	.923673	1.37	.812241	4.61	.1874
60	.736109		.923591		.812517		.1871
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1"	Tan.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.736109	3.24	9.923591	1.37	9.812517	4.51	0.187483	60
1	.736303	3.24	.923509	1.37	.812791	4.61	.187206	59
2	.736498	3.24	.923427	1.37	.813070	4.61	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.61	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.37	.813899	4.60	.186101	55
6	.737274	3.23	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.37	.814452	4.60	.185548	53
8	.737661	3.22	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	9.738048	3.22	9.922768	1.38	9.815280	4.60	0.184720	50
11	.738241	3.22	.922686	1.38	.815555	4.60	.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.59	.184169	48
13	.738627	3.21	.922520	1.38	.816107	4.59	.183893	47
14	.738820	3.21	.922438	1.38	.816382	4.59	.183618	46
15	.739013	3.21	.922355	1.38	.816658	4.59	.183342	45
16	.739206	3.21	.922272	1.38	.816933	4.59	.183067	44
17	.739398	3.21	.922189	1.38	.817209	4.59	.182791	43
18	.739590	3.20	.922106	1.38	.817484	4.59	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.59	.182241	41
20	9.739975	3.20	9.921940	1.39	9.818035	4.59	0.181965	40
21	.740167	3.20	.921857	1.39	.818310	4.58	.181690	39
22	.740359	3.20	.921774	1.39	.818585	4.58	.181415	38
23	.740550	3.19	.921691	1.39	.818860	4.58	.181140	37
24	.740742	3.19	.921607	1.39	.819135	4.58	.180865	36
25	.740934	3.19	.921524	1.39	.819410	4.58	.180590	35
26	.741125	3.19	.921441	1.39	.819684	4.58	.180316	34
27	.741316	3.19	.921357	1.39	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.39	.820234	4.58	.179766	32
29	.741699	3.18	.921190	1.39	.820508	4.58	.179492	31
30	9.741889	3.18	9.921107	1.39	9.820783	4.57	0.179217	30
31	.742080	3.18	.921023	1.39	.821057	4.57	.178943	29
32	.742271	3.18	.920939	1.40	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.17	.920688	1.40	.822154	4.57	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.17	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.16	.920352	1.40	.823251	4.56	.176749	21
40	9.743792	3.16	9.920268	1.40	9.823524	4.56	0.176476	20
41	.743982	3.16	.920184	1.40	.823798	4.56	.176202	19
42	.744171	3.16	.920099	1.40	.824072	4.56	.175928	18
43	.744361	3.15	.920015	1.41	.824345	4.56	.175655	17
44	.744550	3.15	.919931	1.41	.824619	4.56	.175381	16
45	.744739	3.15	.919846	1.41	.824893	4.56	.175107	15
46	.744928	3.15	.919762	1.41	.825166	4.56	.174834	14
47	.745117	3.15	.919677	1.41	.825439	4.56	.174561	13
48	.745306	3.14	.919593	1.41	.825713	4.55	.174287	12
49	.745494	3.14	.919508	1.41	.825986	4.55	.174014	11
50	9.745683	3.14	9.919424	1.41	9.826259	4.55	0.173741	10
51	.745871	3.14	.919339	1.41	.826532	4.55	.173468	9
52	.746060	3.14	.919254	1.41	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.41	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.13	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.54	.171830	3
58	.747187	3.12	.918745	1.42	.828442	4.54	.171558	2
59	.747374	3.12	.918659	1.42	.828715	4.54	.171285	1
60	.747562	3.12	.918574	1.42	.828987	4.54	.171013	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1'.	Tang.	D. 1'.	Cotang.
1	75000	1.42	83000	4.34	0.171013
2	75000	1.42	83000	4.34	.170740
3	75000	1.42	83000	4.34	.170468
4	75000	1.42	83000	4.34	.170195
5	75000	1.42	83000	4.34	.169923
6	75000	1.42	83000	4.34	.169651
7	75000	1.42	83000	4.34	.169379
8	75000	1.42	83000	4.34	.169107
9	75000	1.42	83000	4.34	.168835
10	75000	1.42	83000	4.34	.168563
11	75000	1.42	83000	4.34	0.168291
12	75000	1.42	83000	4.34	.168019
13	75000	1.42	83000	4.34	.167747
14	75000	1.42	83000	4.34	.167475
15	75000	1.42	83000	4.34	.167203
16	75000	1.42	83000	4.34	.166931
17	75000	1.42	83000	4.34	.166659
18	75000	1.42	83000	4.34	.166387
19	75000	1.42	83000	4.34	.166115
20	75000	1.42	83000	4.34	.165843
21	75000	1.42	83000	4.34	0.165571
22	75000	1.42	83000	4.34	.165299
23	75000	1.42	83000	4.34	.165027
24	75000	1.42	83000	4.34	.164755
25	75000	1.42	83000	4.34	.164483
26	75000	1.42	83000	4.34	.164211
27	75000	1.42	83000	4.34	.163939
28	75000	1.42	83000	4.34	.163667
29	75000	1.42	83000	4.34	.163395
30	75000	1.42	83000	4.34	.163123
31	75000	1.42	83000	4.34	0.162851
32	75000	1.42	83000	4.34	.162579
33	75000	1.42	83000	4.34	.162307
34	75000	1.42	83000	4.34	.162035
35	75000	1.42	83000	4.34	.161763
36	75000	1.42	83000	4.34	.161491
37	75000	1.42	83000	4.34	.161219
38	75000	1.42	83000	4.34	.160947
39	75000	1.42	83000	4.34	.160675
40	75000	1.42	83000	4.34	.160403
41	75000	1.42	83000	4.34	0.160131
42	75000	1.42	83000	4.34	.159859
43	75000	1.42	83000	4.34	.159587
44	75000	1.42	83000	4.34	.159315
45	75000	1.42	83000	4.34	.159043
46	75000	1.42	83000	4.34	.158771
47	75000	1.42	83000	4.34	.158499
48	75000	1.42	83000	4.34	.158227
49	75000	1.42	83000	4.34	.157955
50	75000	1.42	83000	4.34	0.157683
51	75000	1.42	83000	4.34	.157411
52	75000	1.42	83000	4.34	.157139
53	75000	1.42	83000	4.34	.156867
54	75000	1.42	83000	4.34	.156595
55	75000	1.42	83000	4.34	.156323
56	75000	1.42	83000	4.34	.156051
57	75000	1.42	83000	4.34	.155779
58	75000	1.42	83000	4.34	.155507
59	75000	1.42	83000	4.34	.155235
60	75000	1.42	83000	4.34	.154963
61	75000	1.42	83000	4.34	.154691
62	75000	1.42	83000	4.34	.154419
63	75000	1.42	83000	4.34	.154147
64	75000	1.42	83000	4.34	.153875
65	75000	1.42	83000	4.34	.153603
66	75000	1.42	83000	4.34	.153331
67	75000	1.42	83000	4.34	.153059
68	75000	1.42	83000	4.34	.152787
69	75000	1.42	83000	4.34	.152515
70	75000	1.42	83000	4.34	.152243
71	75000	1.42	83000	4.34	.151971
72	75000	1.42	83000	4.34	.151699
73	75000	1.42	83000	4.34	.151427
74	75000	1.42	83000	4.34	.151155
75	75000	1.42	83000	4.34	.150883
76	75000	1.42	83000	4.34	.150611
77	75000	1.42	83000	4.34	.150339
78	75000	1.42	83000	4.34	.150067
79	75000	1.42	83000	4.34	.149795
80	75000	1.42	83000	4.34	.149523
81	75000	1.42	83000	4.34	.149251
82	75000	1.42	83000	4.34	.148979
83	75000	1.42	83000	4.34	.148707
84	75000	1.42	83000	4.34	.148435
85	75000	1.42	83000	4.34	.148163
86	75000	1.42	83000	4.34	.147891
87	75000	1.42	83000	4.34	.147619
88	75000	1.42	83000	4.34	.147347
89	75000	1.42	83000	4.34	.147075
90	75000	1.42	83000	4.34	.146803
91	75000	1.42	83000	4.34	.146531
92	75000	1.42	83000	4.34	.146259
93	75000	1.42	83000	4.34	.145987
94	75000	1.42	83000	4.34	.145715
95	75000	1.42	83000	4.34	.145443
96	75000	1.42	83000	4.34	.145171
97	75000	1.42	83000	4.34	.144899
98	75000	1.42	83000	4.34	.144627
99	75000	1.42	83000	4.34	.144355
100	75000	1.42	83000	4.34	.144083

150

M.	Sine.	D. 1'	Cosine.	D. 1'	Tang.	D. 1'	Cotang.	M.
0	9.758591	2.01	9.913365	1.47	9.845227	4.48	0.154773	60
1	.758772	3.00	.913276	1.48	.845196	4.48	.154504	59
2	.758952	3.00	.913187	1.48	.845164	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.845133	4.48	.153967	57
4	.759312	3.00	.913010	1.48	.845102	4.48	.153698	56
5	.759492	3.00	.912922	1.48	.845070	4.48	.153430	55
6	.759672	2.99	.912833	1.48	.845039	4.48	.153161	54
7	.759852	2.99	.912744	1.48	.845008	4.48	.152892	53
8	.760031	2.99	.912655	1.48	.844976	4.47	.152624	52
9	.760211	2.99	.912566	1.48	.844944	4.47	.152356	51
10	9.760300	2.99	9.912477	1.48	9.844913	4.47	0.152087	50
11	.760569	2.99	.912388	1.48	.844881	4.47	.151819	49
12	.760748	2.98	.912299	1.49	.844849	4.47	.151551	48
13	.760927	2.98	.912210	1.49	.844817	4.47	.151283	47
14	.761106	2.98	.912121	1.49	.844786	4.47	.151014	46
15	.761285	2.98	.912031	1.49	.844754	4.47	.150746	45
16	.761464	2.98	.911942	1.49	.844722	4.47	.150478	44
17	.761642	2.97	.911853	1.49	.844690	4.47	.150210	43
18	.761821	2.97	.911763	1.49	.850057	4.46	.149943	42
19	.761999	2.97	.911674	1.49	.850025	4.46	.149675	41
20	9.762177	2.97	9.911584	1.49	9.850059	4.46	0.149407	40
21	.762356	2.97	.911495	1.49	.850026	4.46	.149139	39
22	.762534	2.97	.911405	1.49	.851129	4.46	.148871	38
23	.762712	2.96	.911315	1.50	.851396	4.46	.148604	37
24	.762899	2.96	.911226	1.50	.851664	4.46	.148336	36
25	.763067	2.96	.911136	1.50	.851931	4.46	.148069	35
26	.763245	2.96	.911046	1.50	.852199	4.46	.147801	34
27	.763422	2.96	.910956	1.50	.852466	4.46	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.46	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.46	.146999	31
30	9.763954	2.95	9.910686	1.50	9.853269	4.45	0.146732	30
31	.764131	2.95	.910596	1.50	.853535	4.45	.146465	29
32	.764308	2.95	.910506	1.50	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.51	.854069	4.45	.145931	27
34	.764662	2.94	.910325	1.51	.854336	4.45	.145664	26
35	.764838	2.94	.910235	1.51	.854603	4.45	.145397	25
36	.765015	2.94	.910144	1.51	.854870	4.45	.145130	24
37	.765191	2.94	.910054	1.51	.855137	4.45	.144863	23
38	.765367	2.94	.909963	1.51	.855401	4.45	.144596	22
39	.765544	2.93	.909873	1.51	.855667	4.44	.144329	21
40	9.765720	2.93	9.909782	1.51	9.855939	4.44	0.144062	20
41	.765896	2.93	.909691	1.51	.856204	4.44	.143796	19
42	.766072	2.93	.909601	1.51	.856471	4.44	.143529	18
43	.766247	2.93	.909510	1.51	.856737	4.44	.143263	17
44	.766423	2.93	.909419	1.52	.857004	4.44	.142996	16
45	.766598	2.92	.909328	1.52	.857270	4.44	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.44	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.44	.142197	13
48	.767124	2.92	.909055	1.52	.858069	4.44	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.44	.141664	11
50	9.767475	2.91	9.908873	1.52	9.858602	4.44	0.141398	10
51	.767649	2.91	.908781	1.52	.858869	4.43	.141132	9
52	.767824	2.91	.908690	1.52	.859131	4.43	.140866	8
53	.767999	2.91	.908600	1.52	.859400	4.43	.140600	7
54	.768173	2.91	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.91	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.90	.908324	1.53	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.43	.139270	2
59	.769046	2.90	.908049	1.53	.860995	4.43	.139005	1
60	.769219		.907958		.861261		.138739	0
M.	Cosine.	D. 1'	Sine.	D. 1'	Cotang.	D. 1'		

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.763219	2.90	9.907958	1.53	9.861261	4.43	0.138739	0
1	.769393	2.90	.907866	1.53	.861527	4.43	.138473	1
2	.769566	2.89	.907774	1.53	.861792	4.43	.138208	2
3	.769740	2.89	.907682	1.53	.862058	4.42	.137942	3
4	.769913	2.89	.907590	1.53	.862323	4.42	.137677	4
5	.770087	2.89	.907498	1.53	.862589	4.42	.137411	5
6	.770260	2.89	.907406	1.54	.862854	4.42	.137146	6
7	.770433	2.88	.907314	1.54	.863119	4.42	.136881	7
8	.770606	2.88	.907222	1.54	.863385	4.42	.136615	8
9	.770779	2.88	.907129	1.54	.863650	4.42	.136350	9
10	9.770952	2.89	9.907037	1.54	9.863915	4.42	0.136085	10
11	.771125	2.89	.906945	1.54	.864180	4.42	.135820	11
12	.771298	2.88	.906852	1.54	.864445	4.42	.135555	12
13	.771470	2.87	.906760	1.54	.864710	4.42	.135290	13
14	.771643	2.87	.906667	1.54	.864975	4.42	.135025	14
15	.771815	2.87	.906575	1.54	.865240	4.41	.134760	15
16	.771987	2.87	.906482	1.55	.865505	4.41	.134495	16
17	.772159	2.87	.906389	1.55	.865770	4.41	.134230	17
18	.772331	2.87	.906296	1.55	.866035	4.41	.133965	18
19	.772503	2.86	.906204	1.55	.866300	4.41	.133700	19
20	9.772675	2.86	9.906111	1.55	9.866564	4.41	0.133436	20
21	.772847	2.86	.906018	1.55	.866829	4.41	.133171	21
22	.773018	2.86	.905925	1.55	.867094	4.41	.132906	22
23	.773190	2.86	.905832	1.55	.867358	4.41	.132642	23
24	.773361	2.85	.905739	1.55	.867623	4.41	.132377	24
25	.773533	2.85	.905645	1.55	.867887	4.41	.132113	25
26	.773704	2.85	.905552	1.55	.868152	4.41	.131848	26
27	.773875	2.85	.905459	1.56	.868416	4.41	.131584	27
28	.774046	2.85	.905366	1.56	.868680	4.40	.131320	28
29	.774217	2.85	.905272	1.56	.868945	4.40	.131055	29
30	9.774388	2.84	9.905179	1.56	9.869209	4.40	0.130791	30
31	.774558	2.84	.905085	1.56	.869473	4.40	.130527	31
32	.774729	2.84	.904992	1.56	.869737	4.40	.130263	32
33	.774899	2.84	.904898	1.56	.870001	4.40	.129999	33
34	.775070	2.84	.904804	1.56	.870265	4.40	.129735	34
35	.775240	2.84	.904711	1.56	.870529	4.40	.129471	35
36	.775410	2.83	.904617	1.56	.870793	4.40	.129207	36
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	37
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	38
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	39
40	9.776090	2.83	9.904241	1.57	9.871849	4.40	0.128151	40
41	.776259	2.83	.904147	1.57	.872112	4.39	.127888	41
42	.776429	2.82	.904053	1.57	.872376	4.39	.127624	42
43	.776598	2.82	.903959	1.57	.872640	4.39	.127360	43
44	.776768	2.82	.903864	1.57	.872903	4.39	.127097	44
45	.776937	2.82	.903770	1.57	.873167	4.39	.126833	45
46	.777106	2.82	.903676	1.57	.873430	4.39	.126570	46
47	.777275	2.82	.903581	1.57	.873694	4.39	.126306	47
48	.777444	2.81	.903487	1.57	.873957	4.39	.126043	48
49	.777613	2.81	.903392	1.58	.874220	4.39	.125780	49
50	9.777781	2.81	9.903298	1.58	9.874481	4.39	0.125516	50
51	.777950	2.81	.903203	1.58	.874747	4.39	.125253	51
52	.778119	2.81	.903108	1.58	.875010	4.39	.124990	52
53	.778287	2.81	.903014	1.58	.875273	4.39	.124727	53
54	.778455	2.80	.902919	1.58	.875537	4.38	.124463	54
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	55
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	56
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	57
58	.779128	2.80	.902539	1.59	.876589	4.38	.123411	58
59	.779295	2.80	.902444	1.59	.876852	4.38	.123148	59
60	.779463	2.79	.902349	1.59	.877114	4.38	.122886	60
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.779463	2.79	9.902319	1.59	9.877114	4.38	0.122886	60
1	.779631	2.79	.902253	1.59	.877377	4.38	.122623	59
2	.779798	2.79	.902158	1.59	.877640	4.38	.122360	58
3	.779966	2.79	.902063	1.59	.877903	4.38	.122097	57
4	.780133	2.79	.901967	1.59	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.59	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.59	.878691	4.38	.121309	54
7	.780634	2.78	.901681	1.59	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.59	.879216	4.37	.120784	52
9	.780968	2.78	.901490	1.60	.879478	4.37	.120522	51
10	9.781134	2.78	9.901394	1.60	9.879741	4.37	0.120259	50
11	.781301	2.77	.901298	1.60	.880003	4.37	.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.37	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948	45
16	.782132	2.77	.900818	1.60	.881314	4.37	.118686	44
17	.782298	2.76	.900722	1.60	.881577	4.37	.118423	43
18	.782464	2.76	.900626	1.60	.881839	4.37	.118161	42
19	.782630	2.76	.900529	1.61	.882101	4.37	.117899	41
20	9.782796	2.76	9.900433	1.61	9.882363	4.37	0.117637	40
21	.782961	2.76	.900337	1.61	.882625	4.37	.117375	39
22	.783127	2.76	.900240	1.61	.882887	4.36	.117113	38
23	.783292	2.75	.900144	1.61	.883148	4.36	.116852	37
24	.783458	2.75	.900047	1.61	.883410	4.36	.116590	36
25	.783623	2.75	.899951	1.61	.883672	4.36	.116328	35
26	.783788	2.75	.899854	1.61	.883934	4.36	.116066	34
27	.783953	2.75	.899757	1.61	.884196	4.36	.115804	33
28	.784118	2.75	.899660	1.61	.884457	4.36	.115543	32
29	.784282	2.74	.899564	1.62	.884719	4.36	.115281	31
30	9.784447	2.74	9.899467	1.62	9.884980	4.36	0.115020	30
31	.784612	2.74	.899370	1.62	.885242	4.36	.114758	29
32	.784776	2.74	.899273	1.62	.885504	4.36	.114496	28
33	.784941	2.74	.899176	1.62	.885765	4.36	.114235	27
34	.785105	2.74	.899078	1.62	.886026	4.36	.113974	26
35	.785269	2.73	.898981	1.62	.886288	4.36	.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.36	.113451	24
37	.785597	2.73	.898787	1.62	.886811	4.35	.113189	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.62	.887333	4.35	.112667	21
40	9.786089	2.73	9.898494	1.63	9.887594	4.35	0.112406	20
41	.786252	2.73	.898397	1.63	.887855	4.35	.112145	19
42	.786416	2.72	.898299	1.63	.888116	4.35	.111884	18
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	17
44	.786742	2.72	.898104	1.63	.888639	4.35	.111361	16
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100	15
46	.787069	2.72	.897908	1.63	.889161	4.35	.110839	14
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.71	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.71	.897614	1.63	.889943	4.35	.110057	11
50	9.787720	2.71	9.897516	1.64	9.890204	4.35	0.109796	10
51	.787883	2.71	.897418	1.64	.890465	4.35	.109535	9
52	.788045	2.71	.897320	1.64	.890725	4.34	.109275	8
53	.788208	2.71	.897222	1.64	.890986	4.34	.109014	7
54	.788370	2.70	.897123	1.64	.891247	4.34	.108753	6
55	.788532	2.70	.897025	1.64	.891507	4.34	.108493	5
56	.788694	2.70	.896926	1.64	.891768	4.34	.108232	4
57	.788856	2.70	.896828	1.64	.892028	4.34	.107972	3
58	.789018	2.70	.896729	1.64	.892289	4.34	.107711	2
59	.789180	2.70	.896631	1.64	.892549	4.34	.107451	1
60	.789342	2.70	.896532	1.64	.892810	4.34	.107190	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.789312	2.69	9.896532	1.65	9.892810	4.34	0.107190
1	.789504	2.69	.896433	1.65	.893070	4.34	.106930
2	.789665	2.69	.896335	1.65	.893331	4.34	.106669
3	.789827	2.69	.896236	1.65	.893591	4.34	.106409
4	.789988	2.69	.896137	1.65	.893851	4.34	.106149
5	.790149	2.69	.896038	1.65	.894111	4.34	.105889
6	.790310	2.69	.895939	1.65	.894372	4.34	.105628
7	.790471	2.68	.895840	1.65	.894632	4.34	.105368
8	.790632	2.68	.895741	1.65	.894892	4.33	.105108
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848
10	9.790954	2.68	9.895542	1.66	9.895412	4.33	0.104588
11	.791115	2.68	.895443	1.66	.895672	4.33	.104328
12	.791275	2.67	.895343	1.66	.895932	4.33	.104068
13	.791436	2.67	.895244	1.66	.896192	4.33	.103808
14	.791596	2.67	.895145	1.66	.896452	4.33	.103548
15	.791757	2.67	.895045	1.66	.896712	4.33	.103288
16	.791917	2.67	.894945	1.66	.896971	4.33	.103029
17	.792077	2.67	.894846	1.66	.897231	4.33	.102769
18	.792237	2.67	.894746	1.66	.897491	4.33	.102509
19	.792397	2.66	.894646	1.66	.897751	4.33	.102249
20	9.792557	2.66	9.894546	1.67	9.898010	4.33	0.101990
21	.792716	2.66	.894446	1.67	.898270	4.33	.101730
22	.792876	2.66	.894346	1.67	.898530	4.33	.101470
23	.793035	2.66	.894246	1.67	.898789	4.33	.101211
24	.793195	2.66	.894146	1.67	.899049	4.33	.100951
25	.793354	2.65	.894046	1.67	.899308	4.32	.100692
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432
27	.793673	2.65	.893846	1.67	.899827	4.32	.100173
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913
29	.793991	2.65	.893645	1.67	.900346	4.32	.099654
30	9.794150	2.65	9.893544	1.68	9.900605	4.32	0.099395
31	.794308	2.64	.893444	1.68	.900864	4.32	.099136
32	.794467	2.64	.893343	1.68	.901124	4.32	.098876
33	.794626	2.64	.893243	1.68	.901383	4.32	.098617
34	.794784	2.64	.893142	1.68	.901642	4.32	.098358
35	.794942	2.64	.893041	1.68	.901901	4.32	.098099
36	.795101	2.64	.892940	1.68	.902160	4.32	.097840
37	.795259	2.64	.892839	1.68	.902420	4.32	.097580
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321
39	.795575	2.63	.892638	1.68	.902938	4.32	.097062
40	9.795733	2.63	9.892536	1.69	9.903197	4.32	0.096803
41	.795891	2.63	.892435	1.69	.903456	4.32	.096544
42	.796049	2.63	.892334	1.69	.903714	4.31	.096286
43	.796206	2.63	.892233	1.69	.903973	4.31	.096027
44	.796364	2.62	.892132	1.69	.904232	4.31	.095768
45	.796521	2.62	.892030	1.69	.904491	4.31	.095509
46	.796679	2.62	.891929	1.69	.904750	4.31	.095250
47	.796836	2.62	.891827	1.69	.905008	4.31	.094992
48	.796993	2.62	.891726	1.69	.905267	4.31	.094733
49	.797150	2.61	.891624	1.69	.905526	4.31	.094474
50	9.797307	2.61	9.891523	1.70	9.905785	4.31	0.094215
51	.797464	2.61	.891421	1.70	.906043	4.31	.093957
52	.797621	2.61	.891319	1.70	.906302	4.31	.093698
53	.797777	2.61	.891217	1.70	.906560	4.31	.093440
4	.797934	2.61	.891115	1.70	.906819	4.31	.093181
5	.798091	2.61	.891013	1.70	.907077	4.31	.092923
6	.798247	2.61	.890911	1.70	.907336	4.31	.092664
7	.798403	2.60	.890809	1.70	.907594	4.31	.092406
58	.798560	2.60	.890707	1.70	.907853	4.31	.092147
59	.798716	2.60	.890605	1.70	.908111	4.31	.091889
60	.798872		.890503	1.70	.908369		.091631
M.	Cosine.	D. 1	Sine.	D. 1".	Cotang.	D. 1".	Tang.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	.9793872	2.60	.9890503	1.71	.9903369	4.30	.0091631	60
1	.7990283	2.60	.890100	1.71	.908628	4.30	.091372	59
2	.799184	2.60	.890298	1.71	.9088-6	4.30	.091114	58
3	.799339	2.59	.890195	1.71	.909144	4.30	.090856	57
4	.799495	2.59	.890093	1.71	.909402	4.30	.090598	56
5	.799651	2.59	.889990	1.71	.909660	4.30	.090340	55
6	.799806	2.59	.889888	1.71	.909918	4.30	.090082	54
7	.799962	2.59	.889785	1.71	.910177	4.30	.089823	53
8	.800117	2.59	.889682	1.71	.910435	4.30	.089565	52
9	.800272	2.59	.889579	1.71	.910693	4.30	.089307	51
10	.800427	2.58	.889477	1.72	.910951	4.30	.089049	50
11	.800582	2.58	.889374	1.72	.911209	4.30	.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.72	.911725	4.30	.088275	47
14	.801047	2.58	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.57	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.72	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.30	.086986	42
19	.801819	2.57	.888548	1.72	.913271	4.30	.086729	41
20	.801973	2.57	.888444	1.73	.913529	4.29	.086471	40
21	.802128	2.57	.888341	1.73	.913787	4.29	.086213	39
22	.802282	2.57	.888237	1.73	.914044	4.29	.085956	38
23	.802436	2.56	.888134	1.73	.914302	4.29	.085698	37
24	.802590	2.56	.888030	1.73	.914560	4.29	.085440	36
25	.802743	2.56	.887926	1.73	.914817	4.29	.085183	35
26	.802897	2.56	.887822	1.73	.915075	4.29	.084925	34
27	.803050	2.56	.887718	1.73	.915332	4.29	.084668	33
28	.803204	2.56	.887614	1.73	.915590	4.29	.084410	32
29	.803357	2.55	.887510	1.74	.915847	4.29	.084153	31
30	.803511	2.55	.887406	1.74	.916104	4.29	.083896	30
31	.803664	2.55	.887302	1.74	.916362	4.29	.083638	29
32	.803817	2.55	.887198	1.74	.916619	4.29	.083381	28
33	.803970	2.55	.887093	1.74	.916877	4.29	.083123	27
34	.804123	2.55	.886989	1.74	.917134	4.29	.082866	26
35	.804276	2.55	.886885	1.74	.917391	4.29	.082609	25
36	.804428	2.54	.886780	1.74	.917648	4.29	.082352	24
37	.804581	2.54	.886676	1.74	.917906	4.29	.082094	23
38	.804734	2.54	.886571	1.74	.918163	4.29	.081837	22
39	.804886	2.54	.886466	1.75	.918420	4.29	.081580	21
40	.805039	2.54	.886362	1.75	.918677	4.28	.081323	20
41	.805191	2.54	.886257	1.75	.918934	4.28	.081066	19
42	.805343	2.54	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.53	.885627	1.75	.920476	4.28	.079524	13
48	.806255	2.53	.885522	1.75	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.76	.920990	4.28	.079010	11
50	.806557	2.52	.885311	1.76	.921247	4.28	.078753	10
51	.806709	2.52	.885205	1.76	.921503	4.28	.078497	9
52	.806860	2.52	.885100	1.76	.921760	4.28	.078240	8
53	.807011	2.52	.884994	1.76	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.76	.922274	4.28	.077726	6
55	.807314	2.52	.884783	1.76	.922530	4.28	.077470	5
56	.807465	2.51	.884677	1.76	.922787	4.28	.077213	4
57	.807615	2.51	.884572	1.76	.923044	4.28	.076956	3
58	.807766	2.51	.884466	1.77	.923300	4.28	.076700	2
59	.807917	2.51	.884360	1.77	.923557	4.28	.076443	1
60	.808067	2.51	.884254	1.77	.923814	4.28	.076186	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.808067	2.51	9.884254	1.77	9.923814	4.23	0.076186	60
1	.808218	2.51	.884143	1.77	.924070	4.23	.075930	59
2	.808368	2.51	.884042	1.77	.924327	4.23	.075673	58
3	.808519	2.50	.883936	1.77	.924583	4.27	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.77	.925352	4.27	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883401	1.77	.925865	4.27	.074135	52
9	.809419	2.50	.883297	1.78	.926122	4.27	.073878	51
10	9.809569	2.49	9.883191	1.78	9.926378	4.27	0.073622	50
11	.809718	2.49	.883084	1.78	.926634	4.27	.073366	49
12	.809868	2.49	.882977	1.78	.926890	4.27	.073110	48
13	.810017	2.49	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.49	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.49	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.79	.928427	4.27	.071573	42
19	.810912	2.48	.882229	1.79	.928684	4.27	.071316	41
20	9.811061	2.48	9.882121	1.79	9.928940	4.27	0.071060	40
21	.811210	2.48	.882014	1.79	.929196	4.27	.070804	39
22	.811358	2.48	.881907	1.79	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.79	.929708	4.27	.070292	37
24	.811655	2.47	.881692	1.79	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.79	.930220	4.27	.069780	35
26	.811952	2.47	.881477	1.79	.930475	4.26	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.26	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.26	.069013	32
29	.812396	2.47	.881153	1.80	.931243	4.26	.068757	31
30	9.812544	2.46	9.881046	1.80	9.931499	4.26	0.068501	30
31	.812692	2.46	.880938	1.80	.931755	4.26	.068245	29
32	.812840	2.46	.880830	1.80	.932010	4.26	.067990	28
33	.812988	2.46	.880722	1.80	.932266	4.26	.067734	27
34	.813135	2.46	.880613	1.80	.932522	4.26	.067478	26
35	.813283	2.46	.880505	1.80	.932778	4.26	.067222	25
36	.813430	2.46	.880397	1.81	.933033	4.26	.066967	24
37	.813578	2.45	.880289	1.81	.933289	4.26	.066711	23
38	.813725	2.45	.880180	1.81	.933545	4.26	.066455	22
39	.813872	2.45	.880072	1.81	.933800	4.26	.066200	21
40	9.814019	2.45	9.879963	1.81	9.934056	4.26	0.065944	20
41	.814166	2.45	.879855	1.81	.934311	4.26	.065689	19
42	.814313	2.45	.879746	1.81	.934567	4.26	.065433	18
43	.814460	2.45	.879637	1.81	.934822	4.26	.065178	17
44	.814607	2.44	.879529	1.81	.935078	4.26	.064922	16
45	.814753	2.44	.879420	1.81	.935333	4.26	.064667	15
46	.814900	2.44	.879311	1.82	.935589	4.26	.064411	14
47	.815046	2.44	.879202	1.82	.935844	4.26	.064156	13
48	.815193	2.44	.879093	1.82	.936100	4.26	.063900	12
49	.815339	2.44	.878984	1.82	.936355	4.26	.063645	11
50	9.815485	2.44	9.878875	1.82	9.936611	4.26	0.063389	10
51	.815632	2.43	.878766	1.82	.936866	4.26	.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.26	.062879	8
53	.815924	2.43	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.82	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.83	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.25	.061858	4
57	.816507	2.43	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.42	.877999	1.83	.938653	4.25	.061347	2
59	.816797	2.42	.877890	1.83	.938908	4.25	.061092	1
60	.816943	2.42	.877780	1.83	.939163	4.25	.060837	0
M.	Cosine	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1 st .	Cosine.	D. 1 st .	Tang.	D. 1 st .	Cotang.	M.
0	9.816913	2.42	9.877740	1.85	9.939163	4.25	0.060837	60
1	.817088	2.42	.877670	1.85	.939118	4.25	.060882	59
2	.817233	2.42	.877560	1.85	.939073	4.25	.060927	58
3	.817379	2.42	.877450	1.85	.939028	4.25	.060972	57
4	.817524	2.42	.877340	1.85	.940183	4.25	.059817	56
5	.817669	2.41	.877230	1.85	.940439	4.25	.059561	55
6	.817813	2.41	.877120	1.85	.940691	4.25	.059306	54
7	.817958	2.41	.877010	1.85	.940949	4.25	.059051	53
8	.818103	2.41	.876899	1.85	.941204	4.25	.058796	52
9	.818247	2.41	.876789	1.85	.941459	4.25	.058541	51
10	9.818392	2.41	9.876673	1.85	9.941713	4.25	0.058287	50
11	.818536	2.41	.876563	1.85	.941968	4.25	.058032	49
12	.818681	2.40	.876457	1.85	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.85	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.25	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.39	.875682	1.85	.944007	4.25	.055993	41
20	9.819832	2.39	9.875571	1.85	9.944262	4.25	0.055738	40
21	.819976	2.39	.875459	1.85	.944517	4.25	.055483	39
22	.820120	2.39	.875348	1.85	.944771	4.24	.055228	38
23	.820263	2.39	.875237	1.85	.945026	4.24	.054974	37
24	.820406	2.39	.875126	1.85	.945281	4.24	.054719	36
25	.820550	2.39	.875014	1.85	.945535	4.24	.054465	35
26	.820693	2.38	.874903	1.85	.945790	4.24	.054210	34
27	.820836	2.38	.874791	1.85	.946045	4.24	.053955	33
28	.820979	2.38	.874680	1.85	.946299	4.24	.053701	32
29	.821122	2.38	.874568	1.85	.946554	4.24	.053446	31
30	9.821265	2.38	9.874456	1.85	9.946808	4.24	0.053192	30
31	.821407	2.38	.874344	1.85	.947063	4.24	.052937	29
32	.821550	2.38	.874232	1.85	.947318	4.24	.052682	28
33	.821693	2.37	.874121	1.85	.947572	4.24	.052428	27
34	.821835	2.37	.874009	1.85	.947827	4.24	.052173	26
35	.821977	2.37	.873896	1.85	.948081	4.24	.051919	25
36	.822120	2.37	.873784	1.85	.948335	4.24	.051665	24
37	.822262	2.37	.873672	1.85	.948590	4.24	.051410	23
38	.822404	2.37	.873560	1.85	.948844	4.24	.051156	22
39	.822546	2.37	.873448	1.85	.949099	4.24	.050901	21
40	9.822688	2.37	9.873335	1.85	9.949353	4.24	0.050647	20
41	.822830	2.36	.873223	1.85	.949608	4.24	.050392	19
42	.822972	2.36	.873110	1.85	.949862	4.24	.050138	18
43	.823114	2.36	.872998	1.85	.950116	4.24	.049884	17
44	.823255	2.36	.872885	1.85	.950371	4.24	.049629	16
45	.823397	2.36	.872772	1.85	.950625	4.24	.049375	15
46	.823539	2.36	.872659	1.85	.950879	4.24	.049121	14
47	.823680	2.36	.872547	1.85	.951133	4.24	.048867	13
48	.823821	2.35	.872434	1.85	.951388	4.24	.048612	12
49	.823963	2.35	.872321	1.85	.951642	4.24	.048358	11
50	9.824104	2.35	9.872208	1.85	9.951896	4.24	0.048104	10
51	.824245	2.35	.872095	1.85	.952150	4.24	.047850	9
52	.824386	2.35	.871981	1.85	.952405	4.24	.047595	8
53	.824527	2.35	.871868	1.85	.952659	4.24	.047341	7
54	.824668	2.35	.871755	1.85	.952913	4.24	.047087	6
55	.824808	2.34	.871641	1.85	.953167	4.24	.046833	5
56	.824949	2.34	.871528	1.85	.953421	4.24	.046579	4
57	.825090	2.34	.871414	1.85	.953675	4.23	.046325	3
58	.825230	2.34	.871301	1.85	.953929	4.23	.046071	2
59	.825371	2.34	.871187	1.90	.954183	4.23	.045817	1
60	.825511	2.34	.871073	1.90	.954437	4.23	.045562	
M.	Cosine.	D. 1 st .	Sine.	D. 1 st .	Cotang.	D. 1 st .	T	

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.825511	2.31	9.871073	1.90	9.954437	4.23	0.045563	60
1	.825651	2.31	.870960	1.90	.954691	4.23	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870501	1.90	.955708	4.23	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.90	.956215	4.23	.043785	53
8	.826631	2.33	.870161	1.91	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.91	.956723	4.23	.043277	51
10	9.826910	2.32	9.869933	1.91	9.956977	4.23	0.043023	50
11	.827049	2.32	.869818	1.91	.957231	4.23	.042769	49
12	.827189	2.32	.869704	1.91	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.91	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.91	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.91	.958247	4.23	.041753	45
16	.827745	2.32	.869245	1.91	.958500	4.23	.041500	44
17	.827884	2.31	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.31	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.31	.868900	1.92	.959262	4.23	.040738	41
20	9.828301	2.31	9.868785	1.92	9.959516	4.23	0.040484	40
21	.828439	2.31	.868670	1.92	.959769	4.23	.040231	39
22	.828578	2.31	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.31	.868440	1.92	.960277	4.23	.039723	37
24	.828855	2.31	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.92	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.93	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.23	.038708	33
28	.829407	2.30	.867862	1.93	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.23	.038201	31
30	9.829683	2.30	9.867631	1.93	9.962052	4.23	0.037948	30
31	.829821	2.30	.867515	1.93	.962306	4.23	.037694	29
32	.829959	2.29	.867399	1.93	.962560	4.23	.037440	28
33	.830097	2.29	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.29	.867167	1.93	.963067	4.23	.036933	26
35	.830372	2.29	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.29	.866935	1.91	.963574	4.23	.036426	24
37	.830646	2.29	.866819	1.91	.963828	4.23	.036172	23
38	.830784	2.29	.866703	1.91	.964081	4.23	.035919	22
39	.830921	2.29	.866586	1.91	.964335	4.23	.035665	21
40	9.831058	2.28	9.866470	1.91	9.964588	4.22	0.035412	20
41	.831195	2.28	.866353	1.91	.964842	4.22	.035158	19
42	.831332	2.28	.866237	1.91	.965095	4.22	.034905	18
43	.831469	2.28	.866120	1.91	.965349	4.22	.034651	17
44	.831606	2.28	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.22	.034145	15
46	.831879	2.28	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.27	.865653	1.95	.966362	4.22	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.27	.865419	1.95	.966869	4.22	.033131	11
50	9.832425	2.27	9.865302	1.95	9.967123	4.22	0.032877	10
51	.832561	2.27	.865185	1.95	.967376	4.22	.032624	9
52	.832697	2.27	.865068	1.95	.967629	4.22	.032371	8
53	.832833	2.27	.864950	1.96	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.96	.968136	4.22	.031864	6
55	.833105	2.26	.864716	1.96	.968389	4.22	.031611	5
56	.833241	2.26	.864598	1.96	.968643	4.22	.031357	4
57	.833377	2.26	.864481	1.96	.968896	4.22	.031104	3
58	.833512	2.26	.864363	1.96	.969149	4.22	.030851	2
59	.833648	2.26	.864245	1.96	.969403	4.22	.030597	1
60	.833783	2.26	.864127	1.96	.969656	4.22	.030344	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.833783		9.864127		9.969656		0.030344	60
1	.833919	2.26	.864010	1.96	.969909	4.22	.030091	59
2	.834054	2.26	.863892	1.97	.970162	4.22	.029838	58
3	.834189	2.25	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863533	1.97	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.22	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.25	.863183	1.97	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	9.835134		9.862946		9.972188		0.027812	50
11	.835269	2.24	.862827	1.98	.972441	4.22	.027559	49
12	.835403	2.24	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.24	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.24	.862471	1.98	.973201	4.22	.026799	46
15	.835807	2.24	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.24	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.99	.974466	4.22	.025534	41
20	9.836477		9.861758		9.974720		0.025280	40
21	.836611	2.23	.861638	1.99	.974973	4.22	.025027	39
22	.836745	2.23	.861519	1.99	.975226	4.22	.024774	38
23	.836878	2.23	.861400	1.99	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.99	.975732	4.22	.024268	36
25	.837146	2.22	.861161	1.99	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.99	.976238	4.22	.023762	34
27	.837412	2.22	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	9.837812		9.860562		9.977250		0.022750	30
31	.837945	2.22	.860442	2.00	.977503	4.22	.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.21	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.21	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.21	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.21	.859842	2.01	.978768	4.22	.021232	24
37	.838742	2.21	.859721	2.01	.979021	4.22	.020979	23
38	.838875	2.21	.859601	2.01	.979274	4.22	.020726	22
39	.839007	2.21	.859480	2.01	.979527	4.22	.020473	21
40	9.839140		9.859360		9.979780		0.020220	20
41	.839272	2.20	.859239	2.01	.980033	4.22	.019967	19
42	.839404	2.20	.859119	2.01	.980286	4.22	.019714	18
43	.839536	2.20	.858998	2.01	.980539	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.21	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.21	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.21	.018450	13
48	.840196	2.19	.858393	2.02	.981803	4.21	.018197	12
49	.840328	2.19	.858272	2.02	.982056	4.21	.017944	11
50	9.840459		9.858151		9.982309		0.017691	10
51	.840591	2.19	.858029	2.02	.982562	4.21	.017438	9
52	.840722	2.19	.857908	2.02	.982814	4.21	.017186	8
53	.840854	2.19	.857786	2.03	.983067	4.21	.016933	7
54	.840985	2.19	.857665	2.03	.983320	4.21	.016680	6
55	.841116	2.19	.857543	2.03	.983573	4.21	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.21	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.21	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.21	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.21	.015416	1
60	.841771		.856934		.984837			
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1		

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.841771	2.18	9.856934	2.03	9.981837	4.21	0.015163
1	.841902	2.18	.856812	2.01	.985090	4.21	.014910
2	.842033	2.18	.856690	2.04	.985343	4.21	.014657
3	.842163	2.18	.856563	2.04	.985596	4.21	.014404
4	.842294	2.17	.856416	2.04	.985848	4.21	.014152
5	.842424	2.17	.856323	2.04	.986101	4.21	.013899
6	.842555	2.17	.856201	2.04	.986354	4.21	.013646
7	.842685	2.17	.856078	2.04	.986607	4.21	.013393
8	.842815	2.17	.855956	2.04	.986860	4.21	.013140
9	.842946	2.17	.855833	2.04	.987112	4.21	.012888
10	9.843076	2.17	9.855711	2.05	9.987365	4.21	0.012635
11	.843206	2.17	.855593	2.05	.987618	4.21	.012382
12	.843336	2.16	.855465	2.05	.987871	4.21	.012129
13	.843466	2.16	.855342	2.05	.988123	4.21	.011877
14	.843595	2.16	.855219	2.05	.988376	4.21	.011624
15	.843725	2.16	.855096	2.05	.988629	4.21	.011371
16	.843855	2.16	.854973	2.05	.988882	4.21	.011119
17	.843984	2.16	.854850	2.05	.989134	4.21	.010866
18	.844114	2.16	.854727	2.06	.989387	4.21	.010613
19	.844243	2.16	.854603	2.06	.989640	4.21	.010360
20	9.844372	2.15	9.854440	2.06	9.989893	4.21	0.010107
21	.844502	2.15	.854356	2.06	.990145	4.21	.009855
22	.844631	2.15	.854233	2.06	.990393	4.21	.009602
23	.844760	2.15	.854109	2.06	.990651	4.21	.009349
24	.844889	2.15	.853986	2.06	.990903	4.21	.009097
25	.845018	2.15	.853862	2.06	.991156	4.21	.008844
26	.845147	2.15	.853738	2.06	.991409	4.21	.008591
27	.845276	2.15	.853614	2.06	.991662	4.21	.008338
28	.845405	2.14	.853490	2.07	.991914	4.21	.008086
29	.845533	2.14	.853366	2.07	.992167	4.21	.007833
30	9.845662	2.14	9.853242	2.07	9.992420	4.21	0.007580
31	.845790	2.14	.853118	2.07	.992672	4.21	.007328
32	.845919	2.14	.852994	2.07	.992925	4.21	.007075
33	.846047	2.14	.852869	2.07	.993178	4.21	.006822
34	.846175	2.14	.852745	2.07	.993431	4.21	.006569
35	.846304	2.14	.852620	2.07	.993683	4.21	.006317
36	.846432	2.14	.852496	2.08	.993936	4.21	.006064
37	.846560	2.13	.852371	2.08	.994189	4.21	.005811
38	.846688	2.13	.852247	2.08	.994441	4.21	.005559
39	.846816	2.13	.852122	2.08	.994694	4.21	.005306
40	9.846941	2.13	9.851997	2.08	9.994947	4.21	0.005053
41	.847071	2.13	.851972	2.08	.995199	4.21	.004801
42	.847199	2.13	.851747	2.08	.995452	4.21	.004548
43	.847327	2.13	.851622	2.09	.995705	4.21	.004295
44	.847454	2.12	.851497	2.09	.995957	4.21	.004043
45	.847582	2.12	.851372	2.09	.996210	4.21	.003790
46	.847709	2.12	.851246	2.09	.996463	4.21	.003537
47	.847836	2.12	.851121	2.09	.996715	4.21	.003285
48	.847964	2.12	.850996	2.09	.996968	4.21	.003032
49	.848091	2.12	.850870	2.09	.997221	4.21	.002779
50	9.848218	2.12	9.850745	2.09	9.997473	4.21	0.002527
51	.848345	2.12	.850619	2.10	.997726	4.21	.002274
52	.848472	2.11	.850493	2.10	.997979	4.21	.002021
53	.848599	2.11	.850368	2.10	.998231	4.21	.001769
54	.848726	2.11	.850242	2.10	.998484	4.21	.001516
55	.848852	2.11	.850116	2.10	.998737	4.21	.001263
56	.848979	2.11	.849990	2.10	.998989	4.21	.001011
57	.849106	2.11	.849864	2.10	.999242	4.21	.000758
58	.849232	2.11	.849738	2.10	.999495	4.21	.000505
59	.849359	2.11	.849611	2.11	.999747	4.21	.000253
60	.849485		.849485		0.000000		.000000
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.

TABLE XIV.

TURAL SINES AND COSINES

0		1		2		3		4		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.	
0	0.0000	One.	0.0175	99.985	0.0350	99.939	0.0524	99.963	0.0697	99.936
1	0.0129	One.	0.0177	99.984	0.0519	99.938	0.0526	99.961	0.0700	99.935
2	0.0158	One.	0.0180	99.983	0.0548	99.937	0.0529	99.960	0.0703	99.934
3	0.0187	One.	0.0182	99.983	0.0577	99.936	0.0532	99.958	0.0706	99.933
4	0.0216	One.	0.0186	99.983	0.0606	99.935	0.0535	99.957	0.0709	99.932
5	0.0245	One.	0.0189	99.982	0.0635	99.934	0.0539	99.955	0.0712	99.931
6	0.0275	One.	0.0192	99.982	0.0664	99.933	0.0543	99.954	0.0715	99.930
7	0.0304	One.	0.0195	99.981	0.0693	99.932	0.0547	99.952	0.0718	99.929
8	0.0333	One.	0.0197	99.980	0.0722	99.931	0.0551	99.951	0.0721	99.928
9	0.0362	One.	0.0200	99.979	0.0751	99.930	0.0555	99.949	0.0724	99.927
10	0.0391	One.	0.0203	99.978	0.0780	99.929	0.0559	99.947	0.0727	99.926
11	0.0420	99.999	0.0206	99.977	0.0809	99.928	0.0563	99.946	0.0730	99.925
12	0.0449	99.998	0.0209	99.976	0.0838	99.926	0.0567	99.944	0.0733	99.924
13	0.0478	99.997	0.0212	99.975	0.0867	99.925	0.0571	99.942	0.0736	99.923
14	0.0507	99.996	0.0215	99.974	0.0896	99.924	0.0575	99.941	0.0739	99.922
15	0.0536	99.995	0.0218	99.973	0.0925	99.923	0.0579	99.939	0.0742	99.921
16	0.0565	99.994	0.0221	99.972	0.0954	99.922	0.0583	99.938	0.0745	99.920
17	0.0594	99.993	0.0224	99.971	0.0983	99.921	0.0587	99.936	0.0748	99.919
18	0.0623	99.992	0.0227	99.970	0.1012	99.919	0.0591	99.934	0.0751	99.918
19	0.0652	99.991	0.0230	99.969	0.1041	99.918	0.0595	99.933	0.0754	99.917
20	0.0681	99.990	0.0233	99.968	0.1070	99.917	0.0599	99.931	0.0757	99.916
21	0.0710	99.989	0.0236	99.967	0.1100	99.916	0.0603	99.929	0.0760	99.915
22	0.0739	99.988	0.0239	99.966	0.1129	99.915	0.0607	99.927	0.0763	99.914
23	0.0768	99.987	0.0242	99.965	0.1158	99.913	0.0611	99.926	0.0766	99.913
24	0.0797	99.986	0.0245	99.964	0.1187	99.912	0.0615	99.924	0.0769	99.912
25	0.0826	99.985	0.0248	99.963	0.1217	99.911	0.0619	99.922	0.0772	99.911
26	0.0855	99.984	0.0251	99.962	0.1246	99.910	0.0623	99.921	0.0775	99.910
27	0.0884	99.983	0.0254	99.961	0.1275	99.909	0.0627	99.919	0.0778	99.909
28	0.0913	99.982	0.0257	99.960	0.1304	99.907	0.0631	99.917	0.0781	99.908
29	0.0942	99.981	0.0260	99.959	0.1333	99.906	0.0635	99.915	0.0784	99.907
30	0.0971	99.980	0.0263	99.958	0.1362	99.905	0.0639	99.913	0.0787	99.906
31	0.1000	99.979	0.0266	99.957	0.1391	99.904	0.0643	99.912	0.0790	99.905
32	0.1029	99.978	0.0269	99.956	0.1420	99.902	0.0647	99.910	0.0793	99.904
33	0.1058	99.977	0.0272	99.955	0.1449	99.901	0.0651	99.908	0.0796	99.903
34	0.1087	99.976	0.0275	99.954	0.1478	99.900	0.0655	99.906	0.0799	99.902
35	0.1116	99.975	0.0278	99.953	0.1507	99.898	0.0659	99.904	0.0802	99.901
36	0.1145	99.974	0.0281	99.952	0.1536	99.897	0.0663	99.903	0.0805	99.900
37	0.1174	99.973	0.0284	99.951	0.1565	99.896	0.0667	99.901	0.0808	99.899
38	0.1203	99.972	0.0287	99.950	0.1594	99.894	0.0671	99.899	0.0811	99.898
39	0.1232	99.971	0.0290	99.949	0.1623	99.893	0.0675	99.897	0.0814	99.897
40	0.1261	99.970	0.0293	99.948	0.1652	99.892	0.0679	99.895	0.0817	99.896
41	0.1290	99.969	0.0296	99.947	0.1681	99.890	0.0683	99.893	0.0820	99.895
42	0.1319	99.968	0.0299	99.946	0.1710	99.889	0.0687	99.892	0.0823	99.894
43	0.1348	99.967	0.0302	99.945	0.1739	99.887	0.0691	99.890	0.0826	99.893
44	0.1377	99.966	0.0305	99.944	0.1768	99.886	0.0695	99.889	0.0829	99.892
45	0.1406	99.965	0.0308	99.943	0.1797	99.885	0.0699	99.887	0.0832	99.891
46	0.1435	99.964	0.0311	99.942	0.1826	99.884	0.0703	99.886	0.0835	99.890
47	0.1464	99.963	0.0314	99.941	0.1855	99.883	0.0707	99.884	0.0838	99.889
48	0.1493	99.962	0.0317	99.940	0.1884	99.882	0.0711	99.883	0.0841	99.888
49	0.1522	99.961	0.0320	99.939	0.1913	99.881	0.0715	99.881	0.0844	99.887
50	0.1551	99.960	0.0323	99.938	0.1942	99.880	0.0719	99.880	0.0847	99.886
51	0.1580	99.959	0.0326	99.937	0.1971	99.879	0.0723	99.879	0.0850	99.885
52	0.1609	99.958	0.0329	99.936	0.2000	99.878	0.0727	99.878	0.0853	99.884
53	0.1638	99.957	0.0332	99.935	0.2029	99.877	0.0731	99.877	0.0856	99.883
54	0.1667	99.956	0.0335	99.934	0.2058	99.876	0.0735	99.876	0.0859	99.882
55	0.1696	99.955	0.0338	99.933	0.2087	99.875	0.0739	99.875	0.0862	99.881
56	0.1725	99.954	0.0341	99.932	0.2116	99.874	0.0743	99.874	0.0865	99.880
57	0.1754	99.953	0.0344	99.931	0.2145	99.873	0.0747	99.873	0.0868	99.879
58	0.1783	99.952	0.0347	99.930	0.2174	99.872	0.0751	99.872	0.0871	99.878
59	0.1812	99.951	0.0350	99.929	0.2203	99.871	0.0755	99.871	0.0874	99.877
60	0.1841	99.950	0.0353	99.928	0.2232	99.870	0.0759	99.870	0.0877	99.876
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.	

5°		6°		7°		8°		9°	
Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.
1.08716	.99619	1.0153	.99152	1.2187	.99255	1.3917	.99027	1.5613	.98769
1.08745	.99617	1.0182	.99145	1.2216	.99251	1.3946	.99023	1.5672	.98764
1.08771	.99614	1.0511	.99146	1.2245	.99248	1.3975	.99019	1.5701	.98760
1.08803	.99612	1.0540	.99143	1.2274	.99244	1.4004	.99015	1.5730	.98755
1.08831	.99609	1.0569	.99140	1.2302	.99240	1.4033	.99011	1.5758	.98751
1.08861	.99607	1.0597	.99137	1.2331	.99237	1.4061	.99006	1.5787	.98746
1.08889	.99604	1.0626	.99134	1.2360	.99233	1.4090	.99002	1.5816	.98741
1.08918	.99602	1.0655	.99131	1.2389	.99230	1.4119	.98998	1.5845	.98737
1.08947	.99599	1.0684	.99128	1.2418	.99226	1.4148	.98994	1.5873	.98732
1.08976	.99596	1.0713	.99124	1.2447	.99222	1.4177	.98990	1.5902	.98728
1.09005	.99594	1.0742	.99121	1.2476	.99219	1.4205	.98986	1.5931	.98723
1.09031	.99591	1.0771	.99118	1.2504	.99215	1.4234	.98982	1.5959	.98718
1.09063	.99588	1.0800	.99115	1.2533	.99211	1.4263	.98978	1.5988	.98714
1.09092	.99586	1.0829	.99112	1.2562	.99208	1.4292	.98973	1.6017	.98709
1.09121	.99583	1.0858	.99109	1.2591	.99204	1.4320	.98969	1.6046	.98704
1.09150	.99580	1.0887	.99106	1.2620	.99200	1.4349	.98965	1.6074	.98700
1.09179	.99578	1.0916	.99102	1.2649	.99197	1.4378	.98961	1.6103	.98695
1.09208	.99575	1.0945	.99099	1.2678	.99193	1.4407	.98957	1.6132	.98690
1.09237	.99572	1.0973	.99096	1.2706	.99189	1.4436	.98953	1.6160	.98686
1.09266	.99570	1.1002	.99093	1.2735	.99186	1.4464	.98949	1.6189	.98681
1.09295	.99567	1.1031	.99090	1.2764	.99182	1.4493	.98944	1.6218	.98676
1.09324	.99564	1.1060	.99086	1.2793	.99178	1.4522	.98940	1.6246	.98671
1.09353	.99562	1.1089	.99083	1.2822	.99175	1.4551	.98936	1.6275	.98667
1.09382	.99559	1.1118	.99080	1.2851	.99171	1.4580	.98931	1.6304	.98662
1.09411	.99556	1.1147	.99077	1.2880	.99167	1.4608	.98927	1.6333	.98657
1.09440	.99553	1.1176	.99074	1.2909	.99163	1.4637	.98923	1.6361	.98652
1.09469	.99551	1.1205	.99070	1.2937	.99160	1.4666	.98919	1.6390	.98648
1.09498	.99548	1.1234	.99067	1.2966	.99156	1.4695	.98914	1.6419	.98643
1.09527	.99545	1.1263	.99064	1.2995	.99152	1.4723	.98910	1.6447	.98638
1.09556	.99542	1.1291	.99060	1.3024	.99148	1.4752	.98906	1.6476	.98633
1.09585	.99540	1.1320	.99057	1.3053	.99144	1.4781	.98902	1.6505	.98629
1.09614	.99537	1.1349	.99054	1.3081	.99141	1.4810	.98897	1.6533	.98624
1.09642	.99534	1.1378	.99051	1.3110	.99137	1.4838	.98893	1.6562	.98619
1.09671	.99531	1.1407	.99048	1.3139	.99133	1.4867	.98889	1.6591	.98614
1.09700	.99528	1.1436	.99044	1.3168	.99129	1.4896	.98884	1.6620	.98609
1.09729	.99526	1.1465	.99041	1.3197	.99125	1.4925	.98880	1.6648	.98604
1.09758	.99523	1.1494	.99037	1.3226	.99122	1.4954	.98876	1.6677	.98600
1.09787	.99520	1.1523	.99034	1.3254	.99118	1.4982	.98871	1.6706	.98595
1.09816	.99517	1.1552	.99031	1.3283	.99114	1.5011	.98867	1.6734	.98590
1.09845	.99514	1.1580	.99027	1.3312	.99110	1.5040	.98863	1.6763	.98585
1.09874	.99511	1.1609	.99024	1.3341	.99106	1.5069	.98858	1.6792	.98581
1.09903	.99508	1.1638	.99020	1.3370	.99102	1.5097	.98854	1.6820	.98575
1.09932	.99506	1.1667	.99017	1.3399	.99098	1.5126	.98850	1.6849	.98570
1.09961	.99503	1.1696	.99014	1.3427	.99094	1.5155	.98845	1.6878	.98565
1.09990	.99501	1.1725	.99010	1.3456	.99091	1.5184	.98841	1.6906	.98561
1.10019	.99497	1.1754	.99007	1.3485	.99087	1.5212	.98836	1.6935	.98556
1.10048	.99494	1.1783	.99003	1.3514	.99083	1.5241	.98832	1.6964	.98551
1.10077	.99491	1.1812	.99000	1.3543	.99079	1.5270	.98827	1.6992	.98546
1.10106	.99488	1.1840	.99007	1.3572	.99075	1.5299	.98823	1.7021	.98541
1.10135	.99485	1.1869	.99003	1.3600	.99071	1.5327	.98818	1.7050	.98536
1.10164	.99482	1.1898	.99000	1.3629	.99067	1.5356	.98814	1.7078	.98531
1.10192	.99479	1.1927	.99006	1.3658	.99063	1.5385	.98809	1.7107	.98526
1.10221	.99476	1.1956	.99003	1.3687	.99059	1.5414	.98805	1.7136	.98521
1.10250	.99473	1.1985	.99007	1.3716	.99055	1.5442	.98800	1.7164	.98516
1.10279	.99470	1.2014	.99004	1.3744	.99051	1.5471	.98796	1.7193	.98511
1.10308	.99467	1.2043	.99000	1.3773	.99047	1.5500	.98791	1.7222	.98506
1.10337	.99464	1.2071	.99000	1.3802	.99043	1.5529	.98787	1.7250	.98501
1.10366	.99461	1.2100	.99005	1.3831	.99039	1.5557	.98782	1.7279	.98496
1.10395	.99458	1.2129	.99002	1.3860	.99035	1.5586	.98778	1.7308	.98491
1.10424	.99455	1.2158	.99005	1.3889	.99031	1.5615	.98773	1.7336	.98486
1.10453	.99452	1.2187	.99000	1.3917	.99027	1.5643	.98769	1.7365	.98481
Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.
81°		83°		85°		81°		80°	

M.	10°		11°		12°		13°		14°	
	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine
0	.17365	.98481	.19138	.98163	.20791	.97815	.22435	.97437	.24192	.96957
1	.17393	.98476	.19169	.98157	.20820	.97809	.22463	.97430	.24220	.96950
2	.17422	.98471	.19198	.98152	.20848	.97803	.22491	.97424	.24249	.96944
3	.17451	.98466	.19227	.98146	.20877	.97797	.22519	.97417	.24277	.96938
4	.17479	.98461	.19255	.98140	.20905	.97791	.22546	.97411	.24305	.96932
5	.17508	.98455	.19284	.98135	.20933	.97784	.22573	.97404	.24333	.96926
6	.17536	.98450	.19312	.98129	.20962	.97778	.22601	.97398	.24362	.96920
7	.17565	.98445	.19341	.98124	.20990	.97772	.22628	.97391	.24390	.96914
8	.17594	.98440	.19369	.98118	.21019	.97766	.22656	.97384	.24418	.96908
9	.17623	.98435	.19398	.98112	.21047	.97760	.22683	.97378	.24446	.96902
10	.17651	.98430	.19426	.98107	.21076	.97754	.22711	.97371	.24474	.96896
11	.17680	.98425	.19455	.98101	.21104	.97748	.22738	.97365	.24502	.96890
12	.17708	.98420	.19483	.98096	.21132	.97742	.22766	.97358	.24530	.96884
13	.17737	.98414	.19512	.98090	.21161	.97735	.22793	.97352	.24558	.96878
14	.17766	.98409	.19540	.98084	.21189	.97729	.22821	.97345	.24586	.96872
15	.17794	.98404	.19569	.98079	.21218	.97723	.22848	.97339	.24614	.96866
16	.17823	.98399	.19597	.98073	.21246	.97717	.22876	.97332	.24642	.96860
17	.17852	.98394	.19626	.98067	.21275	.97711	.22903	.97326	.24670	.96854
18	.17880	.98389	.19654	.98061	.21303	.97705	.22931	.97319	.24698	.96848
19	.17909	.98383	.19683	.98056	.21331	.97699	.22958	.97313	.24726	.96842
20	.17937	.98378	.19711	.98050	.21360	.97692	.22986	.97306	.24754	.96836
21	.17966	.98373	.19740	.98044	.21388	.97686	.23013	.97300	.24782	.96830
22	.17995	.98368	.19768	.98039	.21417	.97680	.23041	.97293	.24810	.96824
23	.18023	.98362	.19797	.98033	.21445	.97673	.23068	.97287	.24838	.96818
24	.18052	.98357	.19825	.98027	.21474	.97667	.23096	.97280	.24866	.96812
25	.18081	.98352	.19854	.98021	.21502	.97661	.23123	.97274	.24894	.96806
26	.18109	.98347	.19882	.98016	.21530	.97655	.23151	.97267	.24922	.96800
27	.18138	.98341	.19911	.98010	.21559	.97649	.23178	.97261	.24950	.96794
28	.18166	.98336	.19939	.98004	.21587	.97642	.23206	.97254	.24978	.96788
29	.18195	.98331	.19968	.97998	.21616	.97636	.23233	.97248	.25006	.96782
30	.18224	.98325	.19997	.97992	.21644	.97630	.23261	.97241	.25034	.96776
31	.18252	.98320	.20025	.97987	.21672	.97623	.23288	.97235	.25062	.96770
32	.18281	.98315	.20054	.97981	.21701	.97617	.23316	.97228	.25090	.96764
33	.18309	.98310	.20082	.97975	.21729	.97611	.23343	.97222	.25118	.96758
34	.18338	.98304	.20111	.97969	.21758	.97604	.23371	.97215	.25146	.96752
35	.18367	.98299	.20139	.97963	.21786	.97598	.23398	.97209	.25174	.96746
36	.18395	.98294	.20168	.97957	.21814	.97592	.23426	.97202	.25202	.96740
37	.18424	.98288	.20196	.97952	.21843	.97585	.23453	.97196	.25230	.96734
38	.18452	.98283	.20225	.97946	.21871	.97579	.23481	.97189	.25258	.96728
39	.18481	.98277	.20253	.97940	.21899	.97573	.23508	.97183	.25286	.96722
40	.18509	.98272	.20282	.97934	.21928	.97566	.23536	.97176	.25314	.96716
41	.18538	.98267	.20310	.97928	.21956	.97560	.23563	.97170	.25342	.96710
42	.18567	.98261	.20339	.97922	.21985	.97553	.23591	.97163	.25370	.96704
43	.18595	.98256	.20367	.97916	.22013	.97547	.23618	.97157	.25398	.96698
44	.18624	.98250	.20396	.97910	.22041	.97541	.23646	.97150	.25426	.96692
45	.18652	.98245	.20424	.97905	.22070	.97534	.23673	.97144	.25454	.96686
46	.18681	.98240	.20453	.97899	.22098	.97528	.23701	.97137	.25482	.96680
47	.18710	.98234	.20481	.97893	.22126	.97522	.23728	.97131	.25510	.96674
48	.18738	.98229	.20510	.97887	.22155	.97515	.23756	.97124	.25538	.96668
49	.18767	.98223	.20538	.97881	.22183	.97509	.23783	.97118	.25566	.96662
50	.18795	.98218	.20567	.97875	.22212	.97502	.23811	.97111	.25594	.96656
51	.18824	.98212	.20595	.97869	.22240	.97496	.23838	.97105	.25622	.96650
52	.18852	.98207	.20624	.97863	.22268	.97489	.23866	.97098	.25650	.96644
53	.18881	.98201	.20652	.97857	.22297	.97483	.23893	.97092	.25678	.96638
54	.18910	.98196	.20680	.97851	.22325	.97476	.23921	.97085	.25706	.96632
55	.18938	.98191	.20709	.97845	.22353	.97470	.23948	.97079	.25734	.96626
56	.18967	.98185	.20737	.97839	.22382	.97463	.23976	.97072	.25762	.96620
57	.18995	.98179	.20766	.97833	.22410	.97457	.24003	.97066	.25790	.96614
58	.19024	.98174	.20794	.97827	.22438	.97450	.24031	.97059	.25818	.96608
59	.19052	.98168	.20823	.97821	.22467	.97444	.24058	.97053	.25846	.96602
60	.19081	.98163	.20851	.97815	.22495	.97437	.24086	.97046	.25874	.96596
M.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.
	79°		78°		77°		76°		75°	

TABLE XIV. NATURAL SINES AND COSINES.

223

15°		16°		17°		18°		19°		M.
ne.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
882	.96593	.27561	.96126	.27237	.95630	.26902	.95106	.26557	.94552	60
910	.96535	.27592	.96118	.27265	.95622	.26929	.95097	.26584	.94542	59
938	.96578	.27620	.96110	.27293	.95613	.26957	.95088	.26612	.94533	58
966	.96650	.27618	.96102	.27321	.95605	.26985	.95079	.26639	.94523	57
994	.96692	.27676	.96094	.27318	.95596	.27012	.95070	.26667	.94511	56
1022	.96655	.27704	.96086	.27376	.95588	.27040	.95061	.26694	.94504	55
1050	.96617	.27731	.96078	.27404	.95579	.27068	.95052	.26722	.94495	54
1079	.96580	.27759	.96070	.27432	.95571	.27095	.95043	.26749	.94486	53
1107	.96532	.27787	.96062	.27460	.95562	.27123	.95034	.26777	.94476	52
1135	.96524	.27815	.96054	.27487	.95551	.27151	.95024	.26804	.94466	51
1163	.96517	.27843	.96046	.27515	.95545	.27178	.95015	.26832	.94457	50
1191	.96509	.27871	.96037	.27543	.95536	.27206	.95006	.26859	.94447	49
1219	.96502	.27899	.96029	.27571	.95528	.27233	.94997	.26887	.94438	48
1247	.96494	.27927	.96021	.27599	.95519	.27261	.94988	.26914	.94428	47
1275	.96486	.27955	.96013	.27626	.95511	.27289	.94979	.26942	.94418	46
1303	.96479	.27983	.96005	.27654	.95502	.27316	.94970	.26969	.94409	45
1331	.96471	.28011	.95997	.27682	.95493	.27344	.94961	.26997	.94399	44
1359	.96463	.28039	.95989	.27710	.95485	.27372	.94952	.27024	.94390	43
1387	.96456	.28067	.95981	.27737	.95476	.27399	.94943	.27051	.94380	42
1415	.96448	.28095	.95972	.27765	.95467	.27427	.94933	.27079	.94371	41
1443	.96440	.28123	.95964	.27793	.95459	.27454	.94924	.27106	.94361	40
1471	.96433	.28150	.95956	.27821	.95450	.27482	.94915	.27134	.94351	39
1500	.96425	.28178	.95948	.27849	.95441	.27510	.94906	.27161	.94342	38
1528	.96417	.28206	.95940	.27876	.95433	.27537	.94897	.27189	.94332	37
1556	.96410	.28234	.95931	.27904	.95424	.27565	.94888	.27216	.94322	36
1584	.96402	.28262	.95923	.27932	.95415	.27593	.94879	.27244	.94313	35
1612	.96394	.28290	.95915	.27960	.95407	.27620	.94869	.27271	.94303	34
1640	.96386	.28318	.95907	.27987	.95398	.27648	.94860	.27299	.94293	33
1668	.96379	.28346	.95898	.28015	.95389	.27675	.94851	.27326	.94284	32
1696	.96371	.28374	.95890	.28043	.95380	.27703	.94842	.27353	.94274	31
1724	.96363	.28402	.95882	.28071	.95372	.27730	.94832	.27381	.94264	30
1752	.96355	.28429	.95874	.28098	.95363	.27758	.94823	.27408	.94254	29
1780	.96347	.28457	.95865	.28126	.95354	.27786	.94814	.27436	.94245	28
1808	.96340	.28485	.95857	.28154	.95345	.27813	.94805	.27463	.94235	27
1836	.96332	.28513	.95849	.28182	.95337	.27841	.94795	.27490	.94225	26
1864	.96324	.28541	.95841	.28209	.95328	.27868	.94786	.27518	.94215	25
1892	.96316	.28569	.95832	.28237	.95319	.27896	.94777	.27545	.94206	24
1920	.96308	.28597	.95824	.28265	.95310	.27923	.94768	.27573	.94196	23
1948	.96301	.28625	.95816	.28292	.95301	.27951	.94758	.27600	.94186	22
1976	.96293	.28652	.95807	.28320	.95293	.27979	.94749	.27627	.94176	21
2004	.96285	.28680	.95799	.28348	.95284	.28006	.94740	.27655	.94167	20
2032	.96277	.28708	.95791	.28376	.95275	.28034	.94730	.27682	.94157	19
2060	.96269	.28736	.95782	.28403	.95266	.28061	.94721	.27710	.94147	18
2088	.96261	.28764	.95774	.28431	.95257	.28089	.94712	.27737	.94137	17
2116	.96253	.28792	.95766	.28459	.95248	.28116	.94702	.27765	.94127	16
2144	.96246	.28820	.95757	.28486	.95240	.28144	.94693	.27792	.94118	15
2172	.96238	.28847	.95749	.28514	.95231	.28171	.94684	.27819	.94108	14
2200	.96230	.28875	.95740	.28542	.95222	.28199	.94674	.27846	.94098	13
2228	.96222	.28903	.95732	.28570	.95213	.28227	.94665	.27874	.94088	12
2256	.96214	.28931	.95724	.28597	.95204	.28254	.94656	.27901	.94078	11
2284	.96206	.28959	.95715	.28625	.95195	.28282	.94646	.27929	.94068	10
2312	.96198	.28987	.95707	.28653	.95186	.28309	.94637	.27956	.94058	9
2340	.96190	.29015	.95698	.28680	.95177	.28337	.94627	.27983	.94049	8
2368	.96182	.29042	.95690	.28708	.95168	.28364	.94618	.28011	.94039	7
2396	.96174	.29070	.95681	.28736	.95159	.28392	.94609	.28038	.94029	6
2424	.96166	.29098	.95673	.28763	.95150	.28419	.94599	.28065	.94019	5
2452	.96158	.29126	.95664	.28791	.95141	.28447	.94590	.28093	.94009	4
2480	.96150	.29154	.95655	.28819	.95133	.28474	.94580	.28120	.93999	3
2508	.96142	.29182	.95647	.28846	.95124	.28502	.94571	.28147	.93989	2
2536	.96134	.29210	.95639	.28874	.95115	.28529	.94561	.28175	.93979	1
2564	.96126	.29237	.95630	.28902	.95106	.28557	.94552	.28202	.93969	
sin.	Sine.	Cosin.		Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	

74°

71°

M.	20°		21°		22°		23°		24°	
	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine
0	.34202	.93569	.35767	.93358	.37461	.92718	.39073	.92050	.40674	.91355
1	.34229	.93559	.35761	.93348	.37488	.92707	.39100	.92039	.40701	.91345
2	.34257	.93549	.35751	.93337	.37515	.92697	.39127	.92028	.40727	.91335
3	.34284	.93539	.35741	.93327	.37542	.92686	.39153	.92016	.40753	.91325
4	.34311	.93529	.35731	.93316	.37569	.92675	.39180	.92005	.40780	.91315
5	.34339	.93519	.35721	.93306	.37595	.92664	.39207	.91994	.40806	.91305
6	.34366	.93509	.35710	.93295	.37622	.92653	.39234	.91982	.40833	.91295
7	.34393	.93499	.35699	.93285	.37649	.92642	.39260	.91971	.40860	.91285
8	.34421	.93489	.35689	.93274	.37676	.92631	.39287	.91959	.40886	.91275
9	.34448	.93479	.35678	.93264	.37703	.92620	.39314	.91948	.40913	.91265
10	.34475	.93469	.35668	.93253	.37730	.92609	.39341	.91936	.40939	.91255
11	.34503	.93459	.35657	.93243	.37757	.92598	.39367	.91925	.40966	.91245
12	.34530	.93449	.35646	.93232	.37784	.92587	.39394	.91914	.40992	.91235
13	.34557	.93439	.35635	.93222	.37811	.92576	.39421	.91902	.41019	.91225
14	.34584	.93429	.35624	.93211	.37838	.92565	.39448	.91891	.41045	.91215
15	.34612	.93419	.35613	.93201	.37865	.92554	.39474	.91879	.41072	.91205
16	.34639	.93409	.35602	.93190	.37892	.92543	.39501	.91868	.41098	.91195
17	.34666	.93399	.35591	.93180	.37919	.92532	.39528	.91856	.41125	.91185
18	.34693	.93389	.35580	.93169	.37946	.92521	.39555	.91845	.41151	.91175
19	.34721	.93379	.35569	.93159	.37973	.92510	.39581	.91833	.41178	.91165
20	.34748	.93369	.35558	.93148	.37999	.92499	.39608	.91822	.41204	.91155
21	.34775	.93359	.35547	.93137	.38026	.92488	.39635	.91810	.41231	.91145
22	.34803	.93349	.35536	.93127	.38053	.92477	.39661	.91799	.41257	.91135
23	.34830	.93339	.35525	.93116	.38080	.92466	.39688	.91787	.41284	.91125
24	.34857	.93329	.35514	.93106	.38107	.92455	.39715	.91776	.41310	.91115
25	.34884	.93319	.35503	.93095	.38134	.92444	.39741	.91764	.41337	.91105
26	.34912	.93309	.35492	.93084	.38161	.92433	.39768	.91752	.41363	.91095
27	.34939	.93299	.35481	.93074	.38188	.92422	.39795	.91741	.41390	.91085
28	.34966	.93289	.35470	.93063	.38215	.92410	.39822	.91729	.41416	.91075
29	.34993	.93279	.35459	.93052	.38242	.92399	.39848	.91718	.41443	.91065
30	.35021	.93269	.35448	.93042	.38268	.92388	.39875	.91706	.41469	.91055
31	.35048	.93259	.35437	.93031	.38295	.92377	.39902	.91695	.41496	.91045
32	.35075	.93249	.35426	.93020	.38322	.92366	.39928	.91683	.41522	.91035
33	.35102	.93239	.35415	.93010	.38349	.92355	.39955	.91671	.41549	.91025
34	.35130	.93229	.35404	.92999	.38376	.92344	.39982	.91660	.41575	.91015
35	.35157	.93219	.35393	.92988	.38403	.92332	.40008	.91648	.41602	.91005
36	.35184	.93209	.35382	.92977	.38430	.92321	.40035	.91636	.41628	.90995
37	.35211	.93199	.35371	.92967	.38456	.92310	.40062	.91625	.41655	.90985
38	.35239	.93189	.35360	.92956	.38483	.92299	.40088	.91613	.41681	.90975
39	.35266	.93179	.35349	.92945	.38510	.92287	.40115	.91601	.41707	.90965
40	.35293	.93169	.35338	.92935	.38537	.92276	.40141	.91590	.41734	.90955
41	.35320	.93159	.35327	.92924	.38564	.92265	.40168	.91578	.41760	.90945
42	.35347	.93149	.35316	.92913	.38591	.92254	.40195	.91566	.41787	.90935
43	.35375	.93139	.35305	.92902	.38617	.92243	.40221	.91555	.41813	.90925
44	.35402	.93129	.35294	.92892	.38644	.92231	.40248	.91543	.41840	.90915
45	.35429	.93119	.35283	.92881	.38671	.92220	.40275	.91531	.41866	.90905
46	.35456	.93109	.35272	.92870	.38698	.92209	.40301	.91519	.41892	.90895
47	.35484	.93099	.35261	.92859	.38725	.92198	.40328	.91508	.41919	.90885
48	.35511	.93089	.35250	.92849	.38752	.92186	.40355	.91496	.41945	.90875
49	.35538	.93079	.35239	.92838	.38778	.92175	.40381	.91484	.41972	.90865
50	.35565	.93069	.35228	.92827	.38805	.92164	.40408	.91472	.41998	.90855
51	.35592	.93059	.35217	.92816	.38832	.92152	.40434	.91461	.42024	.90845
52	.35619	.93049	.35206	.92805	.38859	.92141	.40461	.91449	.42051	.90835
53	.35647	.93039	.35195	.92794	.38886	.92130	.40488	.91437	.42077	.90825
54	.35674	.93029	.35184	.92783	.38912	.92119	.40514	.91426	.42104	.90815
55	.35701	.93019	.35173	.92773	.38939	.92107	.40541	.91414	.42130	.90805
56	.35728	.93009	.35162	.92762	.38966	.92096	.40567	.91402	.42156	.90795
57	.35755	.92999	.35151	.92751	.38993	.92085	.40594	.91390	.42183	.90785
58	.35782	.92989	.35140	.92740	.39020	.92073	.40621	.91378	.42209	.90775
59	.35809	.92979	.35129	.92729	.39046	.92062	.40647	.91366	.42235	.90765
60	.35837	.92969	.35118	.92718	.39073	.92050	.40674	.91355	.42262	.90755
M.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.	Cosine	Sine.

25°		26°		27°		28°		29°		M.
no.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
2262	.90631	.43437	.89579	.43389	.89101	.46917	.88235	.48181	.87462	69
2288	.90618	.43463	.89537	.43425	.89087	.46973	.88281	.48206	.87448	59
2315	.90605	.43489	.89491	.43451	.89074	.46989	.88297	.48232	.87431	58
2341	.90591	.43516	.89441	.43477	.89061	.47024	.88251	.48257	.87420	57
2367	.90578	.43542	.89388	.43503	.89019	.47050	.88210	.48283	.87406	56
2394	.90569	.43568	.89346	.43529	.89035	.47076	.88226	.48308	.87391	55
2420	.90557	.43594	.89303	.43555	.89021	.47101	.88213	.48331	.87377	54
2446	.90545	.43620	.89260	.43581	.89008	.47127	.88199	.48359	.87363	53
2473	.90532	.43646	.89217	.43607	.88995	.47153	.88185	.48384	.87349	52
2499	.90520	.43672	.89174	.43632	.88981	.47178	.88172	.48410	.87335	51
2525	.90507	.43698	.89132	.43658	.88968	.47204	.88158	.48435	.87321	50
2552	.90495	.43724	.89089	.43684	.88955	.47229	.88144	.48461	.87306	49
2578	.90483	.43751	.89046	.43710	.88942	.47255	.88130	.48486	.87292	48
2604	.90470	.43777	.89003	.43736	.88929	.47281	.88117	.48511	.87278	47
2631	.90458	.43803	.88960	.43762	.88915	.47306	.88103	.48537	.87264	46
2657	.90446	.43829	.88917	.43787	.88902	.47332	.88089	.48562	.87250	45
2683	.90433	.43855	.88874	.43813	.88889	.47358	.88075	.48588	.87235	44
2709	.90421	.43881	.88831	.43839	.88876	.47383	.88062	.48613	.87221	43
2736	.90408	.43907	.88788	.43865	.88862	.47409	.88048	.48638	.87207	42
2762	.90396	.43933	.88745	.43891	.88849	.47434	.88034	.48661	.87193	41
2788	.90383	.43959	.88702	.43917	.88835	.47460	.88020	.48689	.87178	40
2815	.90371	.43985	.88659	.43942	.88822	.47486	.88006	.48711	.87164	39
2841	.90358	.44011	.88616	.43968	.88808	.47511	.87993	.48738	.87150	38
2867	.90346	.44037	.88573	.43994	.88795	.47537	.87979	.48765	.87136	37
2894	.90333	.44063	.88530	.44020	.88782	.47562	.87965	.48790	.87121	36
2920	.90321	.44089	.88487	.44046	.88768	.47588	.87951	.48816	.87107	35
2946	.90309	.44116	.88444	.44072	.88755	.47614	.87937	.48841	.87093	34
2972	.90296	.44142	.88401	.44098	.88741	.47639	.87923	.48866	.87079	33
2999	.90284	.44168	.88358	.44123	.88728	.47665	.87909	.48892	.87064	32
3025	.90271	.44194	.88315	.44149	.88715	.47690	.87896	.48917	.87050	31
3051	.90259	.44220	.88272	.44175	.88701	.47716	.87882	.48942	.87036	30
3077	.90246	.44246	.88229	.44201	.88688	.47741	.87868	.48968	.87021	29
3104	.90233	.44272	.88186	.44227	.88674	.47767	.87854	.48993	.87007	28
3130	.90221	.44298	.88143	.44252	.88661	.47793	.87840	.49018	.86993	27
3156	.90208	.44324	.88100	.44278	.88647	.47818	.87826	.49041	.86978	26
3182	.90196	.44350	.88057	.44304	.88634	.47844	.87812	.49069	.86964	25
3209	.90183	.44376	.88014	.44330	.88620	.47869	.87798	.49091	.86949	24
3235	.90171	.44402	.87971	.44355	.88607	.47895	.87784	.49119	.86935	23
3261	.90158	.44428	.87928	.44381	.88593	.47920	.87770	.49145	.86921	22
3287	.90146	.44454	.87885	.44407	.88580	.47946	.87756	.49170	.86906	21
3313	.90133	.44480	.87842	.44433	.88566	.47971	.87743	.49195	.86892	20
3340	.90120	.44506	.87799	.44458	.88553	.47997	.87729	.49221	.86878	19
3366	.90108	.44532	.87756	.44484	.88539	.48022	.87715	.49246	.86863	18
3392	.90095	.44558	.87713	.44510	.88526	.48048	.87701	.49271	.86849	17
3418	.90082	.44584	.87670	.44536	.88512	.48073	.87687	.49296	.86834	16
3445	.90070	.44610	.87627	.44561	.88499	.48099	.87673	.49322	.86820	15
3471	.90057	.44636	.87584	.44587	.88485	.48124	.87659	.49347	.86805	14
3497	.90045	.44662	.87541	.44613	.88472	.48150	.87645	.49372	.86791	13
3523	.90032	.44688	.87498	.44639	.88458	.48175	.87631	.49397	.86777	12
3549	.90019	.44714	.87455	.44664	.88445	.48201	.87617	.49423	.86762	11
3575	.90007	.44740	.87412	.44690	.88431	.48226	.87603	.49448	.86748	10
3602	.89994	.44766	.87369	.44716	.88417	.48252	.87589	.49473	.86733	9
3628	.89981	.44792	.87326	.44742	.88404	.48277	.87575	.49498	.86719	8
3654	.89968	.44818	.87283	.44767	.88390	.48303	.87561	.49524	.86704	7
3680	.89956	.44844	.87240	.44793	.88377	.48328	.87546	.49549	.86690	6
3706	.89943	.44870	.87197	.44819	.88363	.48354	.87532	.49574	.86675	5
3733	.89930	.44896	.87154	.44844	.88349	.48379	.87518	.49599	.86661	4
3759	.89918	.44922	.87111	.44870	.88336	.48405	.87504	.49624	.86646	3
3785	.89905	.44948	.87068	.44896	.88322	.48430	.87490	.49650	.86632	2
3811	.89892	.44974	.87025	.44922	.88309	.48456	.87476	.49675	.86617	1
3837	.89879	.45000	.86982	.44947	.88295	.48481	.87462	.49700	.86603	0
sin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
64°		63°		62°		61°				

M.	Sine.	D 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.841771	2.18	9.856934	2.03	9.981837	4.21	0.015163	60
1	.811902	2.18	.856312	2.04	.985090	4.21	.014910	59
2	.842033	2.18	.856690	2.04	.985343	4.21	.014657	58
3	.842163	2.18	.856568	2.04	.985596	4.21	.014404	57
4	.842294	2.17	.856446	2.04	.985848	4.21	.014152	56
5	.842424	2.17	.856323	2.04	.986101	4.21	.013899	55
6	.842555	2.17	.856201	2.04	.986354	4.21	.013646	54
7	.842685	2.17	.856078	2.04	.986607	4.21	.013393	53
8	.842815	2.17	.855956	2.04	.986860	4.21	.013140	52
9	.842946	2.17	.855833	2.04	.987112	4.21	.012888	51
10	9.843076	2.17	9.855711	2.05	9.987365	4.21	0.012635	50
11	.843206	2.17	.855588	2.05	.987618	4.21	.012382	49
12	.843336	2.16	.855465	2.05	.987871	4.21	.012129	48
13	.843466	2.16	.855342	2.05	.988123	4.21	.011877	47
14	.843595	2.16	.855219	2.05	.988376	4.21	.011624	46
15	.843725	2.16	.855096	2.05	.988629	4.21	.011371	45
16	.843855	2.16	.854973	2.05	.988882	4.21	.011118	44
17	.843984	2.16	.854850	2.05	.989134	4.21	.010866	43
18	.844114	2.16	.854727	2.06	.989387	4.21	.010613	42
19	.844243	2.16	.854603	2.06	.989640	4.21	.010360	41
20	9.844372	2.15	9.854480	2.06	9.989993	4.21	0.010107	40
21	.844502	2.15	.854356	2.06	.990145	4.21	.009855	39
22	.844631	2.15	.854233	2.06	.990393	4.21	.009602	38
23	.844760	2.15	.854109	2.06	.990651	4.21	.009349	37
24	.844889	2.15	.853986	2.06	.990903	4.21	.009097	36
25	.845018	2.15	.853862	2.06	.991156	4.21	.008844	35
26	.845147	2.15	.853738	2.06	.991409	4.21	.008591	34
27	.845276	2.15	.853614	2.06	.991662	4.21	.008338	33
28	.845405	2.15	.853490	2.07	.991914	4.21	.008086	32
29	.845533	2.14	.853366	2.07	.992167	4.21	.007833	31
30	9.845662	2.14	9.853242	2.07	9.992420	4.21	0.007580	30
31	.845790	2.14	.853118	2.07	.992672	4.21	.007328	29
32	.845919	2.14	.852994	2.07	.992925	4.21	.007075	28
33	.846047	2.14	.852869	2.07	.993178	4.21	.006822	27
34	.846175	2.14	.852745	2.07	.993431	4.21	.006569	26
35	.846304	2.14	.852620	2.07	.993683	4.21	.006317	25
36	.846432	2.14	.852496	2.08	.993936	4.21	.006064	24
37	.846560	2.13	.852371	2.08	.994189	4.21	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.21	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.21	.005306	21
40	9.846944	2.13	9.851997	2.08	9.994947	4.21	0.005053	20
41	.847071	2.13	.851872	2.08	.995199	4.21	.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.21	.004548	18
43	.847327	2.13	.851622	2.09	.995705	4.21	.004295	17
44	.847454	2.13	.851497	2.09	.995957	4.21	.004043	16
45	.847582	2.12	.851372	2.09	.996210	4.21	.003790	15
46	.847709	2.12	.851246	2.09	.996463	4.21	.003537	14
47	.847836	2.12	.851121	2.09	.996715	4.21	.003285	13
48	.847964	2.12	.850996	2.09	.996968	4.21	.003032	12
49	.848091	2.12	.850870	2.09	.997221	4.21	.002779	11
50	9.849218	2.12	9.850745	2.09	9.997473	4.21	0.002527	10
51	.849345	2.12	.850619	2.09	.997726	4.21	.002274	9
52	.849472	2.12	.850493	2.10	.997979	4.21	.002021	8
53	.849599	2.11	.850368	2.10	.998231	4.21	.001769	7
54	.849726	2.11	.850242	2.10	.998484	4.21	.001516	6
55	.849852	2.11	.850116	2.10	.998737	4.21	.001263	5
56	.849979	2.11	.849990	2.10	.998989	4.21	.001011	4
57	.850106	2.11	.849864	2.10	.999242	4.21	.000758	3
58	.849232	2.11	.849738	2.10	.999495	4.21	.000505	2
59	.849359	2.11	.849611	2.10	.999747	4.21	.000253	1
60	.849485	2.11	.849485	2.11	0.000000	4.21	.000000	0
	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

TABLE XIV.

NATURAL SINES AND COSINES

M.	0°		1°		2°		3°		4°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03722	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99732	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00494	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00523	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00552	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99717	41
20	.00581	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00610	.99998	.02356	.99972	.04100	.99916	.05843	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04158	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99968	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02559	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00843	.99996	.02588	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01163	.99993	.02908	.99958	.04652	.99892	.06395	.99795	.08136	.99668	20
41	.01192	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01512	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01541	.99988	.03286	.99946	.05030	.99873	.06772	.99770	.08513	.99637	7
54	.01570	.99988	.03315	.99945	.05059	.99872	.06801	.99768	.08542	.99635	6
55	.01600	.99987	.03344	.99944	.05088	.99870	.06830	.99766	.08571	.99632	5
56	.01629	.99987	.03373	.99943	.05117	.99869	.06859	.99764	.08600	.99630	4
57	.01658	.99986	.03402	.99942	.05146	.99867	.06888	.99762	.08629	.99627	3
58	.01687	.99986	.03431	.99941	.05175	.99866	.06917	.99760	.08658	.99625	2
59	.01716	.99985	.03460	.99940	.05204	.99864	.06946	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
89°		88°		87°		86°		85°			

TABLE XV.

NATURAL TANGENTS AND COTANGENTS.

M.	10°		11°		12°		13°		14°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
0	.17365	.98481	.19031	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24953	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25039	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
79°		78°		77°		76°		75°			

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 231

M.	40		50		60		70		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.06993	14.3007	.08749	11.4301	.10510	9.51435	.12278	8.11135	60
1	.07022	14.2411	.08778	11.3919	.10540	9.4771	.12308	8.12181	59
2	.07051	14.1821	.08807	11.3540	.10569	9.44141	.12338	8.10336	58
3	.07080	14.1235	.08837	11.3163	.10599	9.40515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.36884	.12397	8.06874	56
5	.07139	14.0079	.08895	11.2417	.10657	9.33307	.12426	8.05156	55
6	.07168	13.9507	.08925	11.2048	.10687	9.29724	.12456	8.03448	54
7	.07197	13.8940	.08954	11.1681	.10716	9.26155	.12485	8.01748	53
8	.07227	13.8378	.08983	11.1316	.10746	9.22599	.12515	7.99958	52
9	.07256	13.7821	.09013	11.0954	.10775	9.19058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.15530	.12574	7.95392	50
11	.07314	13.6719	.09071	11.0237	.10834	9.12016	.12603	7.93618	49
12	.07344	13.6174	.09101	10.9882	.10863	9.08516	.12633	7.91852	48
13	.07373	13.5634	.09130	10.9529	.10893	9.05028	.12662	7.89934	47
14	.07402	13.5098	.09159	10.9178	.10922	9.01554	.12692	7.87995	46
15	.07431	13.4566	.09189	10.8829	.10952	8.98093	.12722	7.86061	45
16	.07461	13.4039	.09218	10.8483	.10981	8.94646	.12751	7.84124	44
17	.07490	13.3515	.09247	10.8139	.11011	8.91211	.12781	7.82128	43
18	.07519	13.2996	.09277	10.7797	.11040	8.87789	.12810	7.80182	42
19	.07548	13.2480	.09306	10.7457	.11070	8.84379	.12840	7.78285	41
20	.07578	13.1969	.09335	10.7119	.11099	8.80983	.12869	7.76335	40
21	.07607	13.1461	.09365	10.6783	.11128	8.77598	.12899	7.74425	39
22	.07636	13.0958	.09394	10.6450	.11158	8.74227	.12929	7.72480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.70867	.12958	7.70515	37
24	.07695	12.9962	.09453	10.5789	.11217	8.67520	.12988	7.68597	36
25	.07724	12.9469	.09482	10.5462	.11246	8.64185	.13017	7.66708	35
26	.07753	12.8981	.09511	10.5136	.11276	8.60862	.13047	7.64866	34
27	.07782	12.8496	.09541	10.4813	.11305	8.57551	.13076	7.62982	33
28	.07812	12.8014	.09570	10.4491	.11335	8.54252	.13106	7.61005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.50969	.13136	7.59087	31
30	.07870	12.7062	.09629	10.3854	.11394	8.47696	.13165	7.56957	30
31	.07899	12.6591	.09658	10.3538	.11423	8.44425	.13195	7.54872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.41172	.13224	7.52767	28
33	.07958	12.5660	.09717	10.2913	.11482	8.37931	.13254	7.50647	27
34	.07987	12.5199	.09746	10.2602	.11511	8.34701	.13284	7.48506	26
35	.08017	12.4742	.09776	10.2294	.11541	8.31482	.13313	7.46352	25
36	.08046	12.4288	.09805	10.1988	.11570	8.28275	.13343	7.44165	24
37	.08075	12.3838	.09834	10.1683	.11600	8.25080	.13372	7.41946	23
38	.08104	12.3390	.09864	10.1381	.11629	8.21893	.13402	7.39654	22
39	.08134	12.2946	.09893	10.1080	.11659	8.18718	.13432	7.37369	21
40	.08163	12.2505	.09923	10.0780	.11688	8.15555	.13461	7.35081	20
41	.08192	12.2067	.09952	10.0483	.11718	8.12402	.13491	7.32742	19
42	.08221	12.1632	.09981	10.0187	.11747	8.09260	.13521	7.30366	18
43	.08251	12.1201	.10011	9.98931	.11777	8.06128	.13550	7.27999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.03007	.13580	7.25639	16
45	.08309	12.0346	.10069	9.93101	.11836	8.00000	.13609	7.23176	15
46	.08339	11.9923	.10099	9.90211	.11865	7.97005	.13639	7.20710	14
47	.08368	11.9504	.10128	9.87338	.11895	7.94070	.13669	7.18240	13
48	.08397	11.9087	.10158	9.84482	.11924	7.91195	.13698	7.15765	12
49	.08427	11.8673	.10187	9.81641	.11954	7.88365	.13728	7.13285	11
50	.08456	11.8262	.10216	9.78817	.11983	7.85551	.13758	7.10800	10
51	.08485	11.7853	.10246	9.76009	.12013	7.82746	.13787	7.08310	9
52	.08514	11.7448	.10275	9.73217	.12042	7.80000	.13817	7.05815	8
53	.08544	11.7045	.10305	9.70441	.12072	7.77265	.13846	7.03315	7
54	.08573	11.6645	.10334	9.67680	.12101	7.74555	.13876	7.00810	6
55	.08602	11.6248	.10363	9.64935	.12131	7.71865	.13906	6.98300	5
56	.08632	11.5853	.10393	9.62205	.12160	7.69185	.13935	6.95785	4
57	.08661	11.5461	.10422	9.59490	.12190	7.66520	.13965	6.93265	3
58	.08690	11.5072	.10452	9.56791	.12219	7.63870	.13995	6.90740	2
59	.08720	11.4685	.10481	9.54106	.12249	7.61235	.14024	6.88210	1
60	.08749	11.4301	.10510	9.51436	.12278	7.58615	.14054	6.85675	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.

232 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

M.	8		9		10		11		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.11551	7.11537	.15838	6.31375	.17633	5.67125	.19438	5.14155	4.5
1	.11084	7.10038	.15681	6.30189	.17663	5.66165	.19468	5.13658	5
2	.11113	7.08546	.15898	6.29007	.17633	5.65205	.19498	5.12662	6
3	.11113	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12068	7
4	.11173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	8
5	.11202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	9
6	.11232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09701	10
7	.11262	6.91174	.16047	6.23169	.17843	5.60452	.19649	5.08921	11
8	.11291	6.90718	.16077	6.22003	.17873	5.59511	.19679	5.08139	12
9	.11321	6.89268	.16107	6.20851	.17903	5.58573	.19710	5.07360	13
10	.11351	6.86823	.16137	6.19703	.17933	5.57638	.19740	5.06584	14
11	.11381	6.85385	.16167	6.18559	.17963	5.56706	.19770	5.05809	15
12	.11410	6.83952	.16196	6.17419	.17993	5.55777	.19801	5.05037	16
13	.11440	6.82525	.16226	6.16283	.18023	5.54851	.19831	5.04267	17
14	.11470	6.81101	.16256	6.15151	.18053	5.53927	.19861	5.03499	18
15	.11499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	19
16	.11529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	20
17	.11559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	21
18	.11588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	22
19	.11618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	23
20	.11648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	24
21	.11678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	25
22	.11707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	26
23	.11737	6.78561	.16525	6.05143	.18323	5.45751	.20133	4.96690	27
24	.11767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	28
25	.11796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	29
26	.11826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	30
27	.11856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	31
28	.11886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.11915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	33
30	.11945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	34
31	.11975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	35
32	.12005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	36
33	.12034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	37
34	.12064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	38
35	.12094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	39
36	.12124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	40
37	.12153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	41
38	.12183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	42
39	.12213	6.57339	.17004	5.88111	.18805	5.31778	.20618	4.85013	43
40	.12243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	44
41	.12272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	45
42	.12302	6.53503	.17093	5.85021	.18895	5.29235	.20709	4.82882	46
43	.12332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	47
44	.12362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	48
45	.12391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	49
46	.12421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	50
47	.12451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	51
48	.12481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	52
49	.12511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	53
50	.12540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	54
51	.12570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76596	55
52	.12600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	56
53	.12630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	57
54	.12660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	58
55	.12690	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	59
56	.12719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	60
57	.12749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	1
58	.12779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.12809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	3
60	.12838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	4
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
1	81		80		79		78		

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 233

12°			13°			14°			15°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	21236	4.70463	23087	4.33118	24833	4.01078	26595	3.73205	28365	3.45245	60
1	21236	4.69791	23117	4.32573	24861	4.00582	26626	3.72771	28392	3.44759	59
2	21236	4.69121	23148	4.32003	24886	4.00086	26657	3.72338	28419	3.44273	58
3	21237	4.68452	23179	4.31430	24916	3.99582	26688	3.71907	28446	3.43787	57
4	21237	4.67786	23209	4.30850	24946	3.99079	26719	3.71476	28473	3.43301	56
5	21238	4.67121	23240	4.30291	24976	3.98577	26750	3.71046	28500	3.42815	55
6	21238	4.66458	23271	4.29724	25006	3.98071	26781	3.70616	28527	3.42329	54
7	21239	4.65797	23301	4.29159	25036	3.97562	26812	3.70188	28554	3.41843	53
8	21239	4.65138	23332	4.28595	25067	3.97053	26843	3.69761	28581	3.41357	52
9	21240	4.64480	23363	4.28032	25097	3.96541	26874	3.69335	28608	3.40871	51
10	21240	4.63825	23393	4.27471	25128	3.96035	26905	3.68909	28635	3.40385	50
11	21240	4.63171	23424	4.26911	25158	3.95528	26936	3.68485	28662	3.39899	49
12	21241	4.62518	23455	4.26352	25189	3.95021	26967	3.68061	28689	3.39413	48
13	21241	4.61868	23485	4.25795	25219	3.94513	26998	3.67638	28716	3.38927	47
14	21242	4.61219	23516	4.25239	25250	3.94006	27029	3.67217	28743	3.38441	46
15	21242	4.60572	23547	4.24685	25280	3.93500	27060	3.66796	28770	3.37955	45
16	21243	4.59927	23578	4.24132	25311	3.93271	27091	3.66376	28797	3.37469	44
17	21243	4.59283	23608	4.23580	25342	3.92793	27122	3.65957	28824	3.36983	43
18	21244	4.58641	23639	4.23030	25373	3.92316	27153	3.65538	28851	3.36497	42
19	21244	4.58001	23670	4.22481	25404	3.91839	27184	3.65121	28878	3.36011	41
20	21245	4.57363	23700	4.21933	25435	3.91361	27215	3.64705	28905	3.35525	40
21	21245	4.56726	23731	4.21387	25466	3.90880	27246	3.64289	28932	3.35039	39
22	21245	4.56091	23762	4.20842	25497	3.90417	27277	3.63874	28959	3.34553	38
23	21246	4.55458	23793	4.20299	25528	3.89945	27308	3.63461	28986	3.34067	37
24	21246	4.54828	23823	4.19756	25559	3.89474	27339	3.63048	29013	3.33581	36
25	21247	4.54196	23854	4.19215	25590	3.89004	27370	3.62636	29040	3.33095	35
26	21247	4.53565	23885	4.18675	25621	3.88536	27401	3.62224	29067	3.32609	34
27	21248	4.52935	23916	4.18137	25652	3.88068	27432	3.61814	29094	3.32123	33
28	21248	4.52306	23946	4.17600	25683	3.87601	27463	3.61405	29121	3.31637	32
29	21249	4.51679	23977	4.17064	25714	3.87136	27494	3.60996	29148	3.31151	31
30	21249	4.51071	24008	4.16530	25745	3.86671	27525	3.60588	29175	3.30665	30
31	22200	4.50451	21030	4.15997	25783	3.86208	27561	3.60181	29211	3.30258	29
32	22201	4.49832	21069	4.15461	25824	3.85745	27595	3.59775	29246	3.29851	28
33	22261	4.49215	21100	4.14931	25855	3.85281	27626	3.59370	29281	3.29444	27
34	22292	4.48600	21131	4.14405	25896	3.84824	27658	3.58966	29316	3.29037	26
35	22322	4.47986	21162	4.13879	25937	3.84361	27690	3.58562	29351	3.28630	25
36	22353	4.47374	21193	4.13350	25978	3.83906	27722	3.58160	29386	3.28223	24
37	22383	4.46761	21223	4.12821	26019	3.83449	27753	3.57758	29421	3.27816	23
38	22411	4.46155	21254	4.12290	26060	3.82992	27785	3.57357	29456	3.27409	22
39	22441	4.45548	21285	4.11778	26101	3.82537	27817	3.56955	29491	3.27002	21
40	22475	4.44942	21316	4.11256	26142	3.82083	27849	3.56553	29526	3.26595	20
41	22505	4.44338	21347	4.10736	26183	3.81630	27881	3.56151	29561	3.26188	19
42	22536	4.43735	21377	4.10216	26225	3.81177	27913	3.55749	29596	3.25781	18
43	22567	4.43134	21408	4.09699	26266	3.80726	27945	3.55347	29631	3.25374	17
44	22597	4.42534	21439	4.09182	26307	3.80276	27977	3.54945	29666	3.24967	16
45	22628	4.41936	21470	4.08666	26348	3.79827	28009	3.54543	29701	3.24560	15
46	22658	4.41340	21501	4.08152	26389	3.79378	28041	3.54141	29736	3.24153	14
47	22689	4.40745	21532	4.07639	26430	3.78931	28073	3.53739	29771	3.23746	13
48	22719	4.40152	21562	4.07127	26471	3.78485	28105	3.53337	29806	3.23339	12
49	22750	4.39560	21593	4.06616	26512	3.78040	28137	3.52935	29841	3.22932	11
50	22781	4.38969	21624	4.06107	26553	3.77595	28169	3.52533	29876	3.22525	10
51	22811	4.38381	21655	4.05599	26595	3.77152	28201	3.52131	29911	3.22118	9
52	22842	4.37793	21686	4.05092	26636	3.76709	28233	3.51729	29946	3.21711	8
53	22872	4.37207	21717	4.04585	26677	3.76268	28265	3.51327	29981	3.21304	7
54	22903	4.36623	21747	4.04078	26718	3.75828	28297	3.50925	30016	3.20897	6
55	22934	4.36040	21778	4.03578	26759	3.75388	28329	3.50523	30051	3.20490	5
56	22964	4.35458	21809	4.03076	26800	3.74950	28361	3.50121	30086	3.20083	4
57	22995	4.34879	21840	4.02574	26841	3.74512	28393	3.49719	30121	3.19676	3
58	23026	4.34300	21871	4.02071	26882	3.74075	28425	3.49317	30156	3.19269	2
59	23056	4.33723	21902	4.01576	26923	3.73640	28457	3.48915	30191	3.18862	1
60	23087	4.33148	21933	4.01078	26965	3.73205					0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
77°			76°			75°			74°		

16			17			18			19		
M	Tang.	Cotang.	M	Tang.	Cotang.	M	Tang.	Cotang.	M	Tang.	Cotang.
0	2.0000	1.0000	0	2.0000	1.0000	0	2.0000	1.0000	0	2.0000	1.0000
1	2.0001	0.9999	1	2.0002	0.9998	1	2.0003	0.9997	1	2.0004	0.9996
2	2.0003	0.9997	2	2.0005	0.9995	2	2.0007	0.9993	2	2.0009	0.9991
3	2.0005	0.9995	3	2.0007	0.9993	3	2.0009	0.9991	3	2.0011	0.9989
4	2.0007	0.9993	4	2.0009	0.9989	4	2.0011	0.9987	4	2.0013	0.9985
5	2.0009	0.9991	5	2.0011	0.9987	5	2.0013	0.9985	5	2.0015	0.9983
6	2.0011	0.9989	6	2.0013	0.9985	6	2.0015	0.9983	6	2.0017	0.9981
7	2.0013	0.9987	7	2.0015	0.9983	7	2.0017	0.9981	7	2.0019	0.9979
8	2.0015	0.9985	8	2.0017	0.9981	8	2.0019	0.9979	8	2.0021	0.9977
9	2.0017	0.9983	9	2.0019	0.9979	9	2.0021	0.9977	9	2.0023	0.9975
10	2.0019	0.9981	10	2.0021	0.9977	10	2.0023	0.9975	10	2.0025	0.9973
11	2.0021	0.9979	11	2.0023	0.9975	11	2.0025	0.9973	11	2.0027	0.9971
12	2.0023	0.9977	12	2.0025	0.9973	12	2.0027	0.9971	12	2.0029	0.9969
13	2.0025	0.9975	13	2.0027	0.9971	13	2.0029	0.9969	13	2.0031	0.9967
14	2.0027	0.9973	14	2.0029	0.9969	14	2.0031	0.9967	14	2.0033	0.9965
15	2.0029	0.9971	15	2.0031	0.9967	15	2.0033	0.9965	15	2.0035	0.9963
16	2.0031	0.9969	16	2.0033	0.9967	16	2.0035	0.9965	16	2.0037	0.9963
17	2.0033	0.9967	17	2.0035	0.9963	17	2.0037	0.9961	17	2.0039	0.9959
18	2.0035	0.9965	18	2.0037	0.9959	18	2.0039	0.9957	18	2.0041	0.9955
19	2.0037	0.9963	19	2.0039	0.9953	19	2.0041	0.9951	19	2.0043	0.9949
20	2.0039	0.9961	20	2.0041	0.9949	20	2.0043	0.9947	20	2.0045	0.9945
21	2.0041	0.9959	21	2.0043	0.9945	21	2.0045	0.9943	21	2.0047	0.9941
22	2.0043	0.9957	22	2.0045	0.9941	22	2.0047	0.9939	22	2.0049	0.9937
23	2.0045	0.9955	23	2.0047	0.9937	23	2.0049	0.9935	23	2.0051	0.9933
24	2.0047	0.9953	24	2.0049	0.9933	24	2.0051	0.9931	24	2.0053	0.9929
25	2.0049	0.9951	25	2.0051	0.9929	25	2.0053	0.9927	25	2.0055	0.9925
26	2.0051	0.9949	26	2.0053	0.9925	26	2.0055	0.9923	26	2.0057	0.9921
27	2.0053	0.9947	27	2.0055	0.9921	27	2.0057	0.9919	27	2.0059	0.9917
28	2.0055	0.9945	28	2.0057	0.9917	28	2.0059	0.9915	28	2.0061	0.9913
29	2.0057	0.9943	29	2.0059	0.9913	29	2.0061	0.9911	29	2.0063	0.9909
30	2.0059	0.9941	30	2.0061	0.9909	30	2.0063	0.9907	30	2.0065	0.9905
31	2.0061	0.9939	31	2.0063	0.9905	31	2.0065	0.9903	31	2.0067	0.9901
32	2.0063	0.9937	32	2.0065	0.9901	32	2.0067	0.9899	32	2.0069	0.9897
33	2.0065	0.9935	33	2.0067	0.9897	33	2.0069	0.9895	33	2.0071	0.9893
34	2.0067	0.9933	34	2.0069	0.9893	34	2.0071	0.9891	34	2.0073	0.9889
35	2.0069	0.9931	35	2.0071	0.9889	35	2.0073	0.9887	35	2.0075	0.9885
36	2.0071	0.9929	36	2.0073	0.9885	36	2.0075	0.9883	36	2.0077	0.9881
37	2.0073	0.9927	37	2.0075	0.9881	37	2.0077	0.9879	37	2.0079	0.9877
38	2.0075	0.9925	38	2.0077	0.9877	38	2.0079	0.9875	38	2.0081	0.9873
39	2.0077	0.9923	39	2.0079	0.9873	39	2.0081	0.9871	39	2.0083	0.9869
40	2.0079	0.9921	40	2.0081	0.9869	40	2.0083	0.9867	40	2.0085	0.9865
41	2.0081	0.9919	41	2.0083	0.9865	41	2.0085	0.9863	41	2.0087	0.9861
42	2.0083	0.9917	42	2.0085	0.9861	42	2.0087	0.9859	42	2.0089	0.9857
43	2.0085	0.9915	43	2.0087	0.9857	43	2.0089	0.9855	43	2.0091	0.9853
44	2.0087	0.9913	44	2.0089	0.9853	44	2.0091	0.9851	44	2.0093	0.9849
45	2.0089	0.9911	45	2.0091	0.9849	45	2.0093	0.9847	45	2.0095	0.9845
46	2.0091	0.9909	46	2.0093	0.9845	46	2.0095	0.9843	46	2.0097	0.9841
47	2.0093	0.9907	47	2.0095	0.9841	47	2.0097	0.9839	47	2.0099	0.9837
48	2.0095	0.9905	48	2.0097	0.9837	48	2.0099	0.9835	48	2.0101	0.9833
49	2.0097	0.9903	49	2.0099	0.9833	49	2.0101	0.9831	49	2.0103	0.9829
50	2.0099	0.9901	50	2.0101	0.9829	50	2.0103	0.9827	50	2.0105	0.9825
51	2.0101	0.9899	51	2.0103	0.9825	51	2.0105	0.9823	51	2.0107	0.9821
52	2.0103	0.9897	52	2.0105	0.9821	52	2.0107	0.9819	52	2.0109	0.9817
53	2.0105	0.9895	53	2.0107	0.9817	53	2.0109	0.9815	53	2.0111	0.9813
54	2.0107	0.9893	54	2.0109	0.9813	54	2.0111	0.9811	54	2.0113	0.9809
55	2.0109	0.9891	55	2.0111	0.9809	55	2.0113	0.9807	55	2.0115	0.9805
56	2.0111	0.9889	56	2.0113	0.9805	56	2.0115	0.9803	56	2.0117	0.9801
57	2.0113	0.9887	57	2.0115	0.9801	57	2.0117	0.9799	57	2.0119	0.9797
58	2.0115	0.9885	58	2.0117	0.9797	58	2.0119	0.9795	58	2.0121	0.9793
59	2.0117	0.9883	59	2.0119	0.9793	59	2.0121	0.9791	59	2.0123	0.9789
60	2.0119	0.9881	60	2.0121	0.9789	60	2.0123	0.9787	60	2.0125	0.9785

M. Cotang. Tang.

Cotang. Tang.

Cotang. Tang.

Cotang. Tang. M.

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 231

M.	40		50		60		70		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.06993	14.3007	.08749	11.4301	.10510	9.51435	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02844	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18025	.12662	7.89734	47
14	.07402	13.5095	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0468	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9982	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9499	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49125	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16399	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	85°		84°		83°		82°		

236 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

24°		25°		26°		27°	
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.41523	2.21604	.46631	2.14451	.48773	2.05030	.50953
1	.44558	2.21128	.46666	2.14288	.48809	2.04579	.50989
2	.44563	2.21252	.46702	2.14125	.48845	2.04123	.51026
3	.44627	2.21077	.46737	2.13963	.48881	2.01577	.51063
4	.44662	2.23902	.46772	2.13801	.48917	2.04126	.51099
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209
8	.44802	2.23204	.46911	2.13154	.49062	2.03825	.51246
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283
10	.44872	2.22857	.46985	2.12832	.49131	2.03526	.51319
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430
14	.45012	2.22161	.47128	2.12190	.49278	2.02929	.51467
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540
17	.45117	2.21647	.47231	2.11711	.49387	2.02483	.51577
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614
19	.45187	2.21301	.47305	2.11392	.49459	2.02187	.51651
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872
26	.45432	2.20108	.47555	2.10283	.49713	2.01155	.51909
27	.45467	2.19933	.47590	2.10126	.49749	2.01008	.51946
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020
30	.45573	2.19430	.47693	2.09654	.49858	2.00569	.52057
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52091
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52163
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242
36	.45781	2.18419	.47912	2.08716	.50076	1.99695	.52279
37	.45814	2.18251	.47948	2.08560	.50113	1.99550	.52316
38	.45849	2.18084	.47984	2.08405	.50149	1.99406	.52353
39	.45883	2.17916	.48019	2.08250	.50185	1.99261	.52390
40	.45921	2.17749	.48055	2.08094	.50222	1.99116	.52427
41	.45956	2.17582	.48091	2.07939	.50258	1.98972	.52461
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52539
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096
59	.46595	2.14611	.48737	2.05182	.50916	1.96402	.53134
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171
M. Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
85°		64°		63°		62°	

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 233

M.	12°		13°		14°		15°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72333	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88535	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	77°		76°		75°		74°		

238 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

M	32°		33°		34°		35°		M
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.60030	.64982	1.53888	.67493	1.48163	.70061	1.42725	59
2	.62563	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42109	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42021	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40023	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
57°		56°		55°		54°			

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 235

M.	20°		21°		22°		23°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40501	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	69°		68°		67°		66°		

240 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

40°		41°		42°		43°	
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.83910	1.19175	.86929	1.15037	.90010	1.11061	.93252
1	.83960	1.19105	.86980	1.14969	.90063	1.10996	.93306
2	.84009	1.19035	.87031	1.14902	.90116	1.10931	.93360
3	.84059	1.18964	.87082	1.14834	.90169	1.10867	.93415
4	.84109	1.18894	.87133	1.14767	.90221	1.10802	.93469
5	.84158	1.18824	.87184	1.14699	.90274	1.10737	.93524
6	.84208	1.18754	.87236	1.14632	.90327	1.10672	.93578
7	.84258	1.18684	.87287	1.14565	.90380	1.10607	.93633
8	.84307	1.18614	.87338	1.14498	.90433	1.10543	.93688
9	.84357	1.18544	.87389	1.14430	.90486	1.10478	.93742
10	.84407	1.18474	.87441	1.14363	.90539	1.10414	.93797
11	.84457	1.18404	.87492	1.14296	.90592	1.10349	.93852
12	.84507	1.18334	.87543	1.14229	.90645	1.10285	.93906
13	.84556	1.18264	.87595	1.14162	.90697	1.10220	.93961
14	.84606	1.18194	.87646	1.14095	.90751	1.10156	.94016
15	.84656	1.18124	.87698	1.14028	.90804	1.10091	.94071
16	.84706	1.18055	.87749	1.13961	.90857	1.10027	.94125
17	.84756	1.17986	.87801	1.13894	.90910	1.09963	.94180
18	.84806	1.17916	.87852	1.13828	.90963	1.09899	.94235
19	.84856	1.17846	.87904	1.13761	.91016	1.09834	.94290
20	.84906	1.17777	.87955	1.13694	.91069	1.09770	.94345
21	.84956	1.17707	.88007	1.13627	.91123	1.09706	.94400
22	.85006	1.17638	.88059	1.13561	.91176	1.09642	.94455
23	.85057	1.17568	.88110	1.13494	.91229	1.09578	.94510
24	.85107	1.17500	.88162	1.13428	.91283	1.09514	.94565
25	.85157	1.17430	.88214	1.13361	.91336	1.09450	.94620
26	.85207	1.17361	.88265	1.13295	.91390	1.09386	.94676
27	.85257	1.17292	.88317	1.13229	.91443	1.09322	.94731
28	.85308	1.17223	.88369	1.13162	.91497	1.09258	.94786
29	.85358	1.17154	.88421	1.13096	.91550	1.09195	.94841
30	.85409	1.17085	.88473	1.13029	.91603	1.09131	.94896
31	.85459	1.17016	.88524	1.12963	.91657	1.09067	.94952
32	.85509	1.16947	.88576	1.12897	.91710	1.09003	.95007
33	.85559	1.16878	.88628	1.12831	.91764	1.08939	.95062
34	.85609	1.16809	.88680	1.12765	.91817	1.08876	.95118
35	.85660	1.16741	.88732	1.12699	.91871	1.08813	.95173
36	.85710	1.16672	.88784	1.12633	.91925	1.08749	.95229
37	.85761	1.16603	.88836	1.12567	.91979	1.08686	.95284
38	.85811	1.16535	.88888	1.12501	.92032	1.08622	.95340
39	.85862	1.16466	.88940	1.12435	.92086	1.08559	.95395
40	.85912	1.16398	.88992	1.12369	.92140	1.08496	.95451
41	.85963	1.16329	.89045	1.12303	.92194	1.08432	.95506
42	.86014	1.16261	.89097	1.12238	.92248	1.08369	.95562
43	.86065	1.16192	.89149	1.12172	.92301	1.08306	.95618
44	.86115	1.16124	.89201	1.12106	.92355	1.08243	.95673
45	.86166	1.16056	.89253	1.12041	.92409	1.08179	.95729
46	.86216	1.15987	.89306	1.11975	.92463	1.08116	.95785
47	.86267	1.15919	.89358	1.11909	.92517	1.08053	.95841
48	.86318	1.15851	.89410	1.11844	.92571	1.07990	.95897
49	.86369	1.15783	.89463	1.11778	.92625	1.07927	.95952
50	.86420	1.15715	.89515	1.11713	.92679	1.07864	.96008
51	.86471	1.15647	.89567	1.11648	.92733	1.07801	.96064
52	.86522	1.15579	.89620	1.11582	.92787	1.07738	.96120
53	.86572	1.15511	.89672	1.11517	.92842	1.07676	.96176
54	.86623	1.15443	.89725	1.11452	.92896	1.07613	.96232
55	.86674	1.15375	.89777	1.11387	.92950	1.07550	.96288
56	.86725	1.15308	.89830	1.11321	.93004	1.07487	.96344
57	.86776	1.15240	.89883	1.11256	.93058	1.07425	.96400
58	.86827	1.15172	.89935	1.11191	.93113	1.07362	.96457
59	.86878	1.15104	.89988	1.11126	.93167	1.07299	.96513
60	.86929	1.15037	.90040	1.11061	.93222	1.07237	.96569
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
40°		41°		42°		43°	

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 237

M	28°		29°		30°		31°		M
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56421	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57461	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73439	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	61°		60°		59°		58°		

TABLE XVI.

RISE PER MILE OF VARIOUS GRADES.

Grade per Station.	Rise per Mile.	Grade per Station.	Rise per Mile.	Grade per Station.	Rise per Mile.	Grade per Station.	Rise per Mile.
.01	.528	.41	21.618	.81	42.768	1.21	63.888
.02	1.056	.42	22.176	.82	43.296	1.22	64.416
.03	1.584	.43	22.704	.83	43.824	1.23	64.944
.04	2.112	.44	23.232	.84	44.352	1.24	65.472
.05	2.640	.45	23.760	.85	44.880	1.25	66.000
.06	3.168	.46	24.288	.86	45.408	1.26	66.528
.07	3.696	.47	24.816	.87	45.936	1.27	67.056
.08	4.224	.48	25.344	.88	46.464	1.28	67.584
.09	4.752	.49	25.872	.89	46.992	1.29	68.112
.10	5.280	.50	26.400	.90	47.520	1.30	68.640
.11	5.808	.51	26.928	.91	48.048	1.31	69.168
.12	6.336	.52	27.456	.92	48.576	1.32	69.696
.13	6.864	.53	27.984	.93	49.104	1.33	70.224
.14	7.392	.54	28.512	.94	49.632	1.34	70.752
.15	7.920	.55	29.040	.95	50.160	1.35	71.280
.16	8.448	.56	29.568	.96	50.688	1.36	71.808
.17	8.976	.57	30.096	.97	51.216	1.37	72.336
.18	9.504	.58	30.624	.98	51.744	1.38	72.864
.19	10.032	.59	31.152	.99	52.272	1.39	73.392
.20	10.560	.60	31.680	1.00	52.800	1.40	73.920
.21	11.088	.61	32.208	1.01	53.328	1.41	74.448
.22	11.616	.62	32.736	1.02	53.856	1.42	74.976
.23	12.144	.63	33.264	1.03	54.384	1.43	75.504
.24	12.672	.64	33.792	1.04	54.912	1.44	76.032
.25	13.200	.65	34.320	1.05	55.440	1.45	76.560
.26	13.728	.66	34.848	1.06	55.968	1.46	77.088
.27	14.256	.67	35.376	1.07	56.496	1.47	77.616
.28	14.784	.68	35.904	1.08	57.024	1.48	78.144
.29	15.312	.69	36.432	1.09	57.552	1.49	78.672
.30	15.840	.70	36.960	1.10	58.080	1.50	79.200
.31	16.368	.71	37.488	1.11	58.608	1.51	79.728
.32	16.896	.72	38.016	1.12	59.136	1.52	80.256
.33	17.424	.73	38.544	1.13	59.664	1.53	80.784
.34	17.952	.74	39.072	1.14	60.192	1.54	81.312
.35	18.480	.75	39.600	1.15	60.720	1.55	81.840
.36	19.008	.76	40.128	1.16	61.248	1.56	82.368
.37	19.536	.77	40.656	1.17	61.776	1.57	82.896
.38	20.064	.78	41.184	1.18	62.304	1.58	83.424
.39	20.592	.79	41.712	1.19	62.832	1.59	83.952
.40	21.120	.80	42.240	1.20	63.360	1.60	84.480

TABLE XVI. RISE PER MILE OF VARIOUS GRADES. 243

Grade per Station.	Rise per Mile.	Grade per Station.	Rise per Mile.	Grade per Station.	Rise per Mile.	Grade per Station.	Rise per Mile.
1.61	85.003	1.81	95.563	2.10	110.880	4.10	216.480
1.62	85.536	1.82	96.096	2.20	116.160	4.20	221.760
1.63	86.064	1.83	96.624	2.30	121.440	4.30	227.040
1.64	86.592	1.84	97.152	2.40	126.720	4.40	232.320
1.65	87.120	1.85	97.680	2.50	132.000	4.50	237.600
1.66	87.648	1.86	98.208	2.60	137.280	4.60	242.880
1.67	88.176	1.87	98.736	2.70	142.560	4.70	248.160
1.68	88.704	1.88	99.264	2.80	147.840	4.80	253.440
1.69	89.232	1.89	99.792	2.90	153.120	4.90	258.720
1.70	89.760	1.90	100.320	3.00	158.400	5.00	264.000
1.71	90.288	1.91	100.848	3.10	163.680	5.10	269.280
1.72	90.816	1.92	101.376	3.20	168.960	5.20	274.560
1.73	91.344	1.93	101.904	3.30	174.240	5.30	279.840
1.74	91.872	1.94	102.432	3.40	179.520	5.40	285.120
1.75	92.400	1.95	102.960	3.50	184.800	5.50	290.400
1.76	92.928	1.96	103.488	3.60	190.080	5.60	295.680
1.77	93.456	1.97	104.016	3.70	195.360	5.70	300.960
1.78	93.984	1.98	104.544	3.80	200.640	5.80	306.240
1.79	94.512	1.99	105.072	3.90	205.920	5.90	311.520
1.80	95.040	2.00	105.600	4.00	211.200	6.00	316.800

THE END

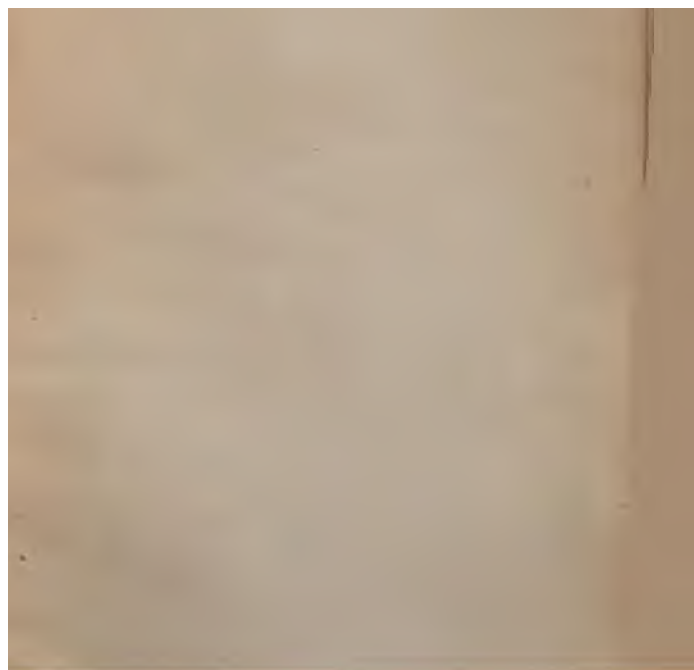


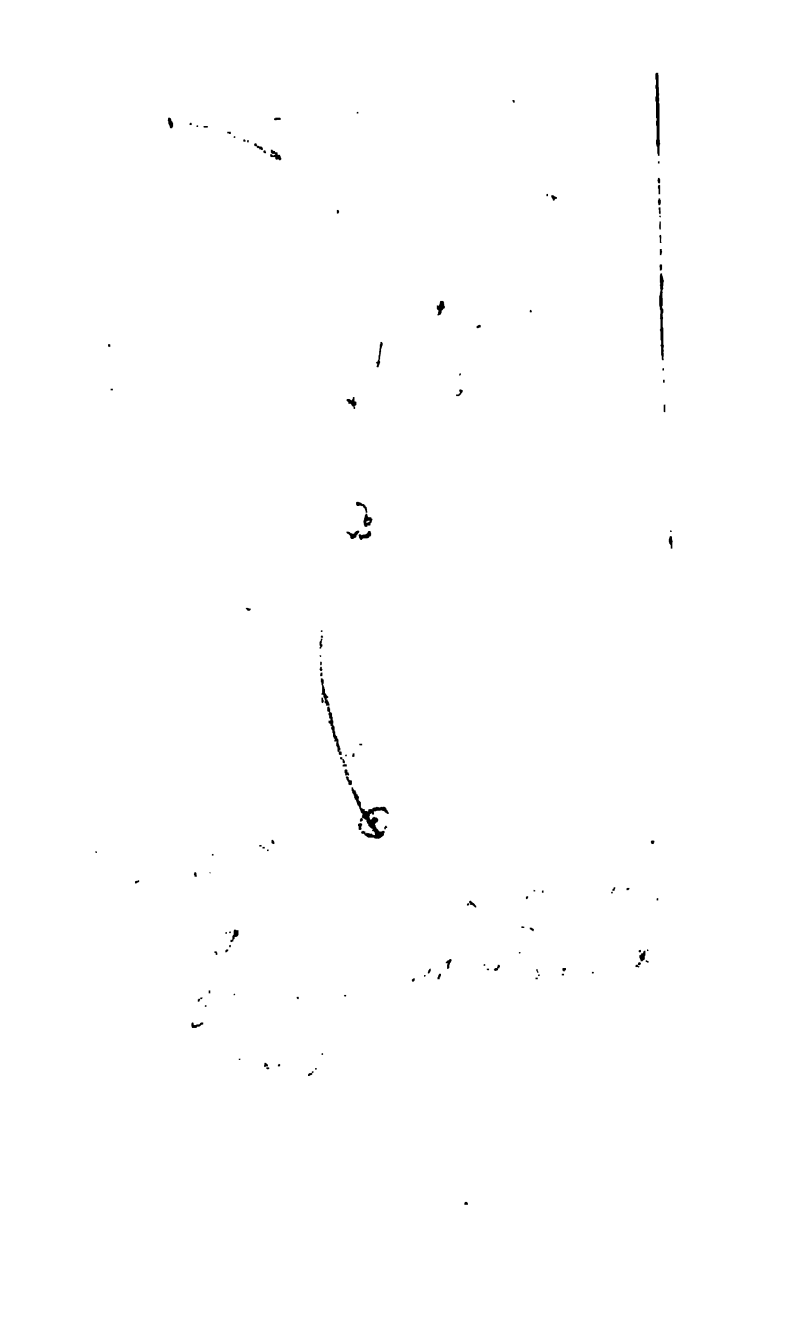
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